MATH299G: Geometry in Physics

Course Description

This course will introduce geometric methods in physics, by studying classical physics on curved spaces. We will start with differential forms on $\mathbb{R}^n$, studied through the lens of electrodynamics and culminating in Maxwell's equations: $d{\star}dA=0$. Then, we will develop symplectic geometry to formalize classical mechanics on manifolds. Finally, we will discuss symmetry through group actions on manifolds, and prove Noether's theorem. This course will emphasize intuition, and the abstraction of that intuition into mathematical formalism.

Course Details

Credits: 1

Seats: 20

Lecture Time: Monday 2:00 - 2:50

Location: MTH 0106

Semester: Fall 2021

Recommended Textbooks (optional):

- John Baez: *Gauge fields, knots, and gravity*
- Vladimir Arnold: *Mathematical methods of Classical Mechanics*, Chapters 7 and 8

Course Facilitator: Elliot Kienzle

Faculty Advisor: Tamás Darvas

Email: Elliot.Kienzle@gmail.com (Please include 'MATH299G' in the subject)

Prerequisites: 1 course with a minimum grade of C- from (MATH241, MATH340); and 1 course with a minimum grade of C- from (MATH240, MATH341, MATH461). Or PHYS373

Recommended: MATH405, MATH410, PHYS410, or PHYS411

This course only requires knowledge of multivariable calculus and linear algebra, but the topic is somewhat advanced. It will be helpful to have some experience with higher level math. Likewise, while upper level physics classes are not expected, they would provide useful context.

Topics

Unit 1: Electrodynamics & differential forms
The level of exposition and topics covered will be similar to *Gauge fields, knots, and gravity*, Section I. See also *Mathematical methods of Classical Mechanics*, chapter 7.

- Differential forms and vectors
  - Wedge product
  - Integration
- Exterior derivative
  - Stokes theorem
  - Vector calculus
  - Hodge star operator
- Maxwell's equations
  - Field strength 2-form
  - Vector potential
  - $d \star dA = 0$

Unit 2: Classical mechanics & symplectic geometry
See *Mathematical methods of Classical Mechanics*, chapter 8.

- Manifolds
  - (co)tangent bundle
- Mechanics on flat space
- Mechanics on cotangent bundles
  - Canonical 1-form
- Symplectic manifolds
  - Symplectic forms
  - Darboux's theorem
- Other structures
  - Hamiltonian vector fields
  - Poisson brackets
  - Liouville theorem
  - Geodesic motion
- Symmetry
  - Lie groups and algebras
  - Lie group actions on manifolds
    - Momentum maps
  - Noether's theorem

Assignments

Homework

A short homework (~ 5 minutes) will be assigned each week, which mostly serves to help you make sure you're following the broad strokes of the lectures. Homeworks will often come with one or more extra credit problems, which may ask you to fill in some gaps of the lecture before, or walk you through an interesting extension of the course contents.

Each homework and associated extra credit is due before the start of the next lecture.

Final Project

The contents of this course unlock a plethora of interesting things to learn about, both in pure math and its application to physics. For the final project, you will try to read a paper on one such thing. You will
You will try to learn about one such thing for your final project. This project consists of a short report on a topic of your choosing, summarizing the main ideas and results of that little area. You are encouraged to "jump in the deep end" so to speak, and take on something out of your comfort zone. As such, you do not need to fully understand your topic to get an A on the project.

Some possible topics include:

- Frobenius theorem, integrable hamiltonian systems, and action angle coordinates
- Geodesics on lie groups, Euler-arnold equations, and the Euler equation in fluid mechanics.
- Shrodinger's equation via symplectic mechanics
- Hamiltonian chaos
- And many more...

Weekly Schedule

Week of:	Topic	Assignment
8/30	Overview of classElectromagnetism, Differential forms as "fluxes"	
9/6	Vectors and differential forms, generalizing the cross product with the wedge product	
9/13	Integrating differential forms, exterior derivative, stokes theorem	
9/20	Application of differential forms to vector calculus, maxwell's equations in standard form	

9/27	Electromagnetic 4-potential, field strength two form, maxwell's equations w/ differential forms: $d\star dA=0$	
10/4	Manifolds, tangent and cotangent bundles.	
10/11	Mechanics on flat space, Hamilton's equations with differential forms.	
10/18	Liouville form & canonical symplectic form, mechanics on cotangent bundle	
10/25	Symplectic forms, Darboux theorem	
11/1	Hamiltonian vector fields, vector field flows, Lie derivatives	
11/8	Poisson bracket, integrals of motion, Liouville's theorem	Final Project topic short list
11/15	Metrics, kinetic-type Hamiltonians, geodesic motion	
11/22	Continuous symmetries, lie groups	Final project topic finalized
11/29	Lie algebras, Lie group actions	
12/6	Momentum map, Noether's theorem	Final project Due

Grading

Grades will be maintained on ELMS. You will be responsible for all material discussed in lecture as well as other standard means of communication (Piazza, email announcements, etc.), including but not limited to deadlines, policies, assignment changes, etc.

Any request for reconsideration of any grading on coursework must be submitted within one week of when it is returned. No requests will be considered afterwards.

Your final course grade will be determined according to the following percentages.

- Weekly assignments (50%)

- Final project (50%)
- Extra credit opportunities (30%)

Grades are assigned according to the following cutoffs:

$$\cdots < 80 \leq \text{B}- < 83 \leq \text{B} < 87 \leq \text{B}+ < 90 \leq \text{A}- < 93 \leq \text{A} \leq 100 \leq \text{A}+$$

Communicating with course staff

Interaction beyond the classroom is encouraged, but should be limited to important or more urgent issues. Topics that need not be addressed immediately can wait till class time. Unless otherwise necessary, contact the facilitator before the instructor.

Facilitator Name and Email:
- Elliot Kienzle: elliot.kienzle@gmail.com

Instructor Name and Email:
- Tamás Darvas: tdarvas@umd.edu

Excused Absence and Academic Accommodations

See the section titled "Attendance, Absences, or Missed Assignments" available at Course Related Policies.

Disability Support Accommodations

See the section titled "Accessibility" available at Course Related Policies.

Academic Integrity

Note that academic dishonesty includes not only cheating, fabrication, and plagiarism, but also includes helping other students commit acts of academic dishonesty by allowing them to obtain copies of your work. In short, all submitted work must be your own. Cases of academic dishonesty will be pursued to the fullest extent possible as stipulated by the Office of Student Conduct. It is very important for you to be aware of the consequences of cheating, fabrication, facilitation, and plagiarism. For more information on the Code of Academic Integrity or the Student Honor Council, please visit http://www.shc.umd.edu.

Course Evaluations

If you have a suggestion for improving this class, don't hesitate to tell the instructor or TAs during the semester. At the end of the semester, please don't forget to provide your feedback using the campus-wide CourseEvalUM system. Your comments will help make this class better.