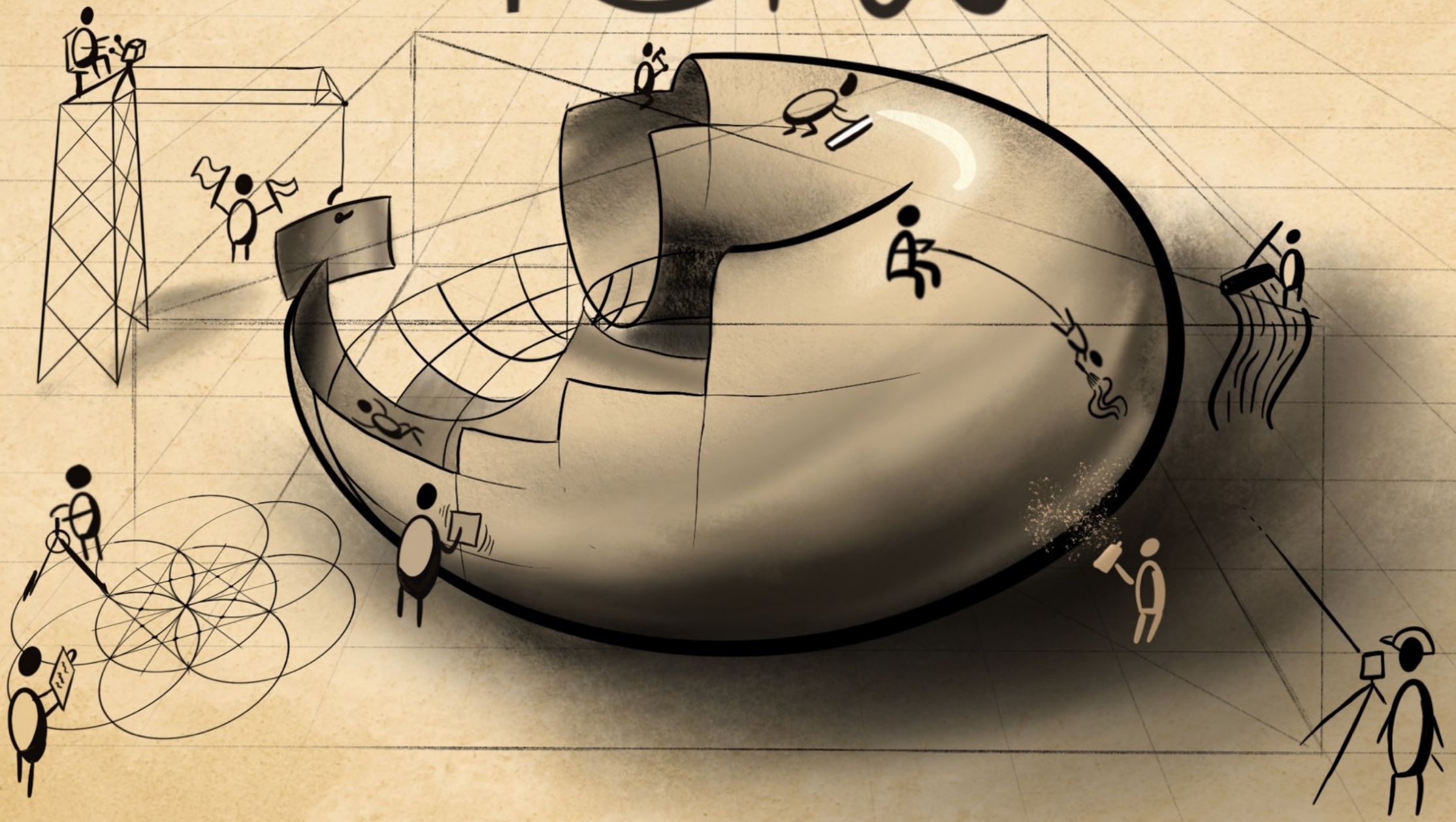
To Draw A Torus



Moduli Stack of genus 2 curves

S_4

$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

