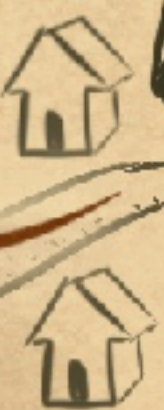


**Vistas of mathematical  
visualization**

**Elliot Kienzle**



*Lion*

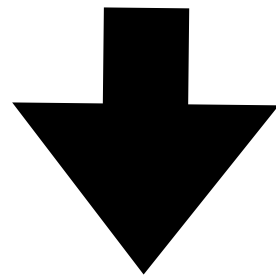


*Swamp*



I want to believe math

Seeing is believing



I want to see math

# **Episode 1:**

# **Sums of squares**

$$(n+1)^2 = n^2 + 2n + 1$$

$$1^2 = 1 = 1$$



$$2^2 = 4 = 1 + 3$$

$$(n+1)^2 - n^2 = 2n + 1$$

$$3^2 = 9 = 1 + 3 + 5$$

$$4^2 = 16 = 1 + 3 + 5 + 7$$

$$5^2 = 25 = 1 + 3 + 5 + 7 + 9$$

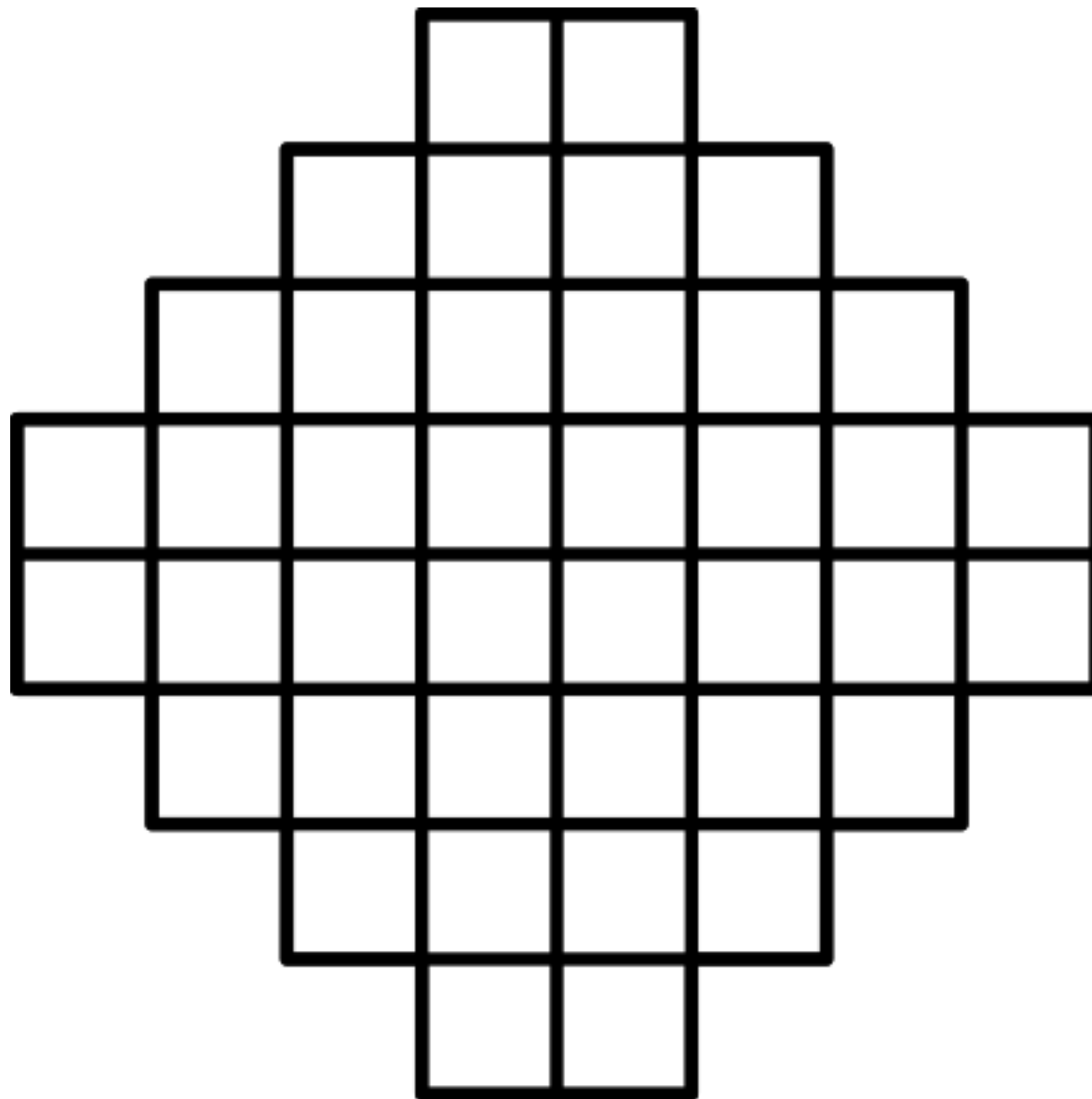
⋮



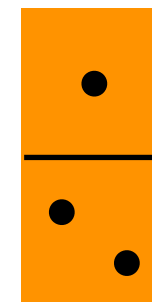
$1^2$   $2^2$   $3^2$   $4^2$   $5^2$



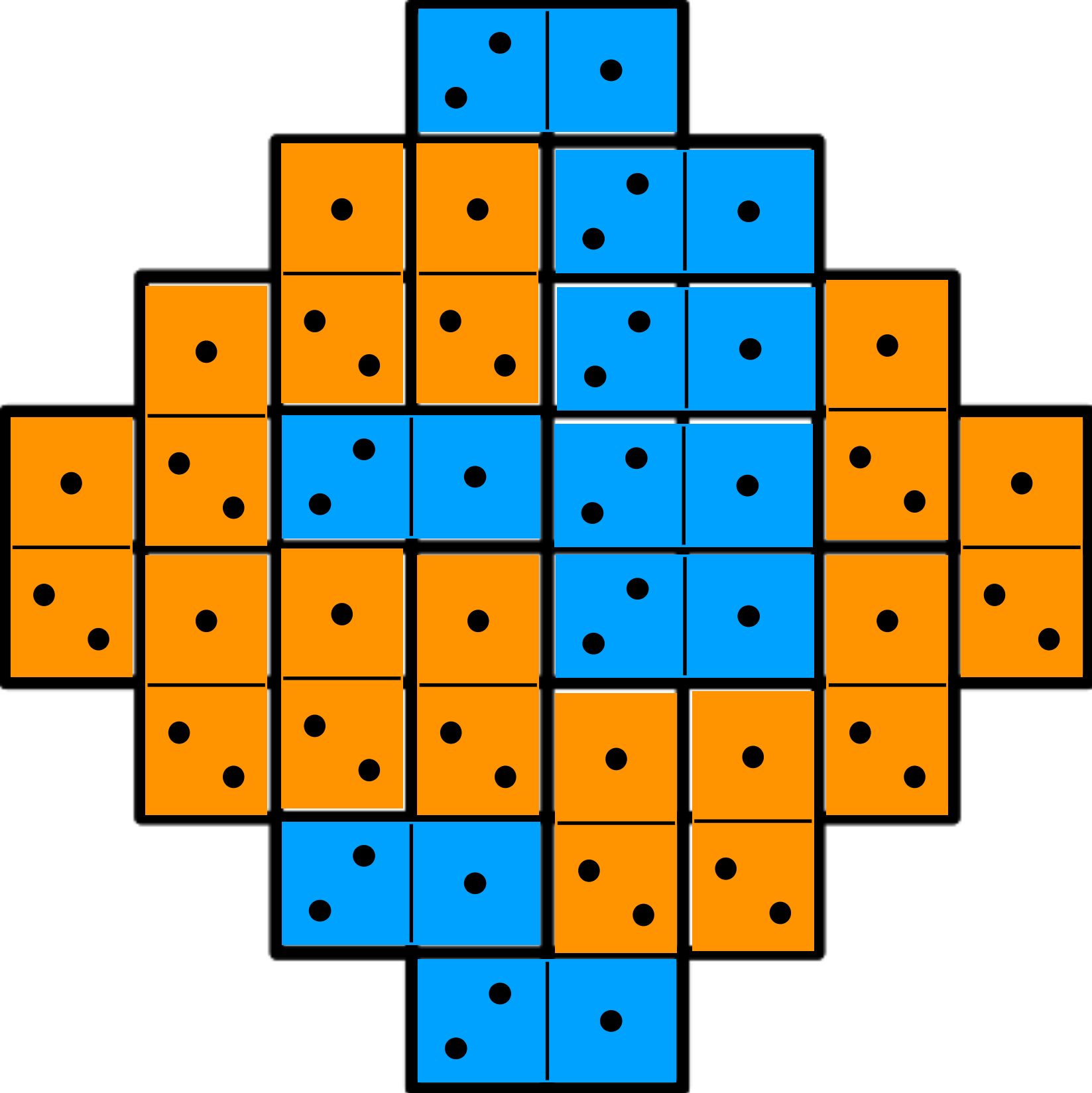
**Episode 2:**  
**Aztec diamond and arctic**  
**circle**



