Discussion section Monday 11/7

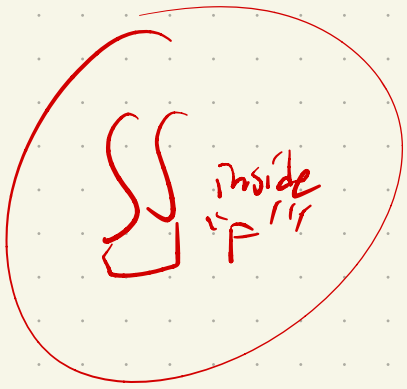
Curl & Divergence!! (Resources @ bottom)

Class so far:

first section: Derivatives of functions

second section: integrals

third section: Put it together



1D $\int_a^b f' = f(a) - f(a)$

line integrals $\int_a^b \nabla f \cdot d\vec{r} = f(a) - f(a)$

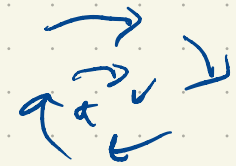
Greens $\iint_{Q_x - P_y} |\nabla \times F| = \oint_C \underline{F} \cdot d\vec{r}$

Derivatives of vector functions

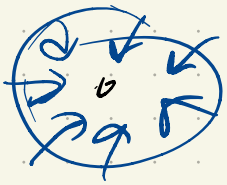
1. gradient — scalar f → vector ∇f
2. curl — vector $\underline{F}$ → vector $\nabla \times \underline{F}$ $[Q_x - P_y$
3. Divergence — vector $\underline{F} = (P, Q, R)$ → scalar $P_x + Q_y + R_z$

What Do they mean tho??

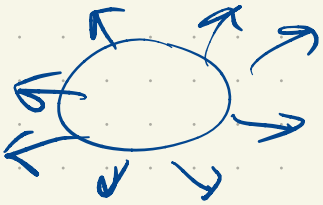
curl is rotation



Divergence is compression



negative Divergence



Positive Divergence?

$f(x,y)$ scalar function

∇f vector field

$(f_x, f_y, 0)$

$\nabla \times \nabla f = (0, 0, Q_x - P_y)$

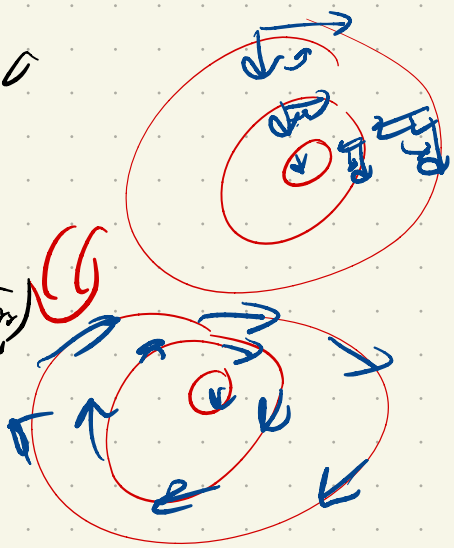
$f_{yx} - f_{xy} = 0$ Clairaut's thm

curl-free vector field!

Or... rotate ∇f by 90°

$F = (-f_y, f_x)$

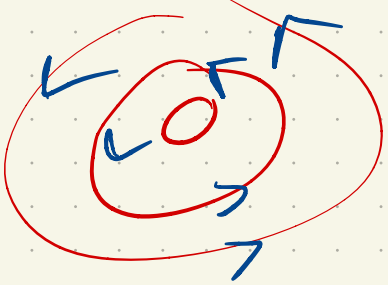
$\nabla \times F = (0, 0, \frac{\partial}{\partial x} f_x - \frac{\partial}{\partial x}(-f_y)) = (0, 0, f_{xx} + f_{yy})$



what is curl?

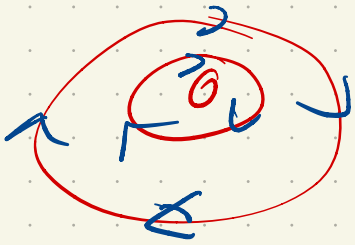
what is Divergence?

around max



positive curl

around min



negative curl

$$\nabla \cdot F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} \qquad F = (-f_y, f_x, 0) \quad P, Q, R$$

$$= -f_{yx} + f_{xy} + 0 = 0 \quad \text{by clairaut's}$$

Divergence-free field!!

resources:
3b1b video on curl snd divergence: https://www.youtube.com/watch?v=rB83DpBJQsE&ab_channel=3Blue1Brown

calc blue video on 2d curl and divergence:
https://www.youtube.com/watch?v=EFODp8HllZI&ab_channel=ProfGhristMath

calc blue video on 3d curl
https://www.youtube.com/watch?v=ntGWiFh0nOU&ab_channel=ProfGhristMath

my little applet on vector field divegence snd curl (will make rhis better)
https://openprocessing.org/sketch/1728195