D_3

$\mathbb{Z}_2$

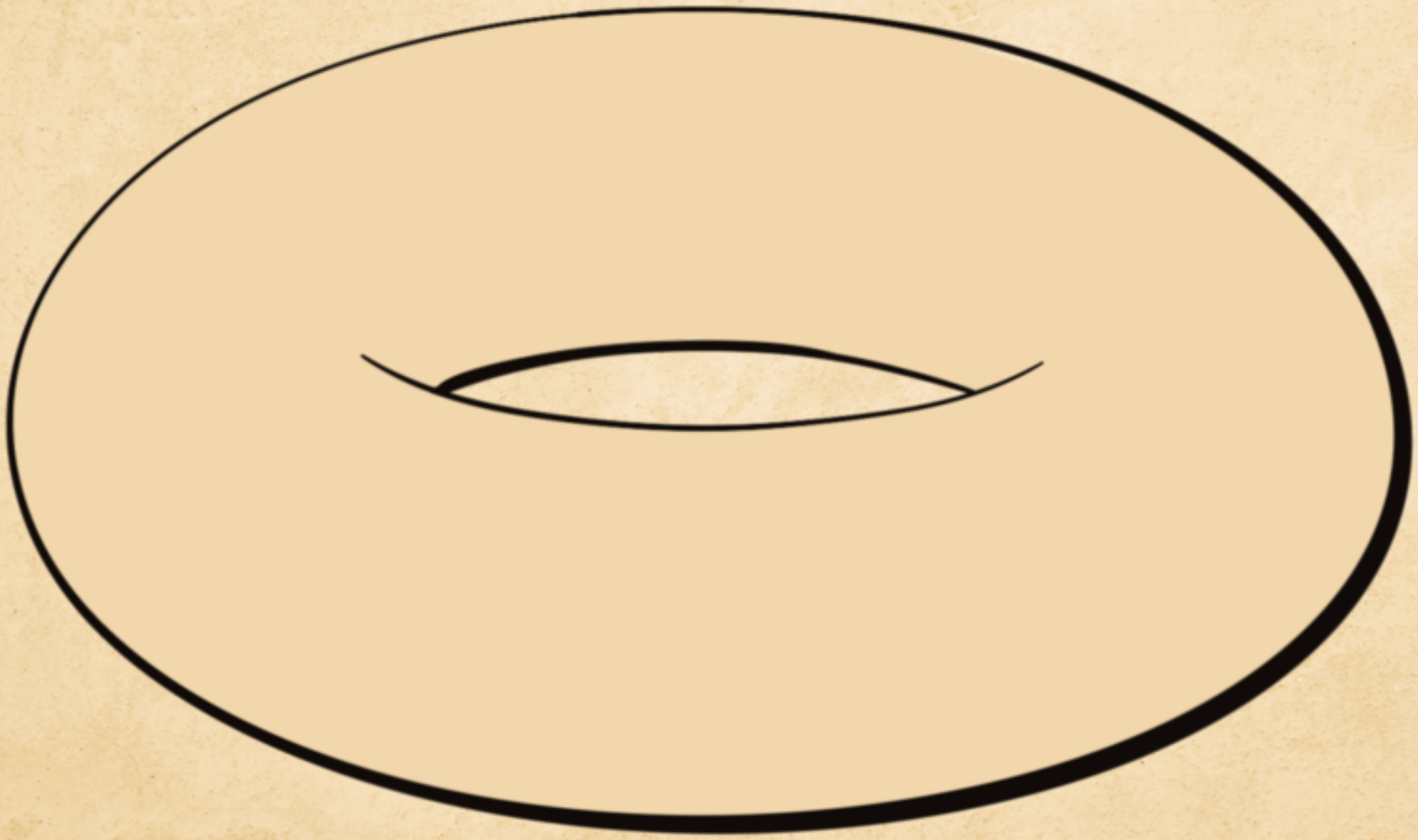
$\mathbb{Z}_3$

$\mathbb{Z}_2$

$\mathbb{Z}_2$

NO Symmetry

