$\int x \sec^2 x \, dx$

$u = x \qquad dv = \sec^2 x \, dx$

$du = \underline{1} \, dx \qquad v = \tan x$

$\int u \, dv = uv - \int v \, du$

$\int x \sec^2 x = x \tan x - \int \tan x \, dx$

$\int \tan = \int \frac{\sin x}{\cos x} dx \qquad u = \cos x$

$du = -\underline{\sin x} \, dx$

$= \int \frac{-du}{u} = -\ln u + C = \ln|\cos x| + C$

$\int x \sec^2 x = x \tan x - (-\ln|\cos x| + C)$

$= x \tan x + \ln|\cos x| + C$:)

$\lim_{x \to 0} x^2 \ln|x| = f(x)$ $\quad 0^2 \ln|0|$

$f(-x)$
$= (-x)^2 \ln|-x|$
$= x^2 \ln|x| = f(x)$

$\lim_{x\to 0} \cancel{\dfrac{\ln|x|}{1/x^2}}$ $\quad \lim_{x \to 0} \dfrac{x^2}{1/\ln|x|}$

L'hopital's rule: if $f, g \to 0$ as $x \to a$

the $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \lim_{x \to a} \dfrac{f'(x)}{g'(x)}$

$\lim_{x \to 0} \dfrac{d/dx\ x^2}{\frac{d}{dx}\ 1/\ln|x|} = \dfrac{2x}{\frac{1}{x} \cdot \frac{-1}{\ln|x|^2}} = -2\ x^2 \ln|x|^2$

Chain rule: $\frac{d}{dx} f(g(x)) = g'(x) \cdot f'(g(x))$

$f = 1/x \quad g = \ln x$

$f' = \dfrac{-1}{x^2} \quad g' = 1/x$

$\frac{d}{dx} \dfrac{1}{\ln x} = \dfrac{1}{x} \cdot \dfrac{-1}{\ln|x|^2}$

Take 2 !!

$\lim_{x \to 0} \dfrac{\ln|x|}{1/x^2}$ $\quad d/dx \ln x = 1/x$

$d/dx\ 1/x^2 = -2/x^3$

$\frac{d}{dx} x^n = n \cdot x^{n-1}$

$\int x^n = \dfrac{x^{n+1}}{n+1}$

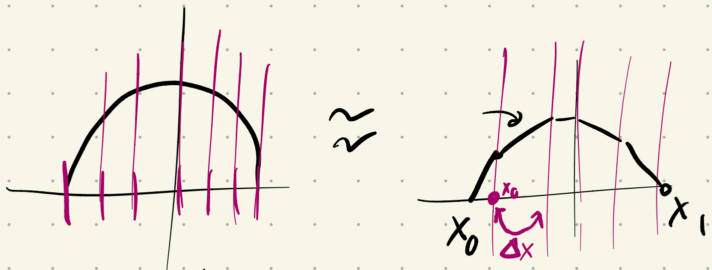
$\lim_{x \to 0} \dfrac{1/x}{-2/x^3} = \lim_{x \to 0} \dfrac{-1}{2} \dfrac{x^{\cancel{3}\,2}}{\cancel{x}} = \lim_{x \to 0} \dfrac{-1}{2} x^2$

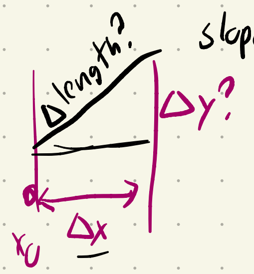
$= 0$:)

Arc length:

Q: how "long" is the graph of a function??

$f(x) = 1 - x^2$

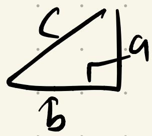




slope $= \frac{dy}{dx} = f'$

$$\frac{\Delta y}{\Delta x} = f' \Rightarrow \Delta y = \underline{\Delta x \cdot f'}$$

Pythagorean's thm



$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$

$$\Delta \text{length} = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{\underline{\Delta x^2} + \underline{\Delta x^2} f'^2}$$
$$= \sqrt{\Delta x^2 (1 + f'^2)} = \Delta x \sqrt{1 + f'^2}$$

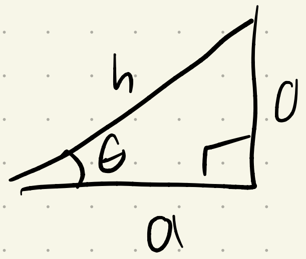
$$\underline{\text{total}} \text{ length} = \sum \Delta \text{lengths} = \sum \underbrace{\Delta x} \underbrace{\sqrt{1 + f'^2}}$$

in the limit ... length $= \int dx \sqrt{1 + f'^2}$

all in all...

arclength from x_0 to x_1 is $\boxed{\int_{x_0}^{x_1} \sqrt{1 + f'(x)^2}\, dx}$

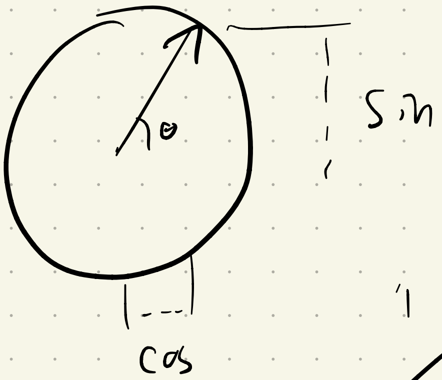
imagine this?!!



$$\tan\theta = \frac{o}{a} = \frac{\sin\theta}{\cos\theta} = \frac{o/h}{a/h}$$

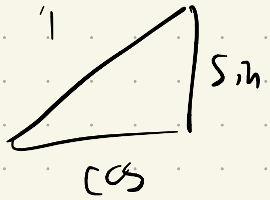
$\sin\theta$ is the height of a vector of length 1 & angle θ

Unit circle



$$\boxed{\sin^2 + \cos^2 = 1^2 = 1}$$

$$\frac{\sin^2}{\cos^2} + 1 = \frac{1}{\cos^2}$$



$$\boxed{\tan^2 + 1 = \sec^2}$$

$\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$$1 + \frac{\cos^2}{\sin^2} = \frac{1}{\sin^2}$$

$$\boxed{1 + \cot^2 = \csc^2}$$

Double angle formula

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\sin(2x) = \sin(x+x) = 2\sin x \cos x$

$\cos(2x) = \cos(x+x) = \cos^2 x - \sin^2 x$

Combine w/ $\cos^2 + \sin^2 = 1$ $\quad \cos^2 = 1 - \sin^2 \quad \sin^2 = 1 - \cos^2$

$$\cos(2x) = \cos^2 - \sin^2$$
$$= (1-\sin^2) - \sin^2 \qquad = \cos^2 - (1-\cos^2)$$
$$= 1 - 2\sin^2 \qquad = 2\cos^2 - 1$$

$$\boxed{\sin(x/2) = \sqrt{\frac{1-\cos x}{2}}}$$

$$\cos(2(x/2)) = 1 - 2\sin^2(x/2)$$
$$\cos(x) = 1 - 2\sin^2(x/2)$$

$$2\sin^2(x/2) + \cos x = 1$$

$$2\sin^2(x/2) = 1 - \cos x$$

$$\sin^2(x/2) = \frac{1-\cos x}{2}$$

$$\sin(x/2) = \sqrt{\frac{1-\cos x}{2}}$$