**Aztec diamond**

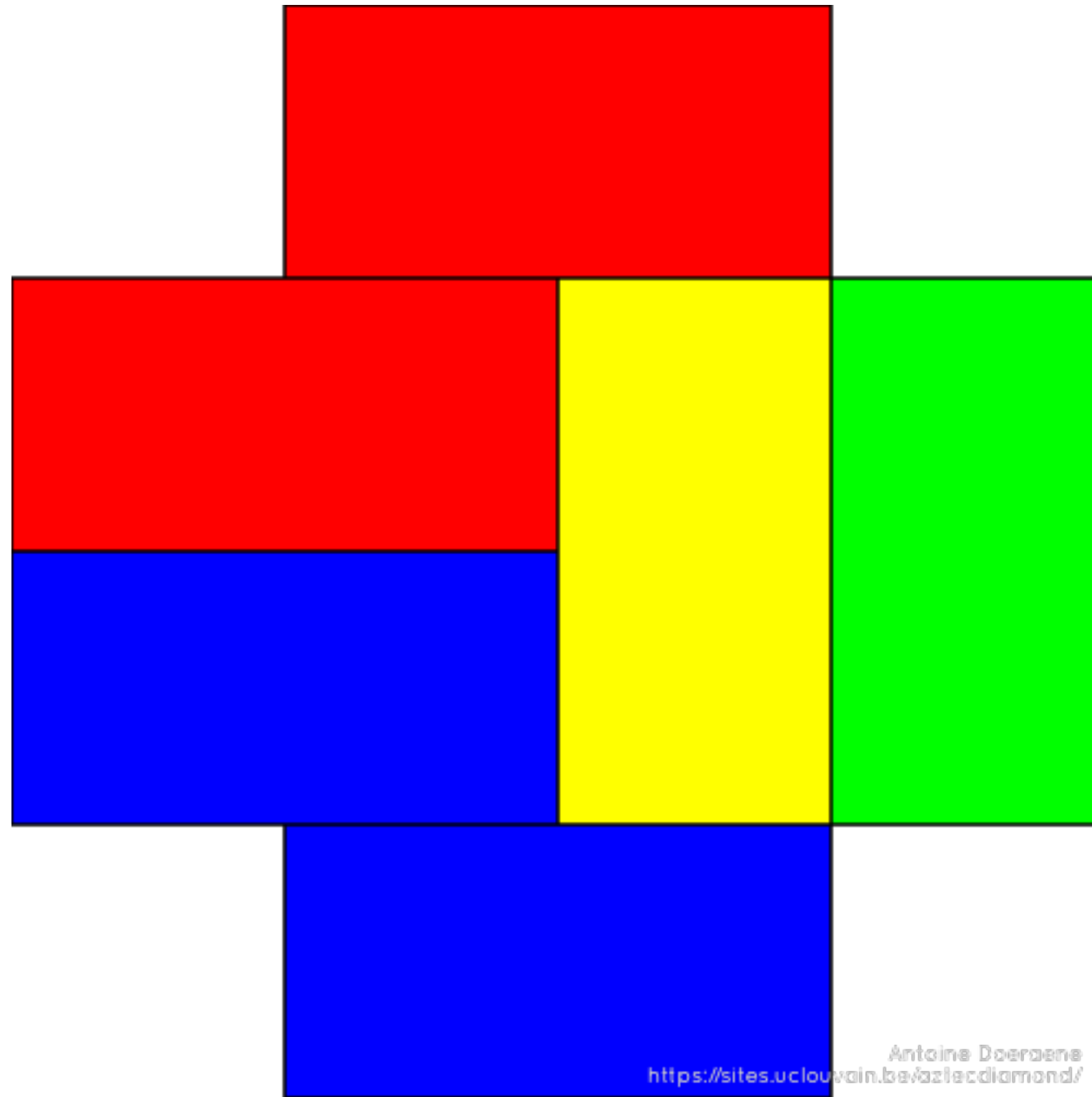


**Domino**

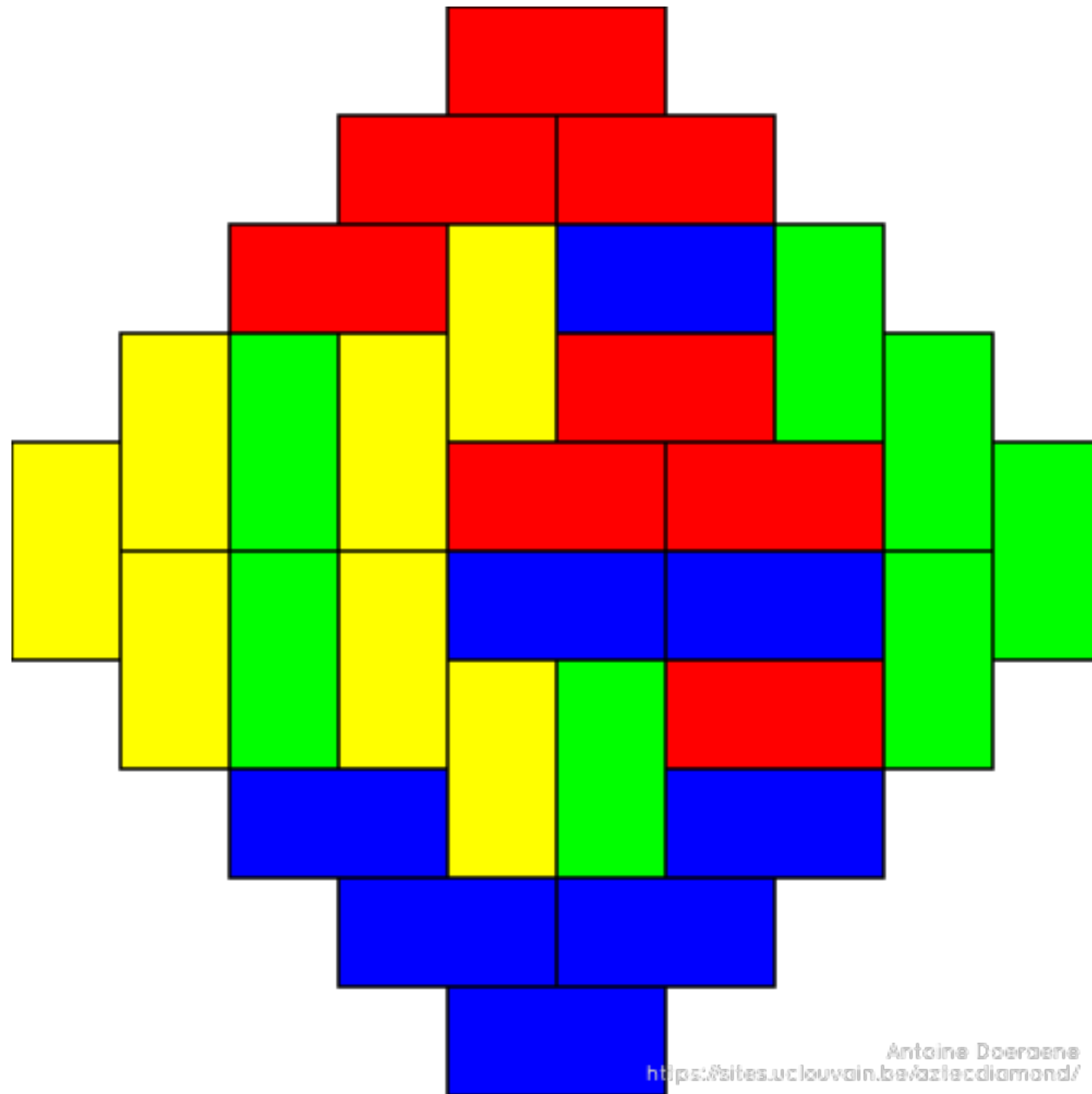




Generated using <https://sites.uclouvain.be/aztecdiamond/>

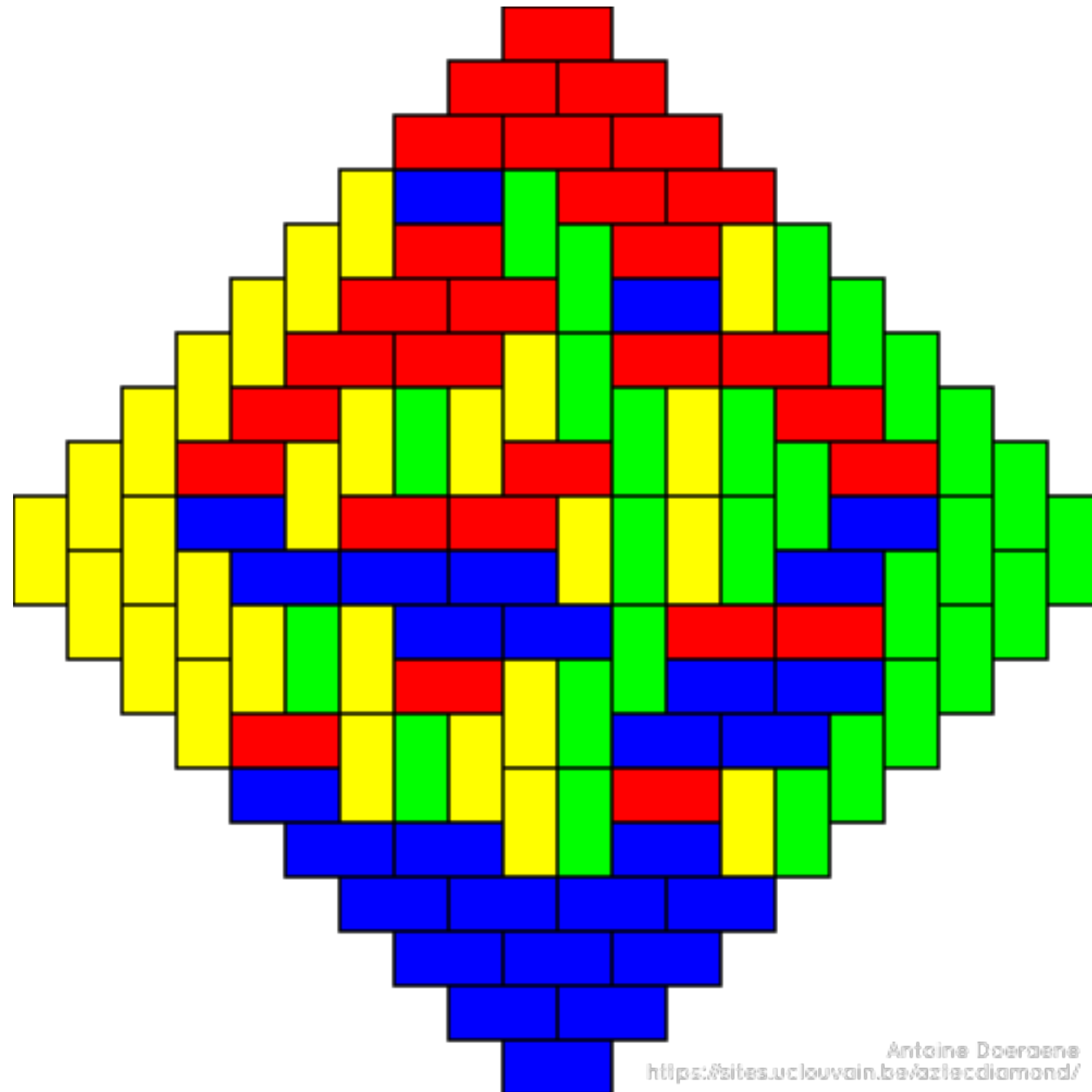


Random domino tiling  
(Size 2)



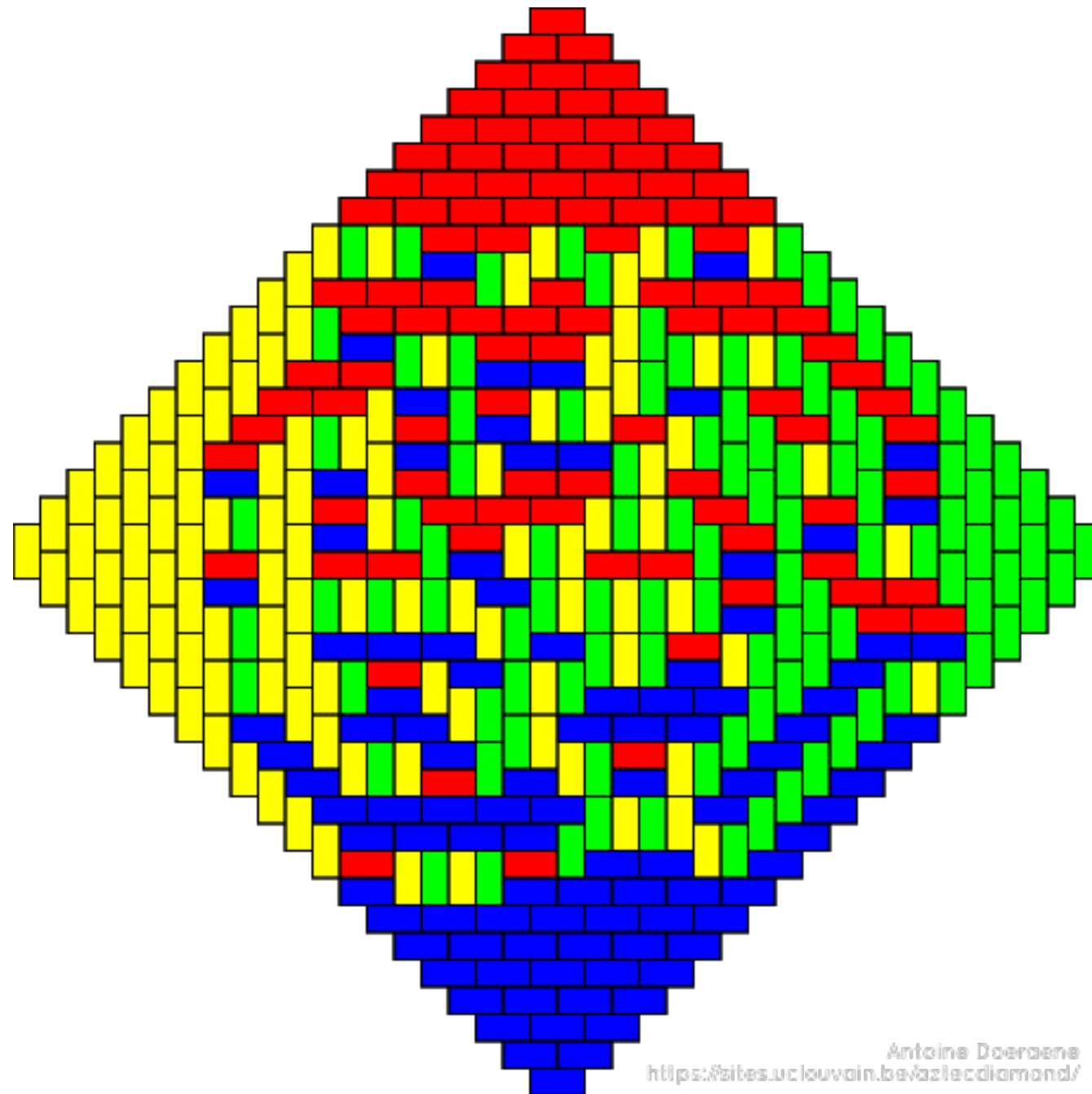
Antoine Doeraene  
<https://sites.uclouvain.be/azlecdiamond/>

Random domino tiling  
(Size 5)



