

Quiz 3: Gradients, Tangent Planes, chain Rule

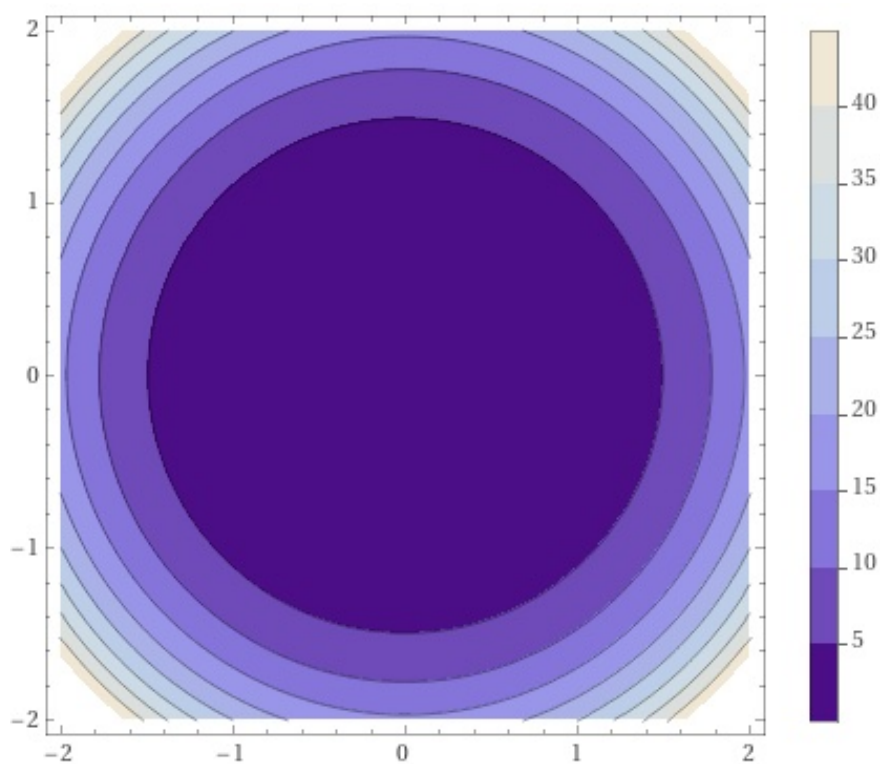
Name:

Section:

All questions on this quiz deals with the function

$$f(x, y) = (x^2 + y^2)^2$$

To save time, here is a contour plot of this function (thanks to wolfram alpha):



1. (5 points) Compute the gradient of f at the point $(1, 0)$. Draw the resulting vector on the contour plot, starting from $(1, 0)$

2. (5 points) Find the equation for the tangent plane to the graph $z = f(x, y)$ at the point $(1, 0, f(1, 0))$.

3. (5 points) Consider the path

$$\gamma(t) = (x(t), y(t)) = (\cos(t), \sin(t))$$

Define the function $g(t) = f \circ \gamma = f(x(t), y(t))$. Compute $\dot{g}(0)$, the first derivative of g with respect to t at $t = 0$. (Hint: Try to directly substitute γ into f)

4. (5 points) Now consider the path

$$\rho(t) = (x(t), y(t)) = (1 + \cos(t), -1 + \sin(t))$$

Define the function $h(t) = f \circ \rho = f(x(t), y(t))$. Compute $\dot{h}(\pi/2)$, the first derivative of h with respect to t at $t = \pi/2$. (Hint: Direct substitution will make you sad. Try and use the chain rule)