1. Outline



2. Shading

3. Shadows

4. Ambient reflections

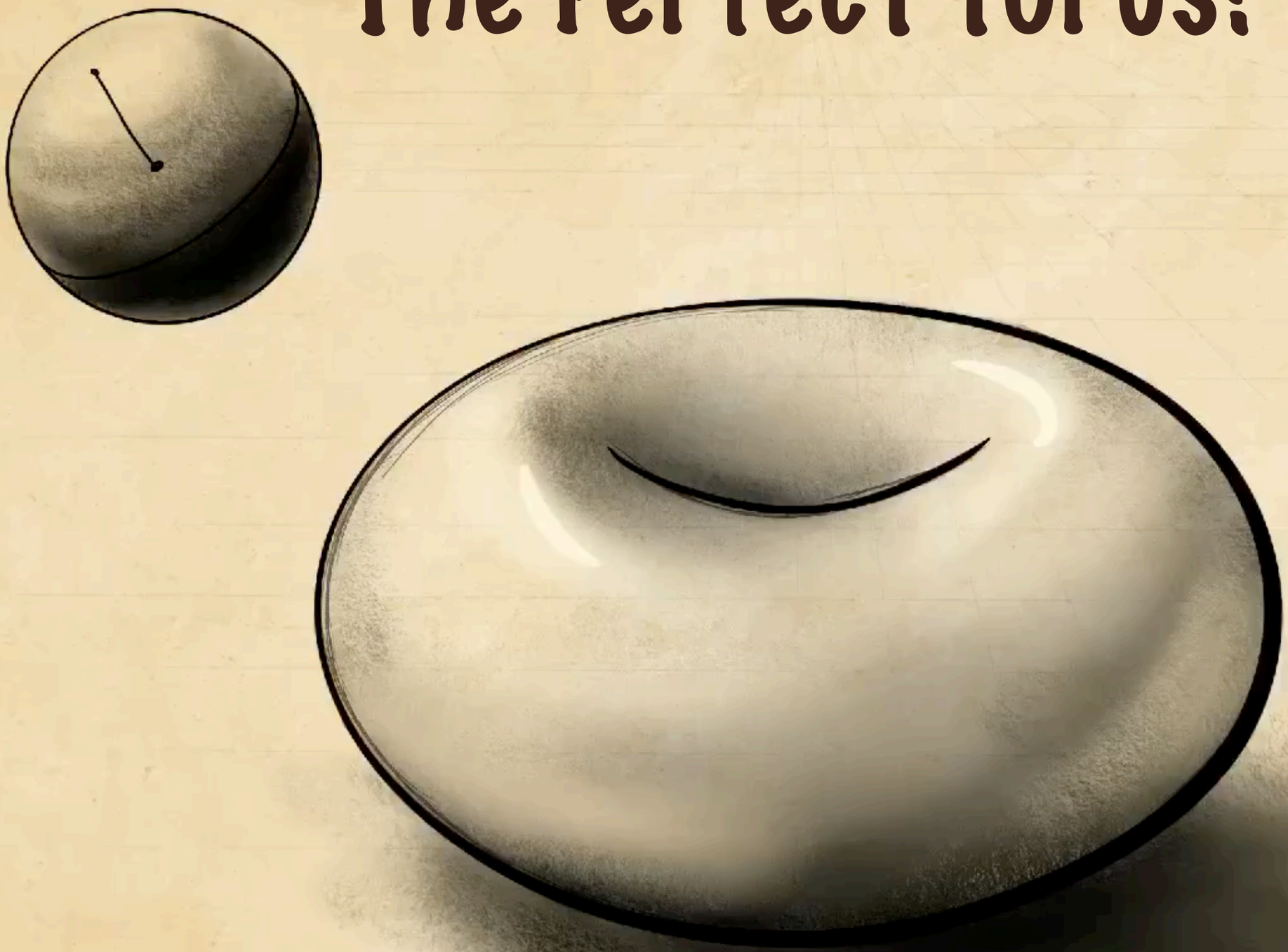
5. Highlights

How do I best draw a Torus?