Antoine Doeraene  
<https://sites.uclouvain.be/azlecdiamond/>

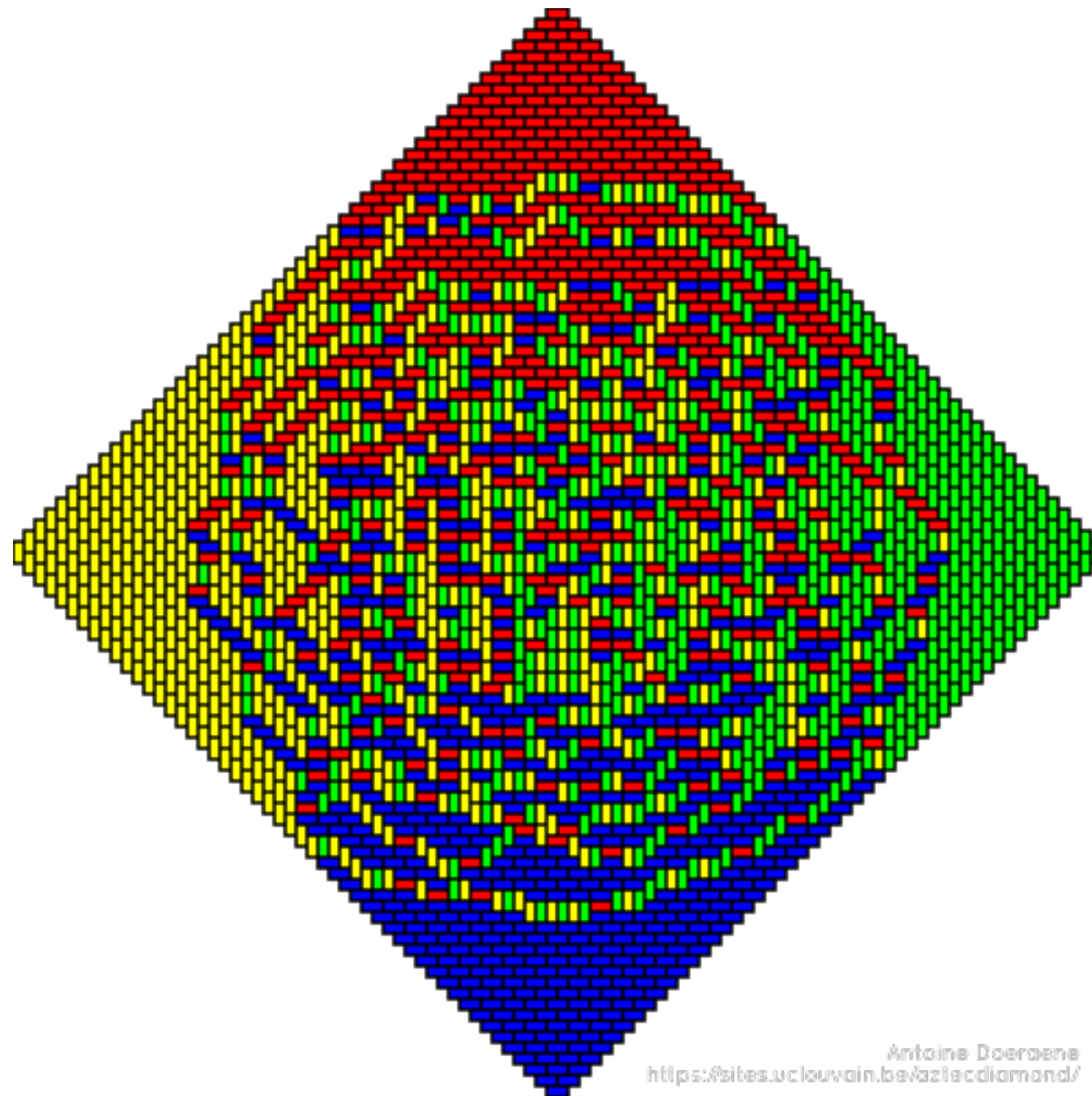
Random domino tiling  
(Size 10)



Antoine Doeraene  
<https://sites.uclouvain.be/aztec/diamond/>

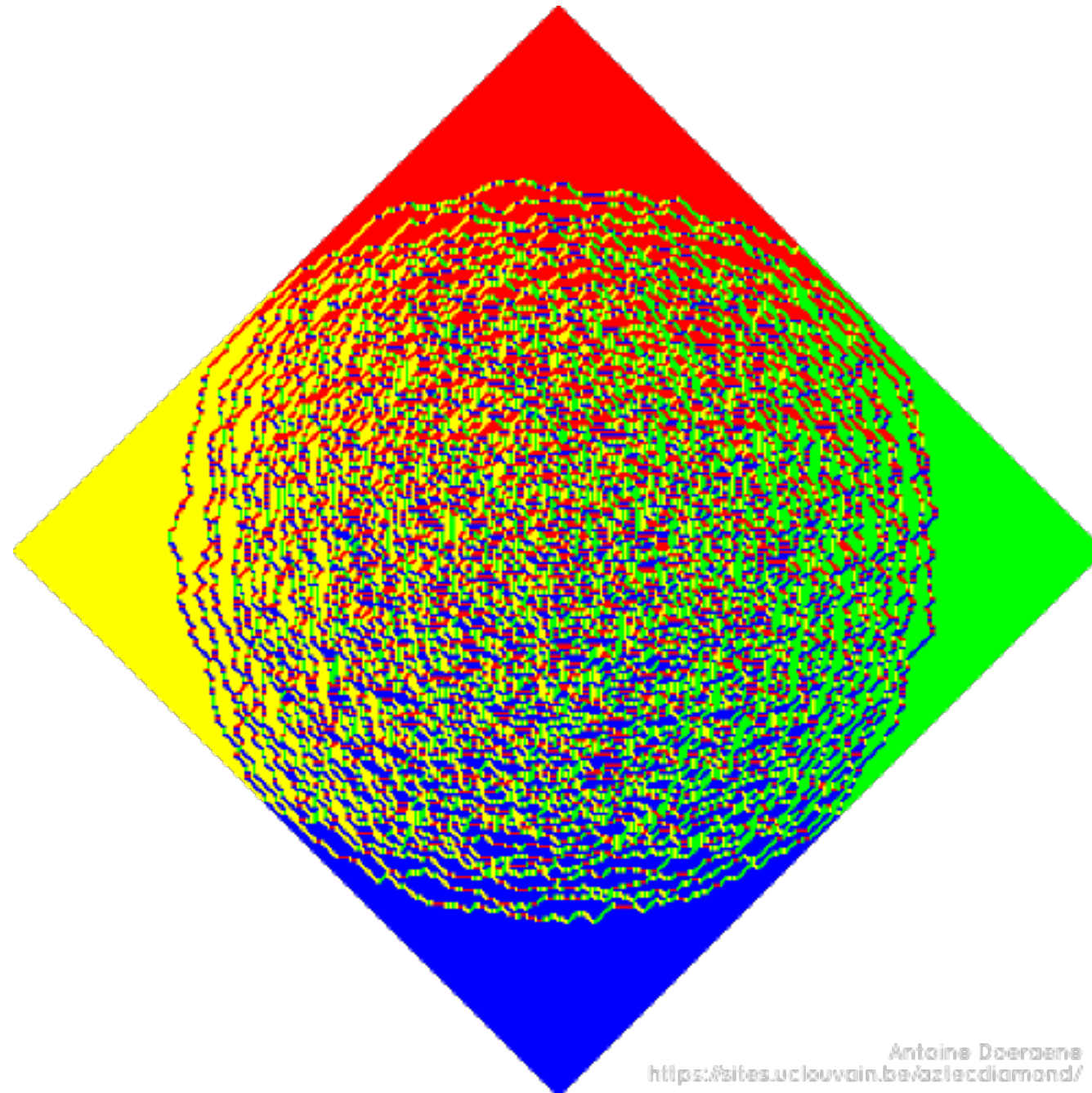
Random domino tiling  
(Size 20)



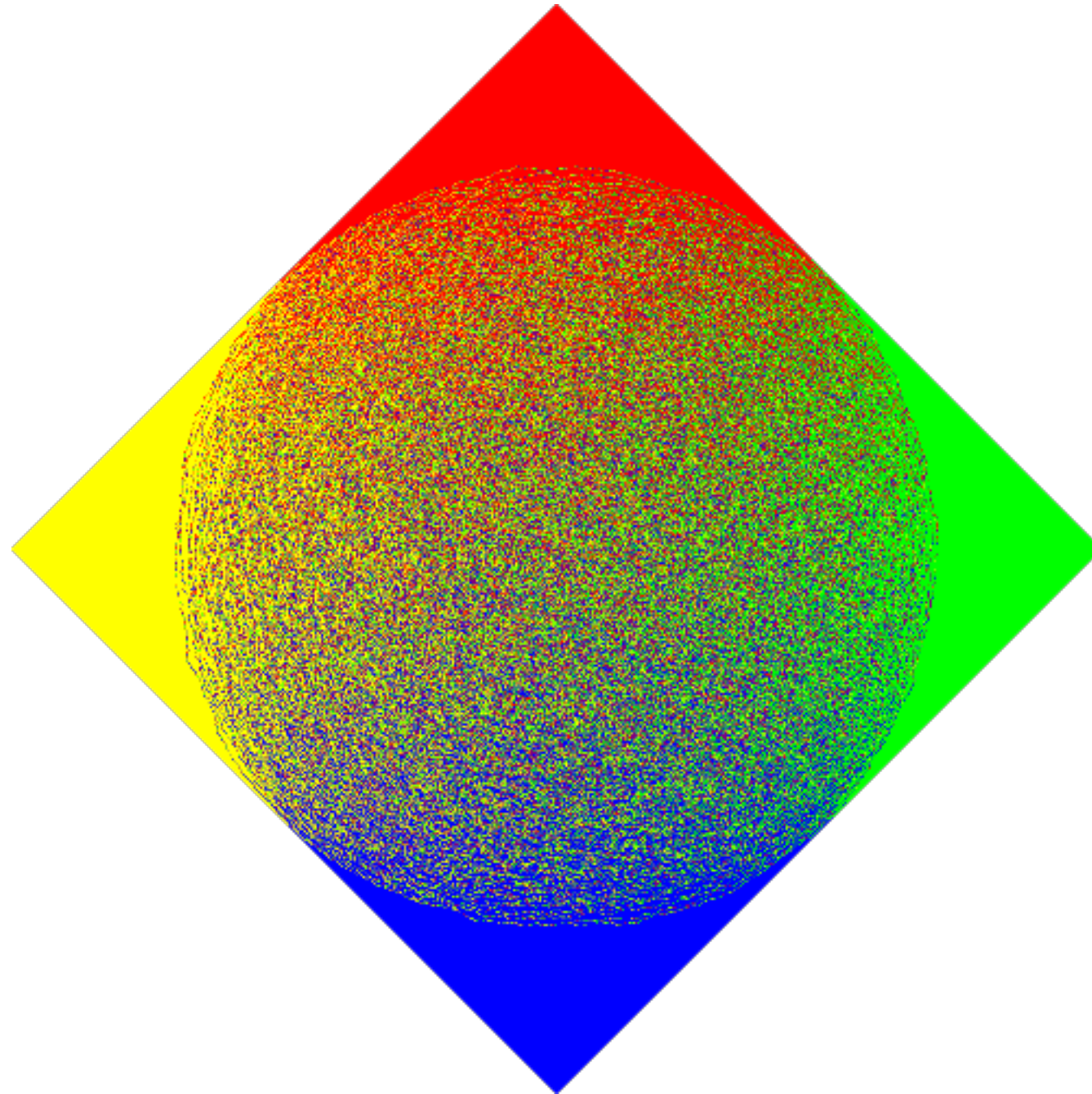


Antoine Doeraene  
<https://sites.uclouvain.be/azlecdiamond/>

Random domino tiling  
(Size 50)



Random domino tiling  
(Size 200)

