

Math 53: Worksheet 0

August 24

1. Define $f : [-2, 1] \rightarrow \mathbb{R}$ by $f(x) = \sqrt{x^2 + x + 1}$.

- (a) Find the equation of the line tangent to f at $(-1, 1)$.
- (b) Find the local and absolute extreme values of f .

2. Evaluate the following limits:

- (a) $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$.
- (b) $\lim_{x \rightarrow 0} x^2 \ln |x|$.
- (c) $\lim_{t \rightarrow 0} \frac{\sqrt{1+t} - 1}{t}$

3. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

- (a) Show that f is continuous at $x = 0$.
 - (b) Is f differentiable as well at $x = 0$?
4. A wire 10 inches long is to be cut into two pieces. A circle is to be formed from one piece and a square from the other. How should the wire be cut to **minimize** the total area enclosed by the two figures?
5. Use the appropriate integration techniques to evaluate the following:
- (a) $\int_9^{25} e^{\sqrt{t}} dt$.
 - (b) $\int x \sec^2(x) dx$.
 - (c) $\int \frac{\cos(\pi/x)}{x^2} dx$.