↳ where should I put the highlights?

↳ what shape are the highlights?

The Perfect torus?



Synthetic construction of torus

Mathematical Percision

⇓

hand-drawn torus

Mathematical Impercision

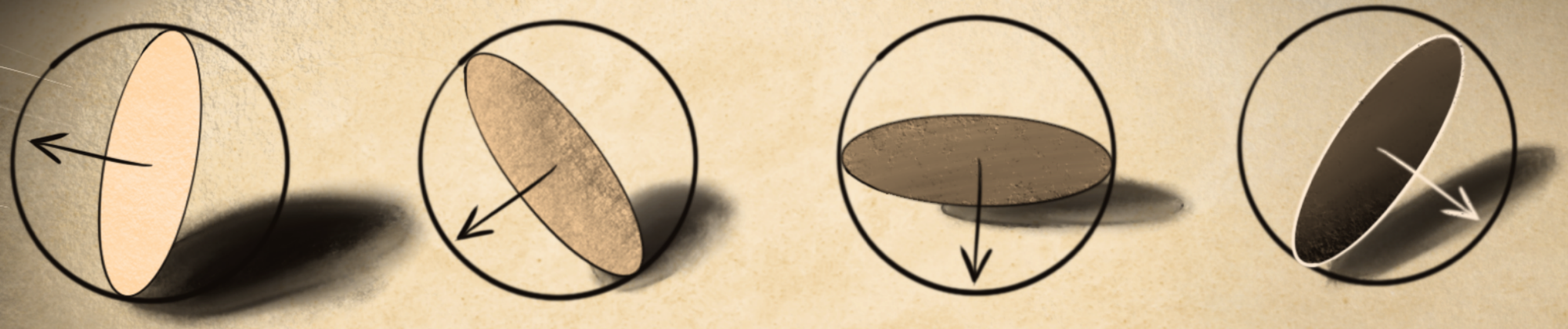
To draw a Torus is to study its

Differential Geometry

Shading determined by:

- Direction of light source
- Direction surface is facing (normal vector)