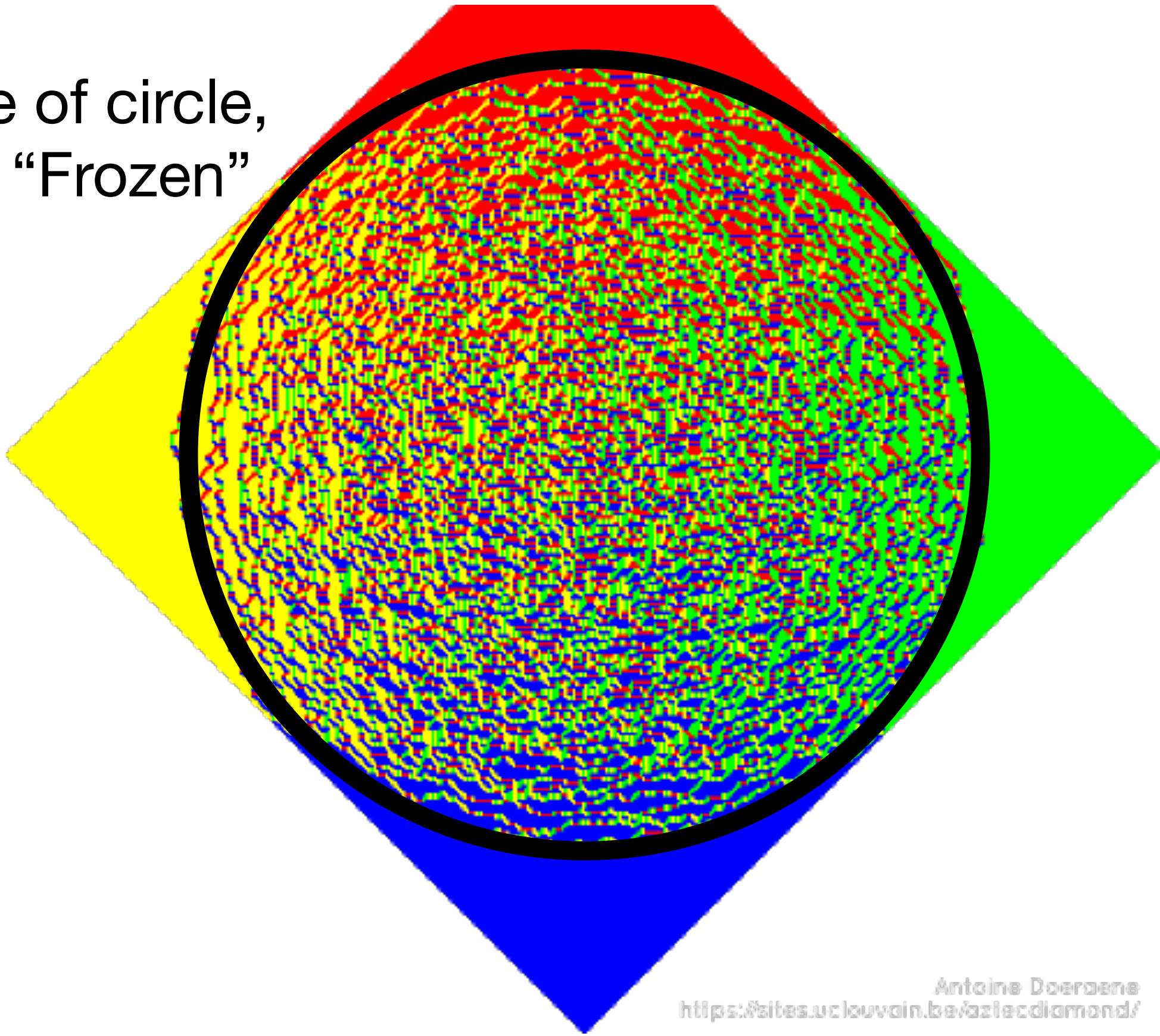


Random domino tiling  
(Size 1000)



# Arctic Circle Theorem

Outside of circle,  
tiling is “Frozen”



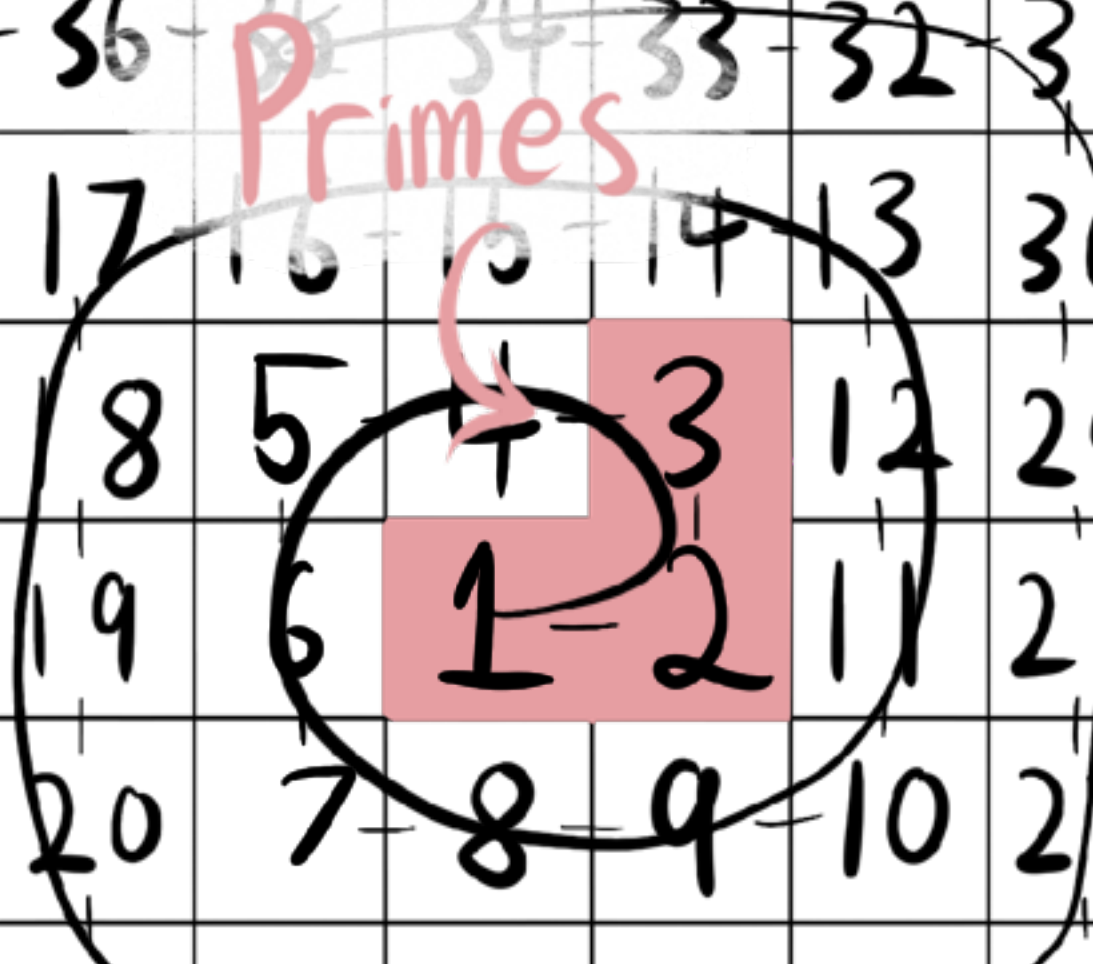


# Episode 3:

## Primes in a spiral

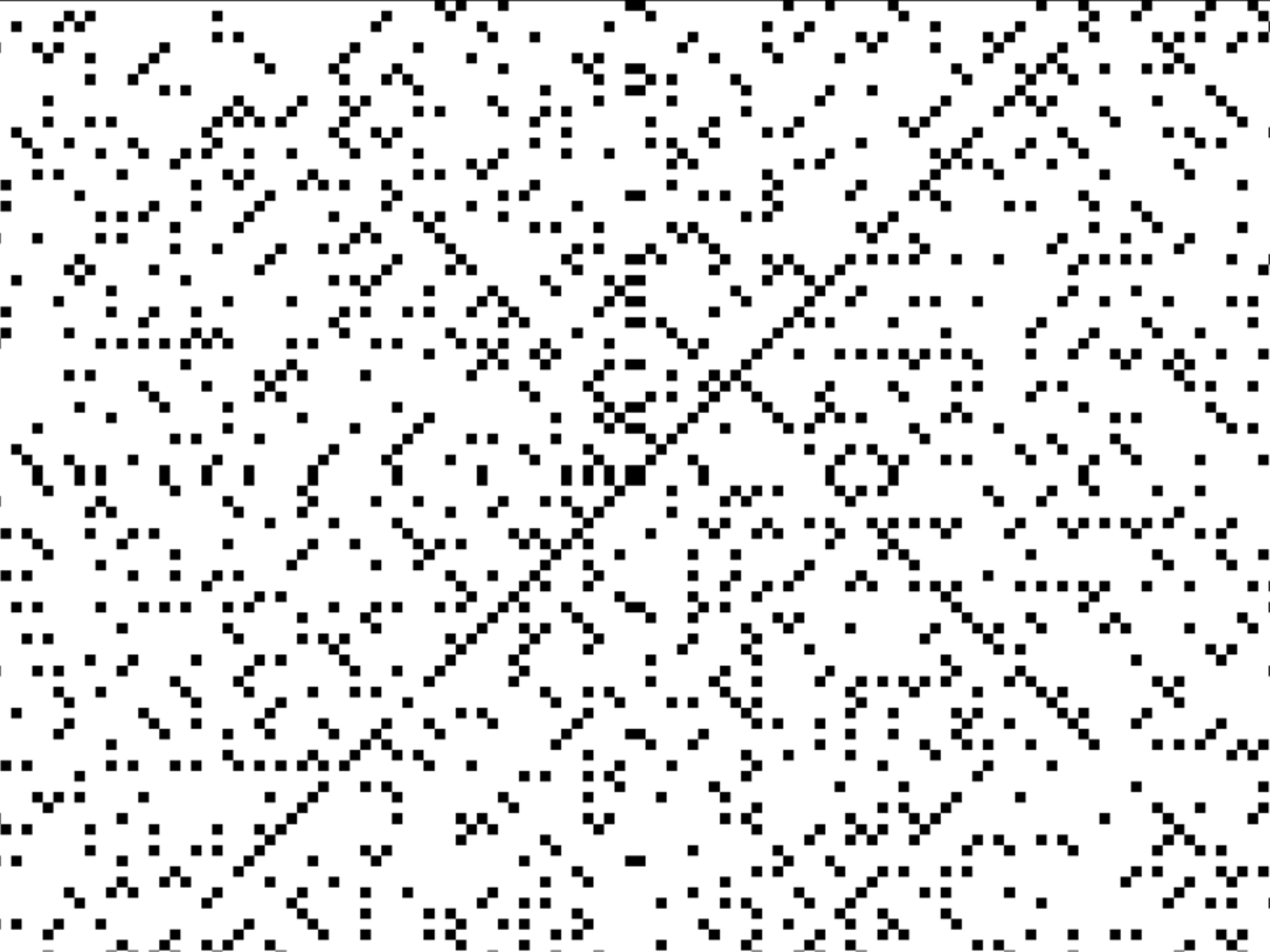
146	101	100	99	98	97	96	95	94	93	92	91	132	179
147	102	65	64	63	62	61	60	59	58	57	90	131	178
148	103	66	37	36	<del>35</del>	34	33	32	31	56	89	130	177
149	104	67	38	17	<del>16</del>	<del>15</del>	14	13	30	55	88	129	176
150	105	68	39	8	5	4	3	12	29	54	87	128	175
151	106	69	40	9	6	1	2	11	28	53	86	127	174
152	107	70	41	20	7	8	9	10	27	52	85	126	173
153	108	71	42	21	22	23	24	25	26	51	84	125	172
154	109	72	43	44	45	46	47	48	49	50	83	124	171
155	110	73	74	75	76	77	78	79	80	81	82	123	170

Primes





146	101	100	99	98	97	96	95	94	93	92	91	132	179
147	102	65	64	63	62	61	60	59	58	57	90	131	178
148	103	66	37	36	35	34	33	32	31	56	89	130	177
149	104	67	38	17	16	15	14	13	30	55	88	129	176
150	105	68	39	18	5	4	3	12	29	54	87	128	175
151	106	69	40	19	6	1	2	11	28	53	86	127	174
152	107	70	41	20	7	8	9	10	27	52	85	126	173
153	108	71	42	21	22	23	24	25	Ulam spiral				
154	109	72	43	44	45	46	47	48					
155	110	73	74	75	76	77	78	Shade in primes					







Random points  
(distributed like primes)

$n^2 - n + 41$

Start at 41

41: Euler's lucky #

$n^2 - n + 41$  prime for

$n = 0, 1, \dots, 40$

# How to computer

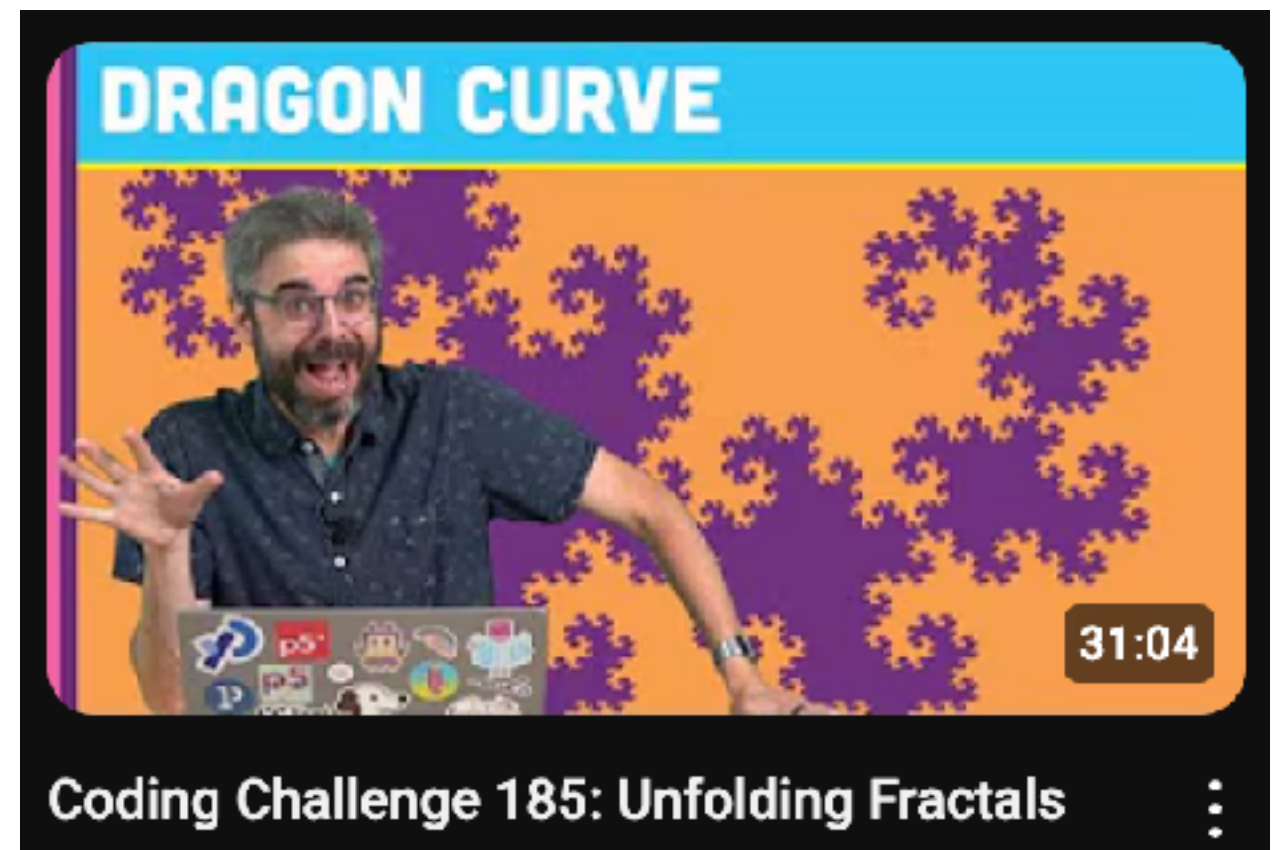
“Creative coding”

p5.js

<https://p5js.org/>



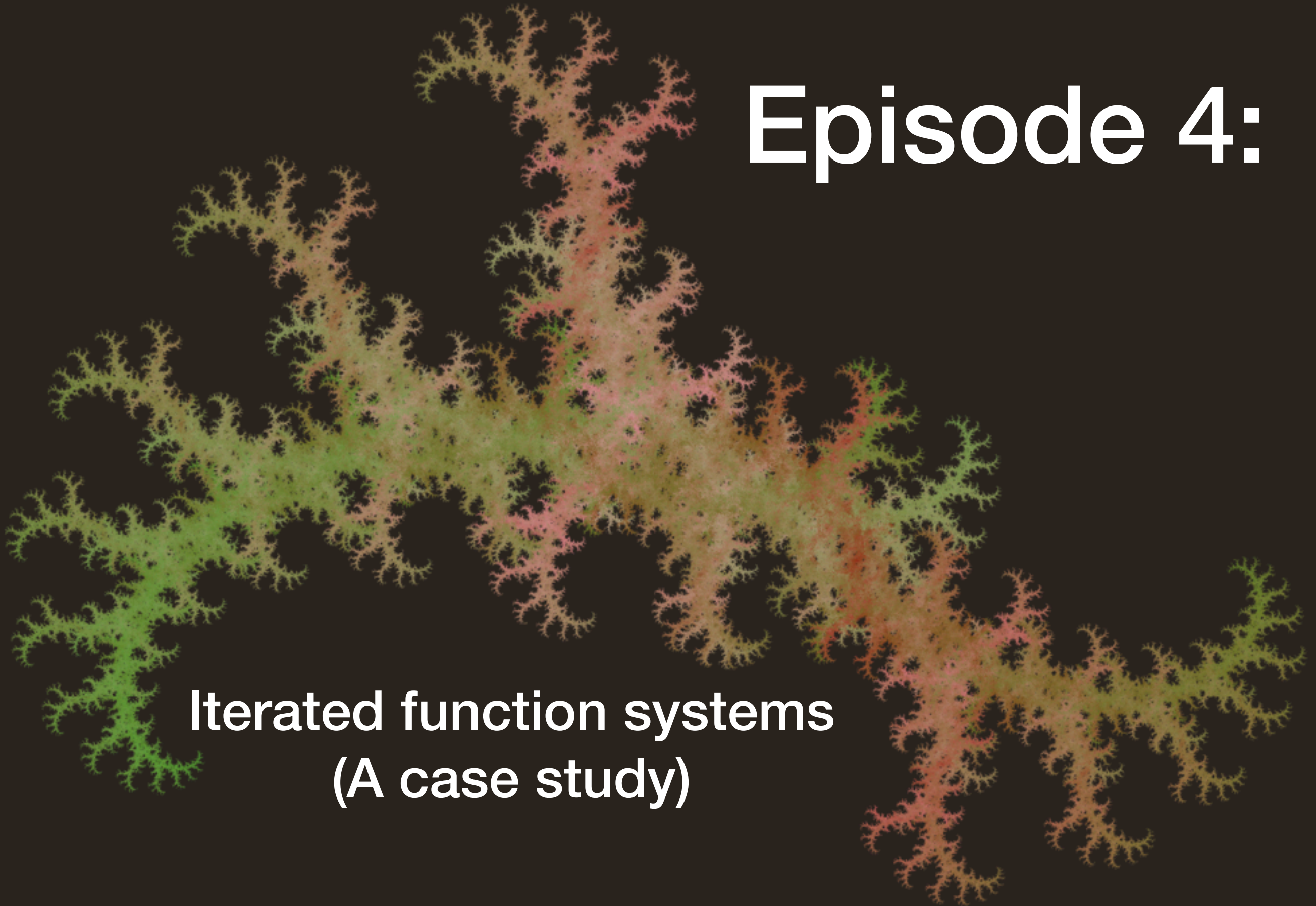
<https://openprocessing.org/user/278206/?o=32&view=sketches>



The coding train (YouTube)

# Episode 4:

Iterated function systems  
(A case study)

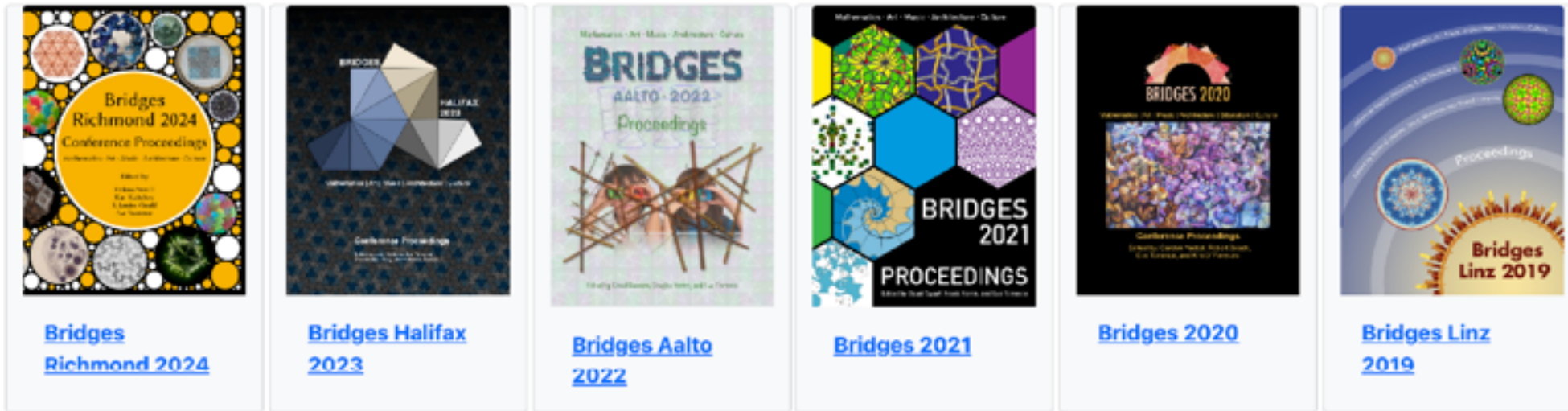




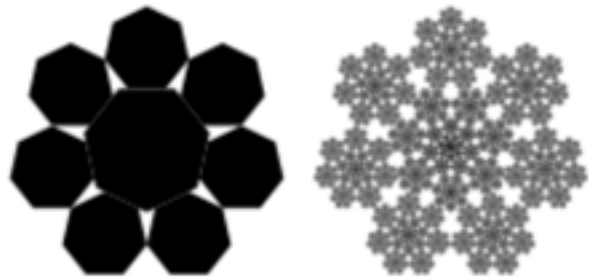
# Find inspiration

**The Bridges Archive** <https://archive.bridgesmathart.org>

Welcome to the Bridges Archive, an online repository of every paper ever published in the annual [Bridges Conference](#) on art and mathematics. Click on the years below or in the sidebar on the left to view the papers for that year.