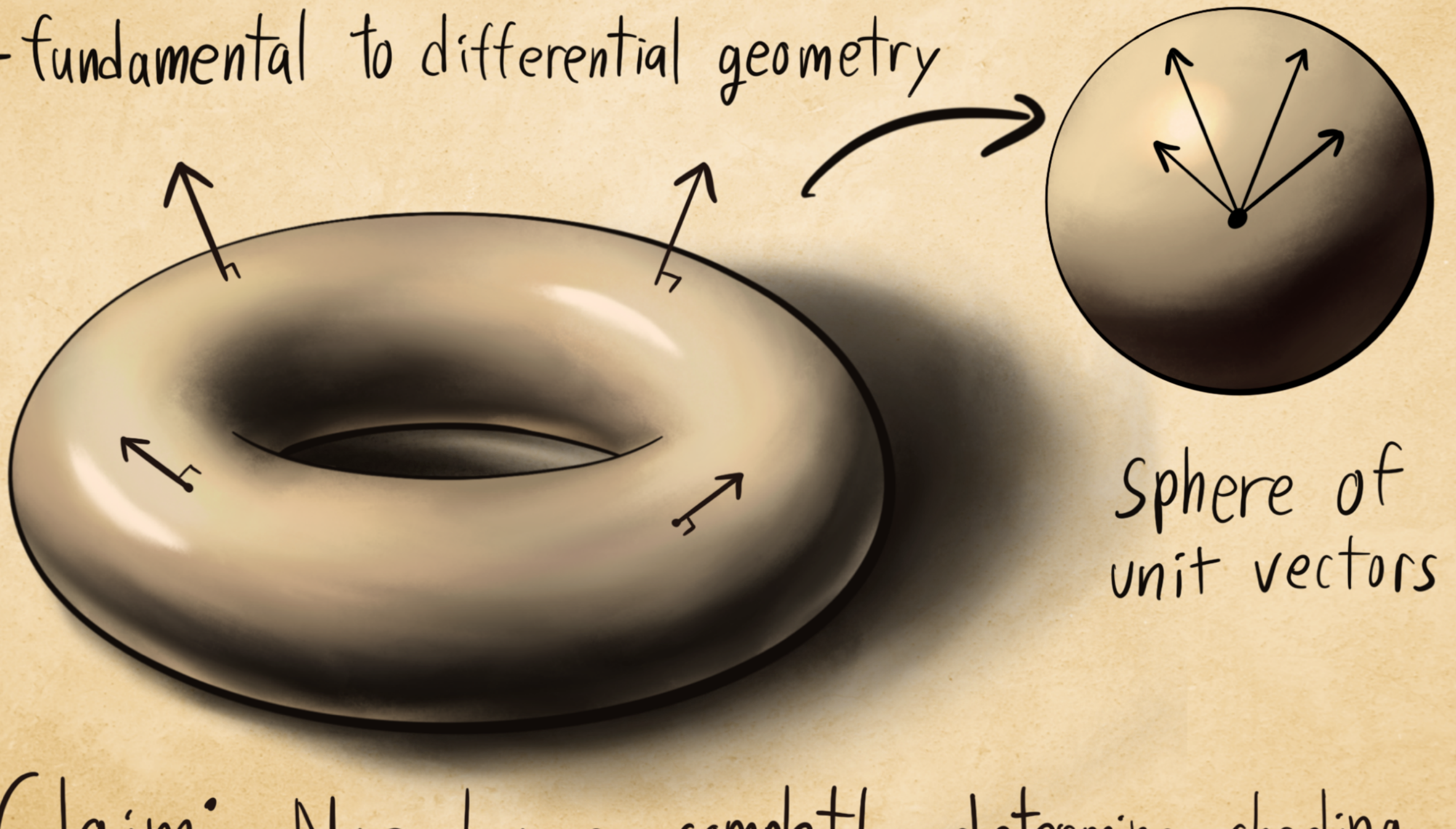
Light ⟷ Dark



Facing Towards light ⟷ Facing away from light

Gauss map / Normal map

- sends each point on surface to its unit normal
- fundamental to differential geometry