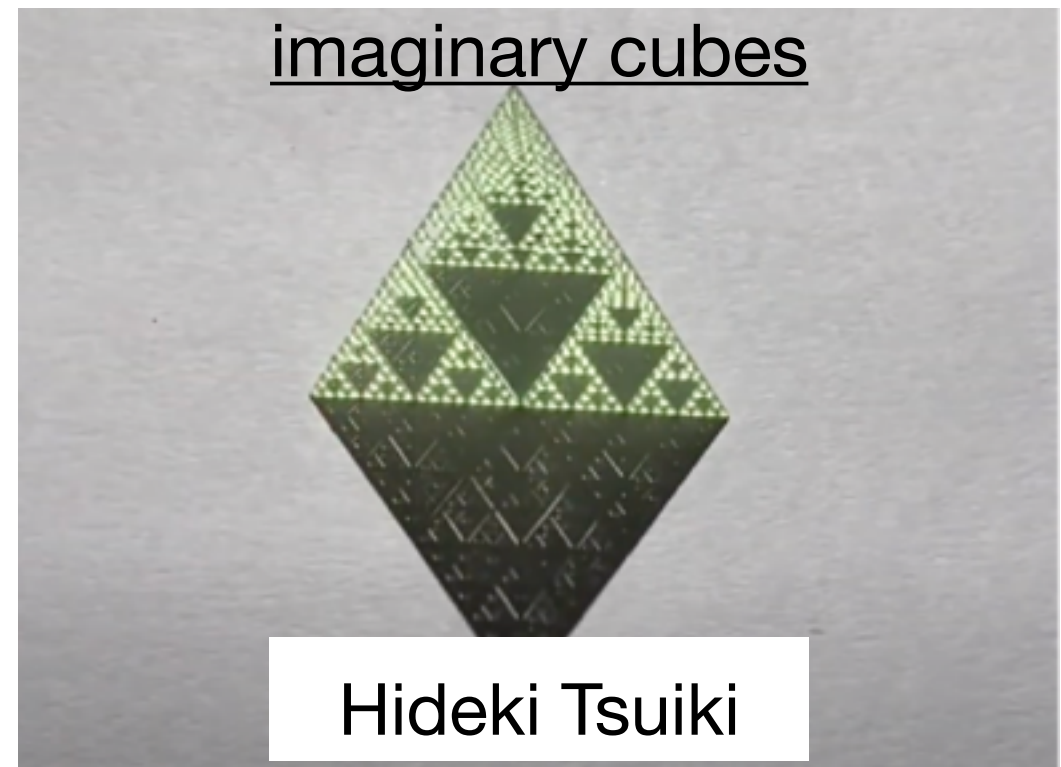


## N-flake variations



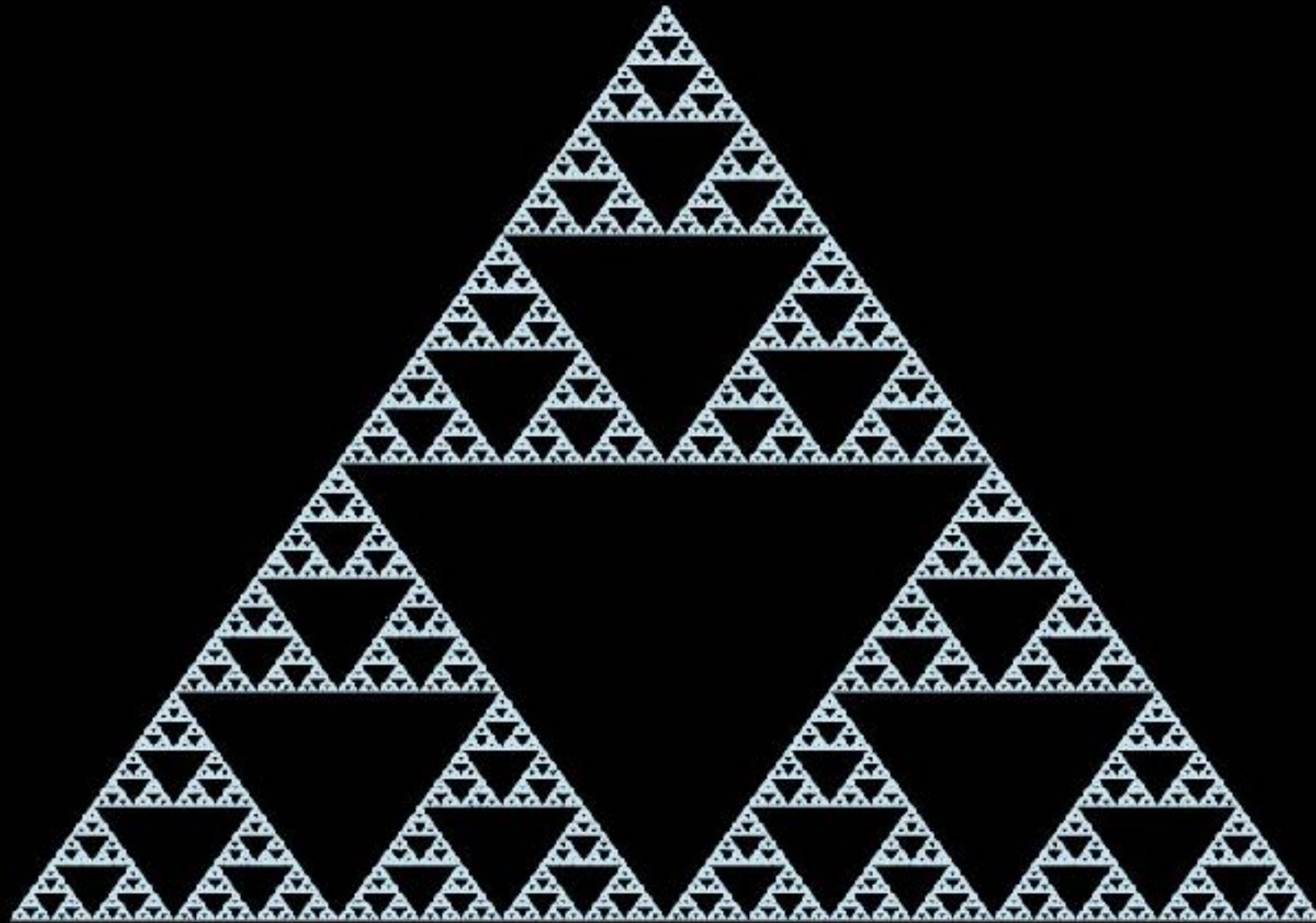
Steven Wilkinson and Blake Settle

## Shadows of fractal imaginary cubes



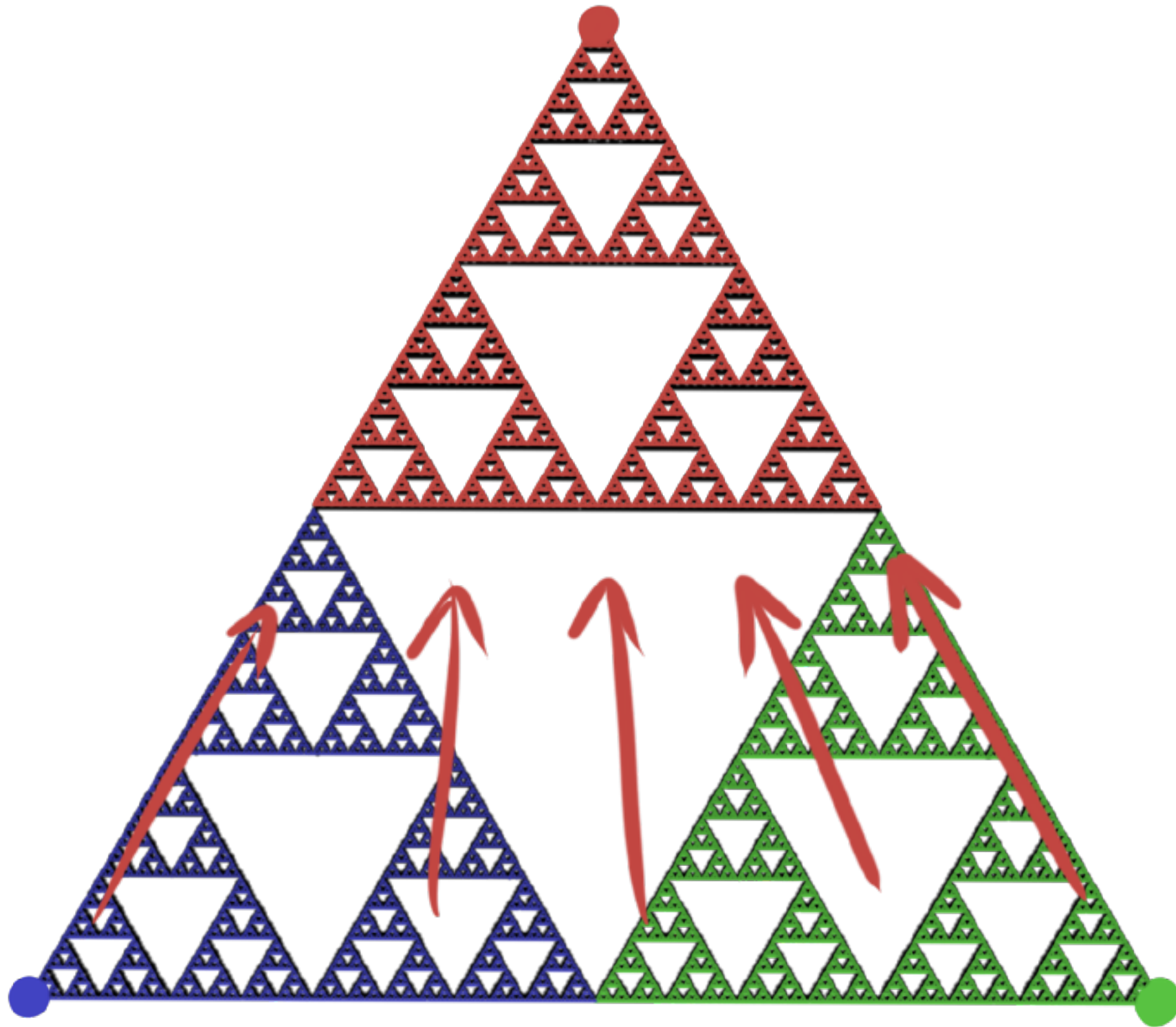
Hideki Tsuiki

# Chaos game



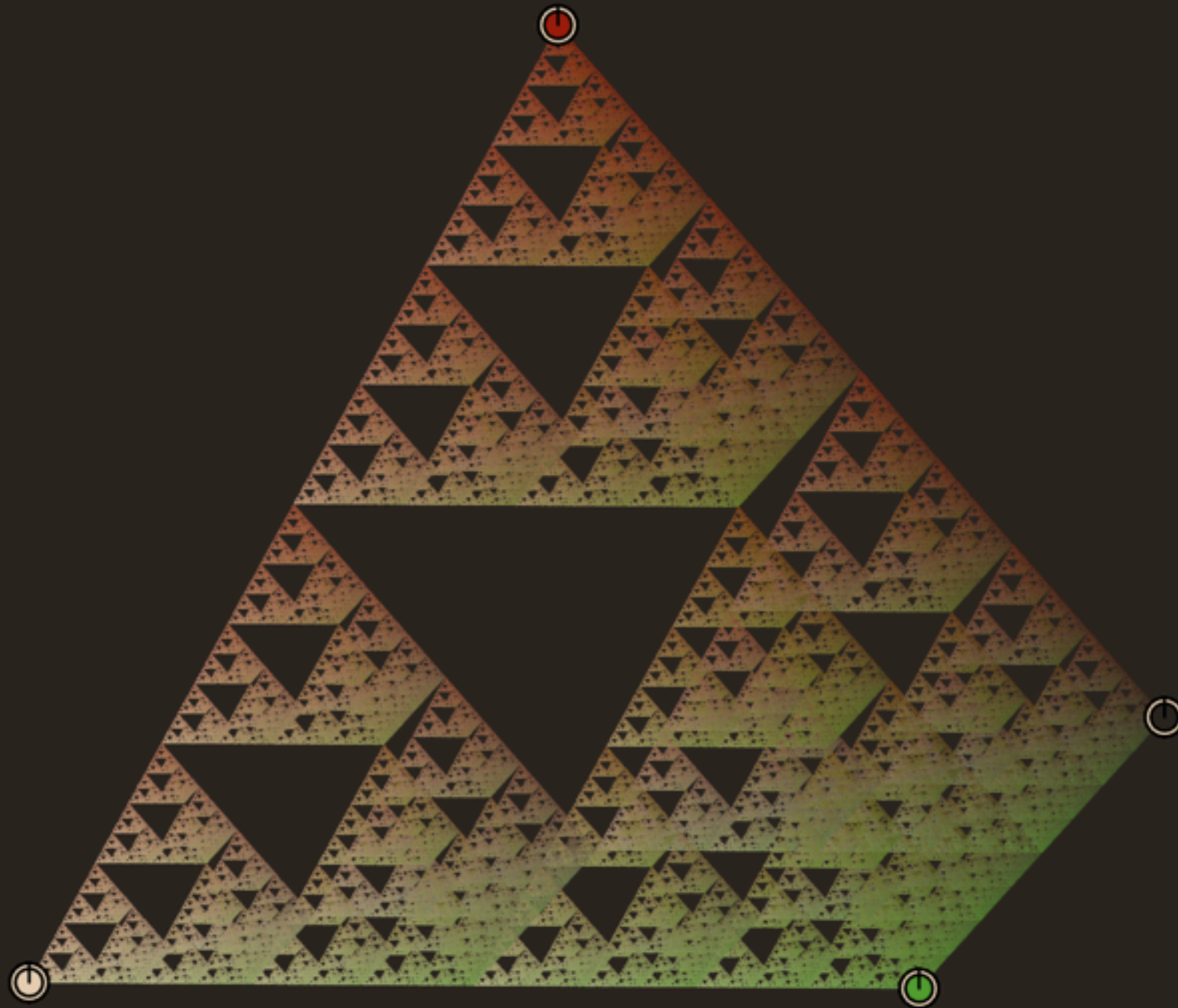
From channel “think twice”

# Sierpinski triangle



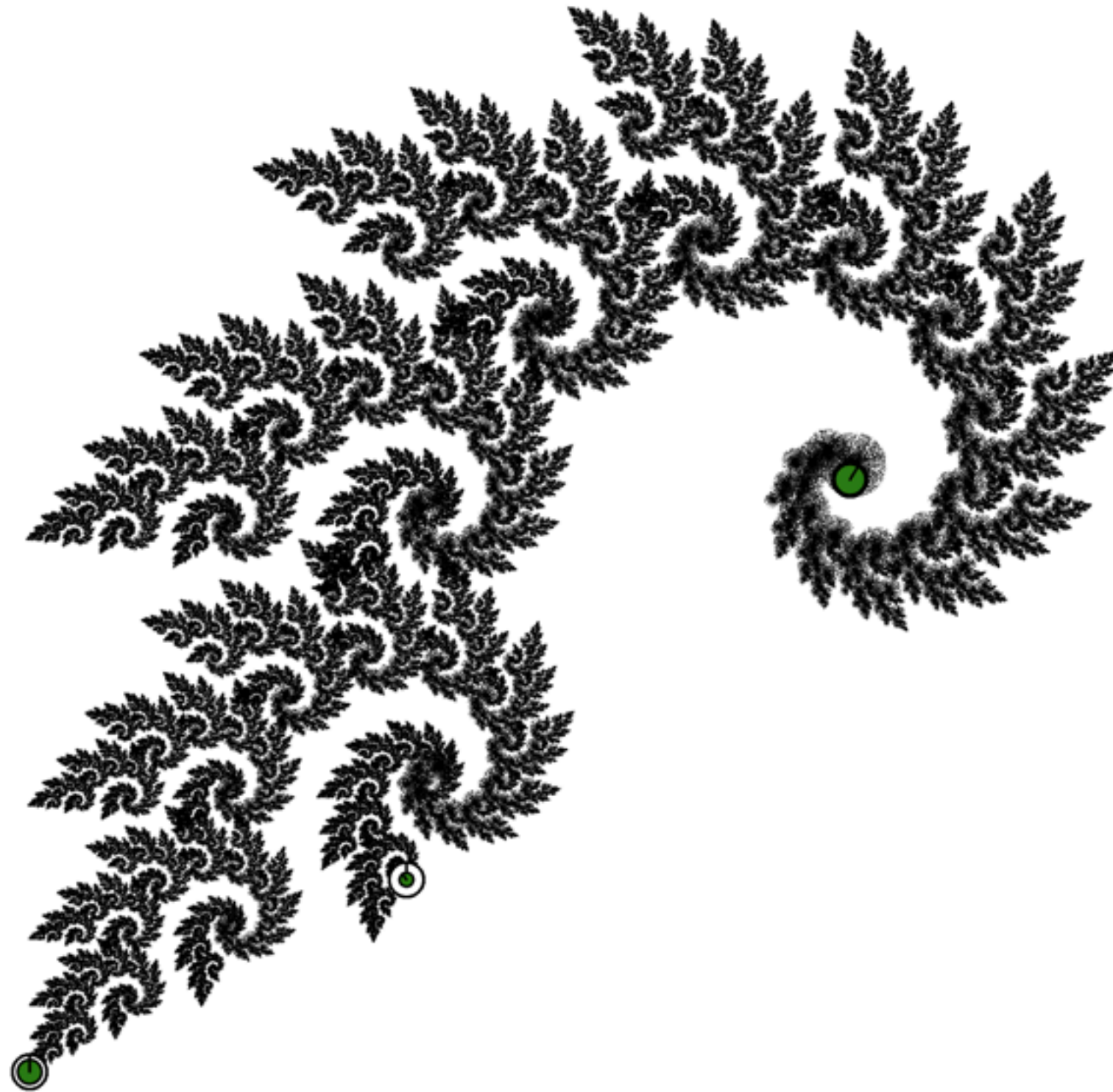


# Chaos game with 4 points



Sierpinski tetrahedron?