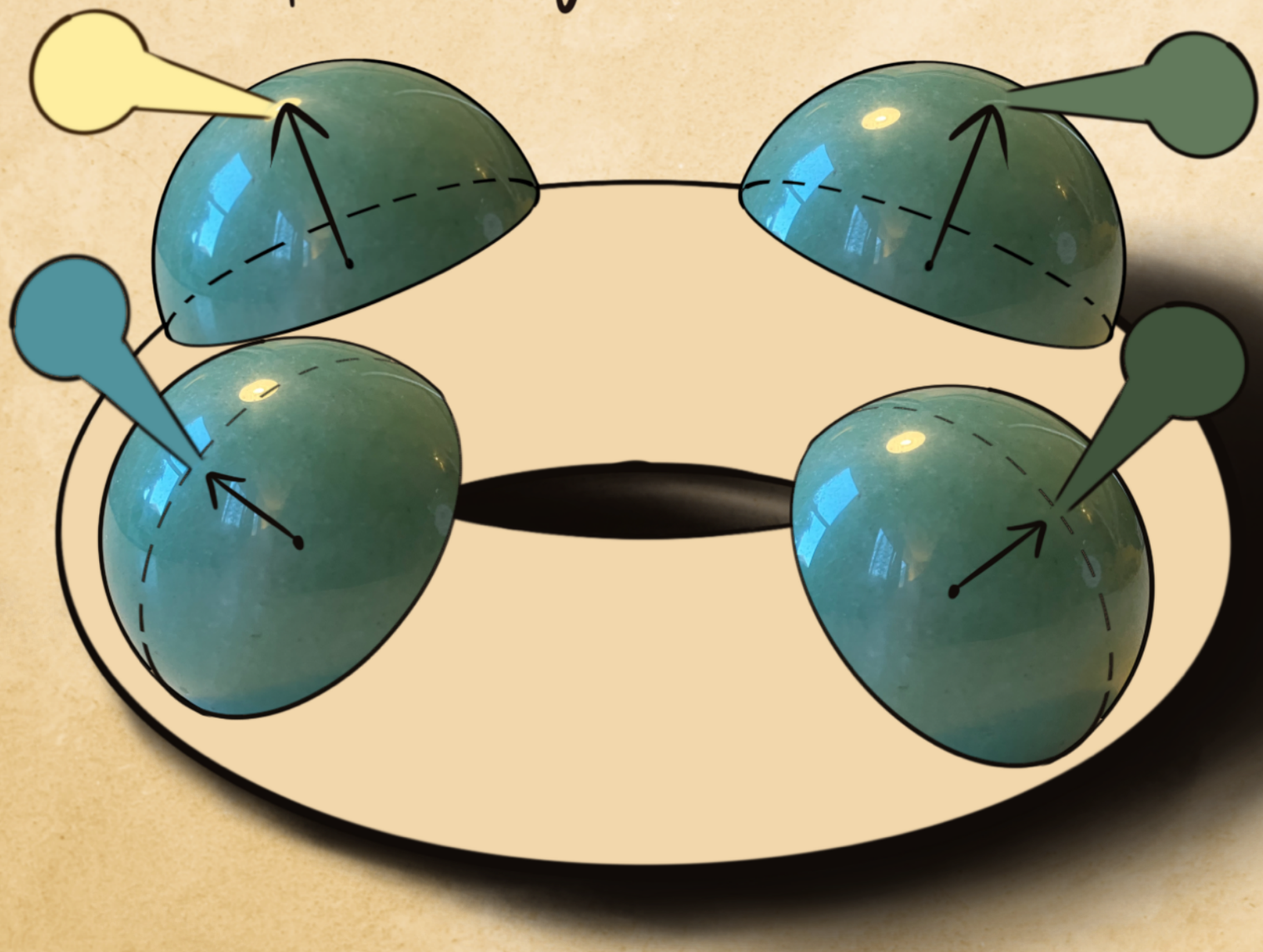


Sphere of unit vectors

Claim: Normal map completly determins shading

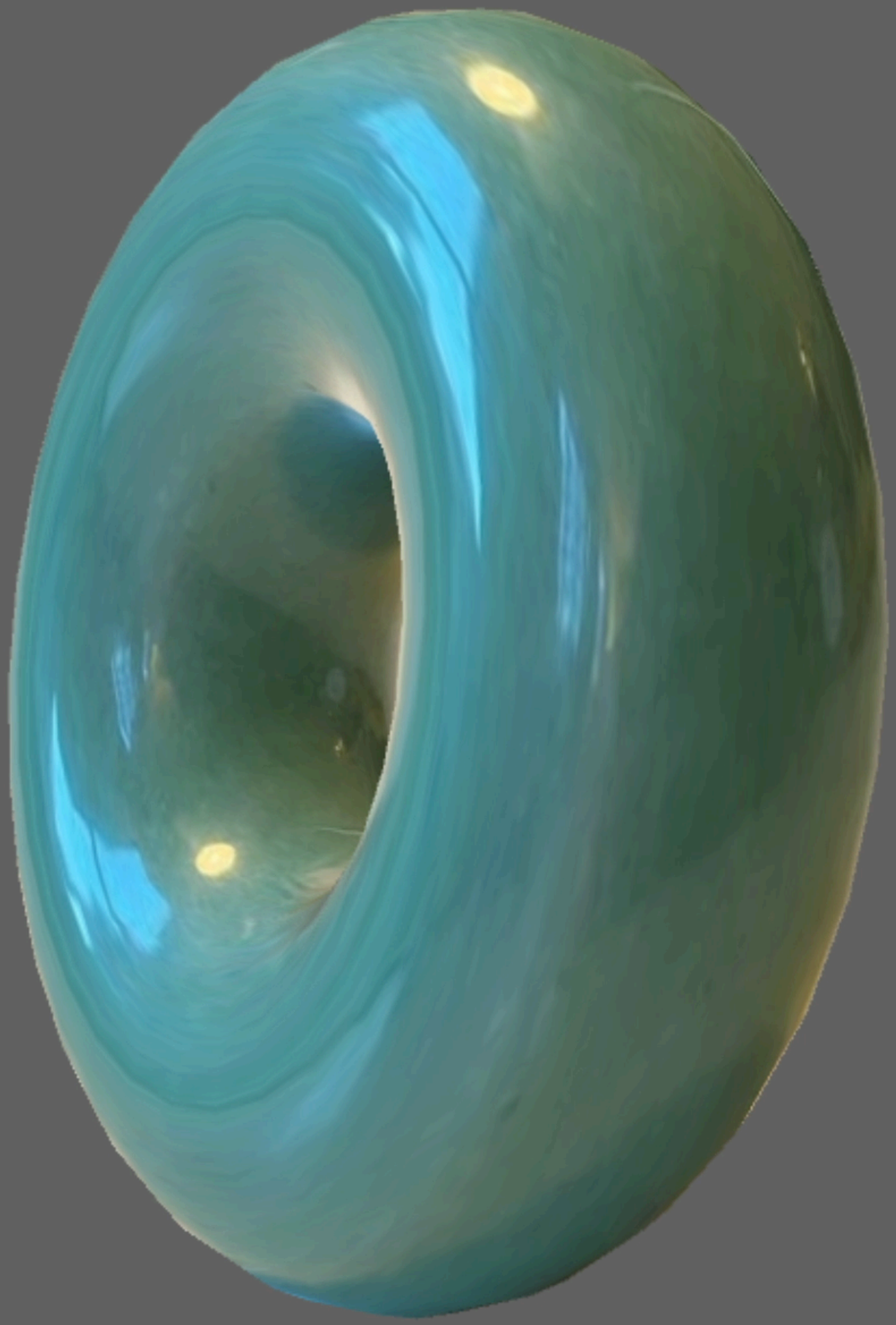
Shading from Normals:

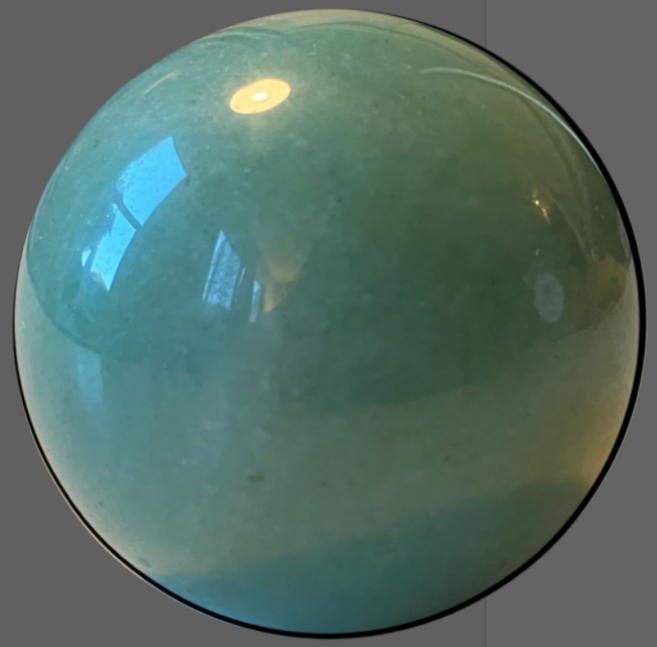
1. create reference for how to shade each normal vector
2. compute normal map
3. lookup shading of each normal



Shaded unit sphere

(picture of sphere)





M.C. Escher

Hand with Reflecting Sphere

M.C. Escher

Hand with Reflecting ~~Sphere~~ Torus

The lighting information is contained in a refrence shading of a unit sphere.