<https://chessapig.github.io/code/ifs>

# The Geometry center

University of Minnesota



R.I.P

1991-1998



# Geometry Labs United

[Experimental Algebra and Geometry Lab](#), University of Texas Rio Grande Valley

[Experimental Mathematics Lab](#), University of Colorado Boulder

[Experimental Mathematics Lab](#), University of Luxembourg

[Heidelberg Experimental Geometry Lab](#), University of Heidelberg

[i-center](#), Kansas State University

[Illinois Mathematics Lab](#), University of Illinois Urbana-Champaign

*(formerly known as the Illinois Geometry Lab)*

[JMU Experimental Mathematics Lab](#), James Madison University

[Knox Math Lab](#), University of Tennessee Knoxville

[Laboratoire de mathématiques UdeS](#), Université de Sherbrooke

[Lab of Experimental Mathematics at Maryland](#), University of Maryland College Park

*(formerly known as the Experimental Geometry Lab)*

[LOG\(M\): Laboratory of Geometry at Michigan](#), University of Michigan

[Madison Experimental Mathematics Lab](#), University of Wisconsin-Madison

[Mason Experimental Geometry Lab](#), George Mason University

[Mathematics Lab](#), Max Plank Institute for Mathematics in the Sciences, Leipzig University

[Nebraska Experiential Math and Outreach \(NEMO\) Lab](#), University of Nebraska-Lincoln

[MSCS Undergraduate Research Laboratory](#), University of Illinois at Chicago

*(replacing the Mathematical Computing Laboratory)*

[quanTA: Centre for Quantum Topology and Its Applications](#), University of Saskatchewan

[Texas Experimental Geometry Lab](#), University of Texas at Austin

[University of Kentucky Math Lab](#), University of Kentucky

[Washington Experimental Mathematics Lab](#), University of Washington

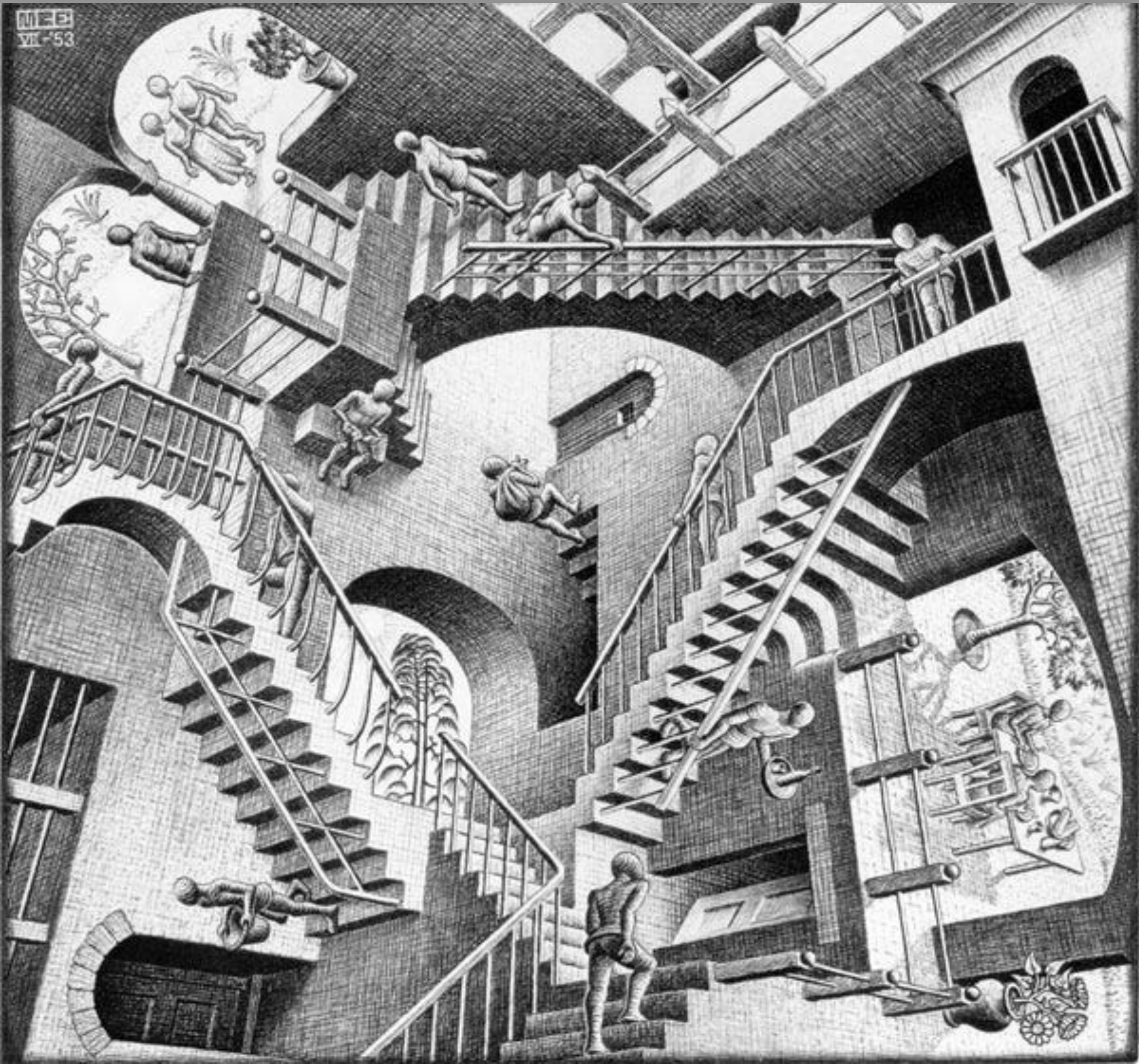
# **Episode 5: Visual artists**





M. C. Escher

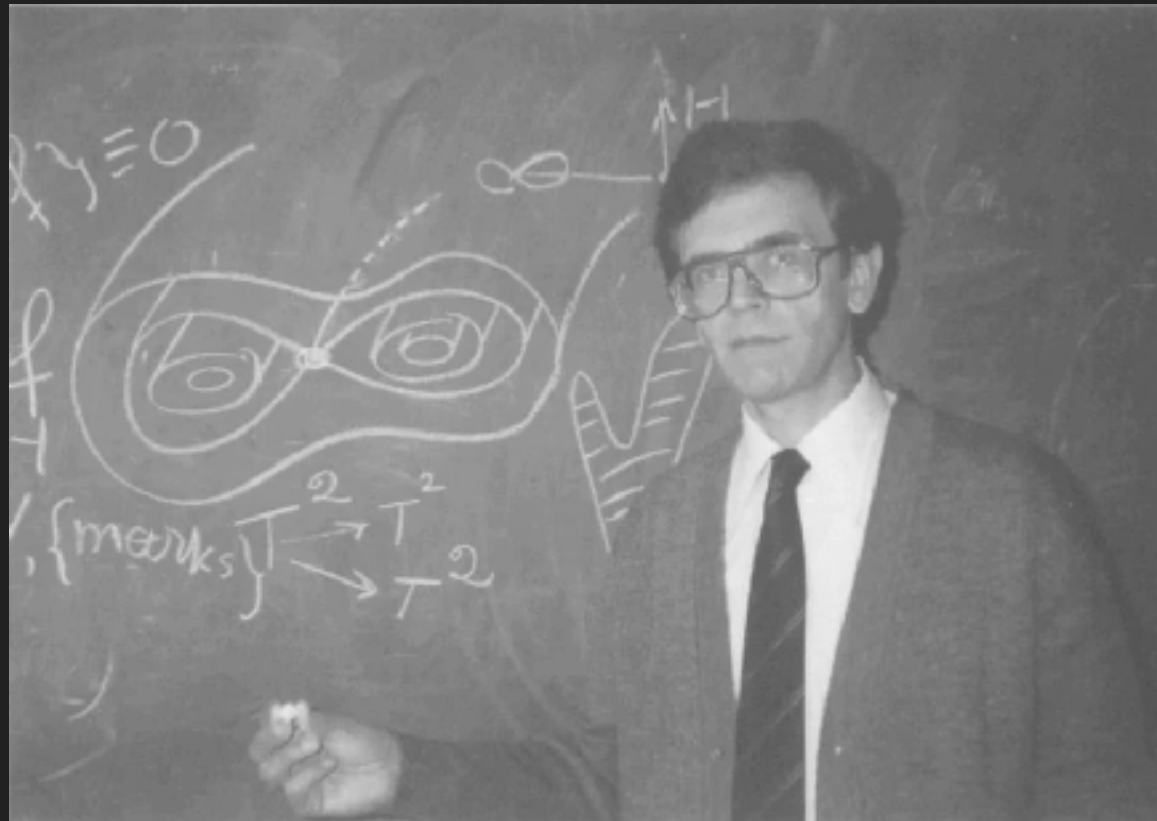








# Anatoly Fomenko

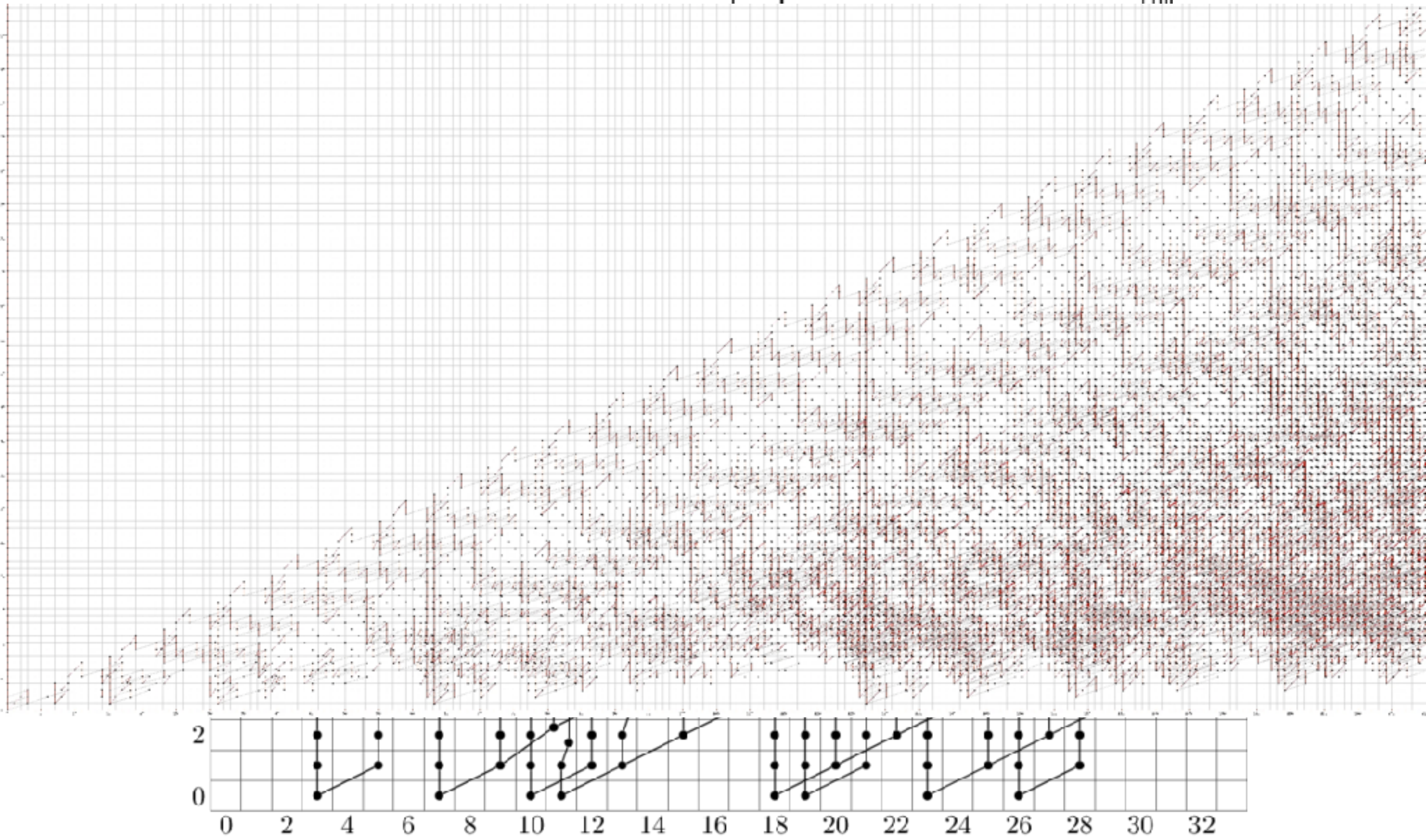


- Topologist
- Painter
- Crackpot



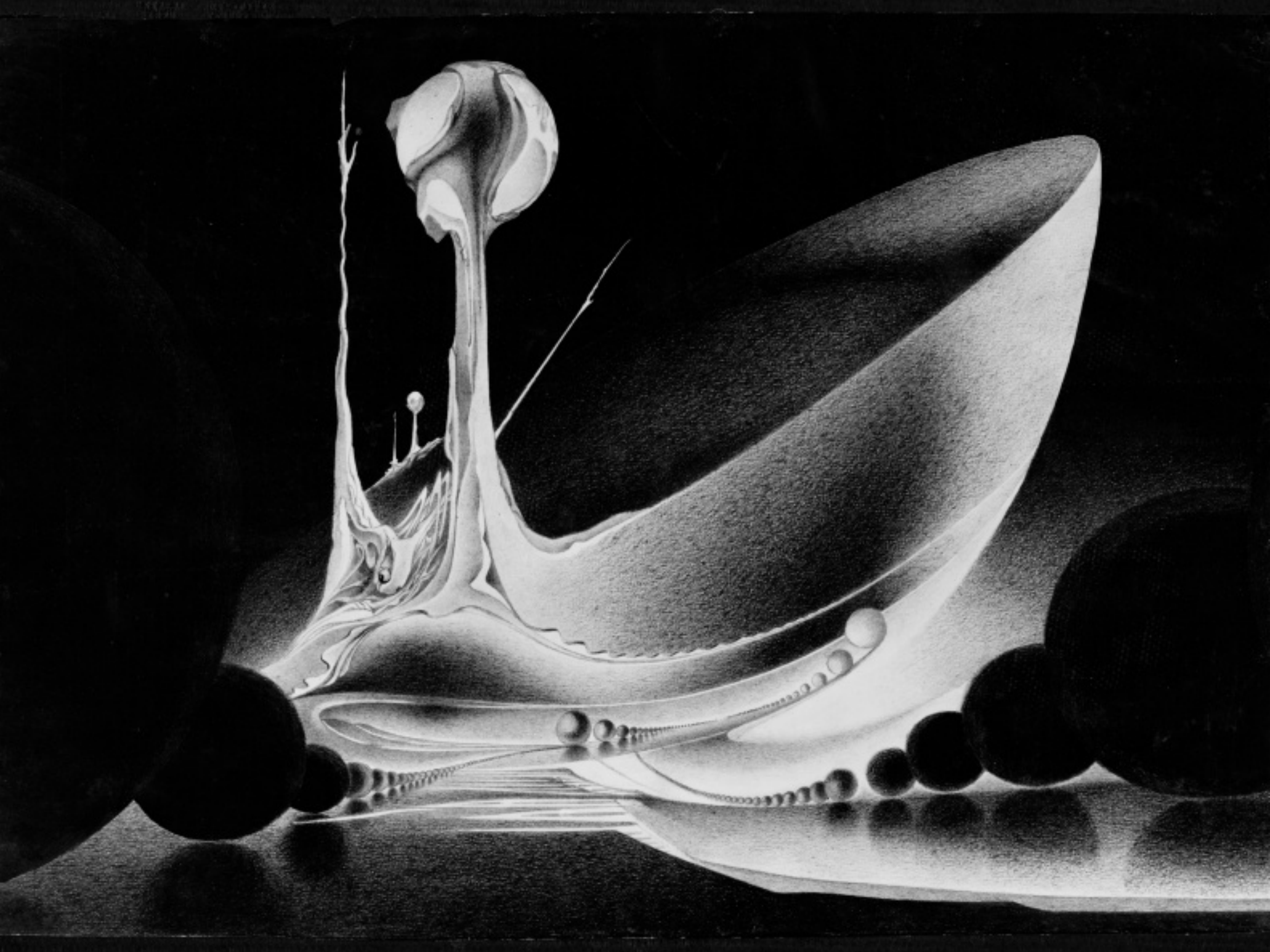


# Spectral sequences





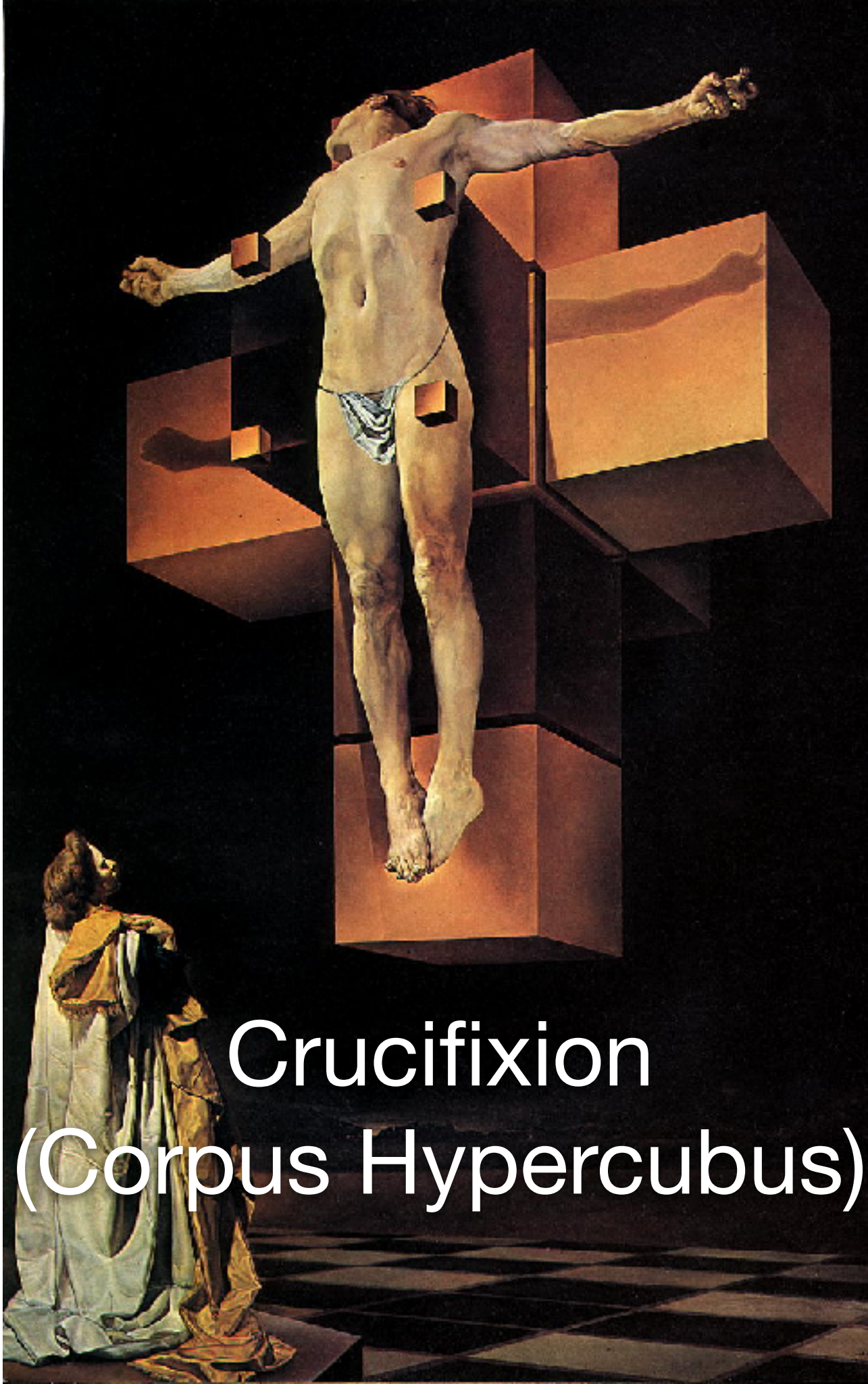




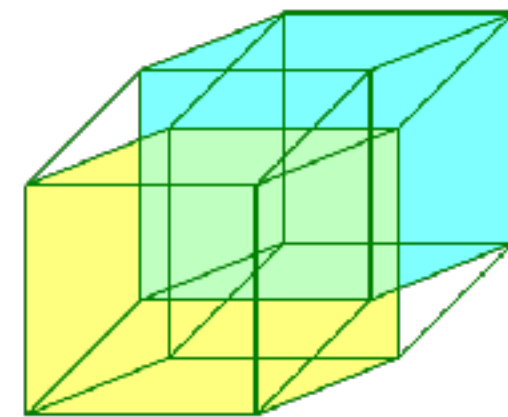
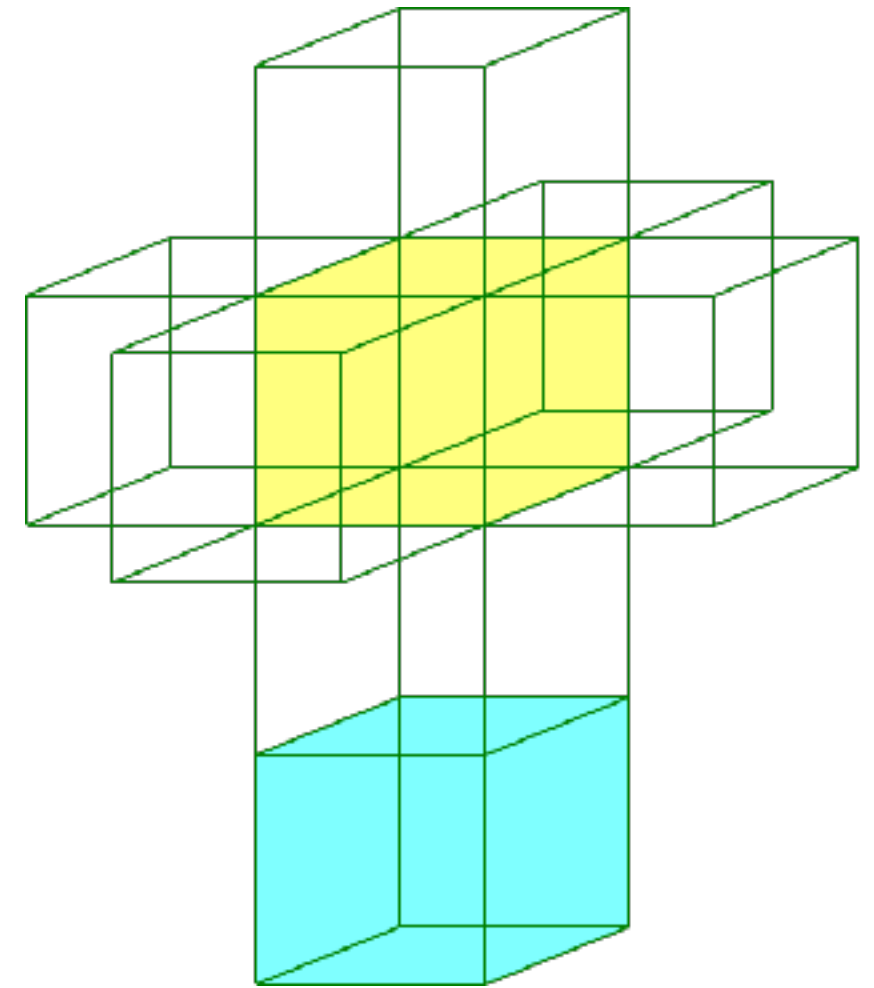


# Salvador Dalí





Crucifixion  
(Corpus Hypercubus)



Unfolded  
hypercube



# The Swallow's Tail





Me!

# The Mirror Symmetry Tree





# How to get involved

- Draw math as you learn
- Give form to your mathematical world



## Illustrating math community:

- Joint mathematics meeting (JMM)
- Discord server