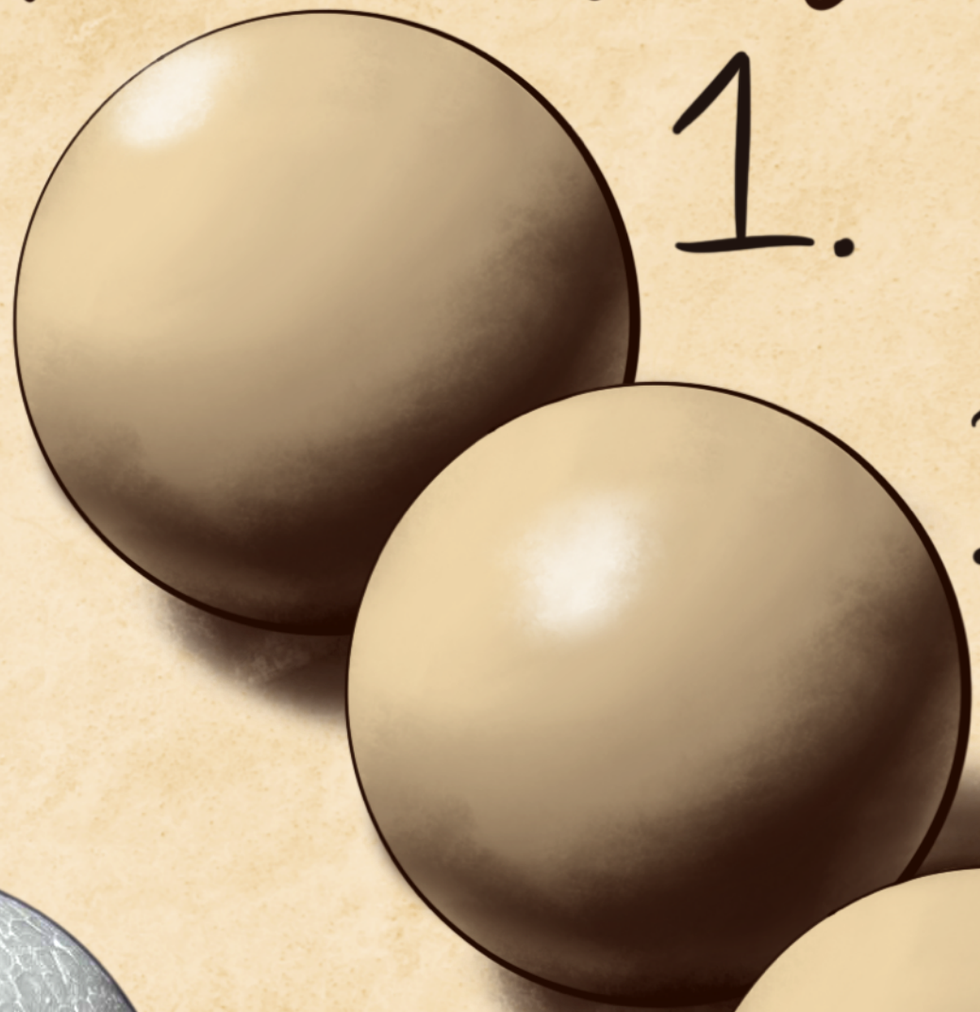
To shade a surface, pull back the refrence shading via the gauss map G

How does this help us draw?

Where to put Highlight?

Single light source

1.



2. ✓

3.

Picture:

Light

Normal vector

highlight

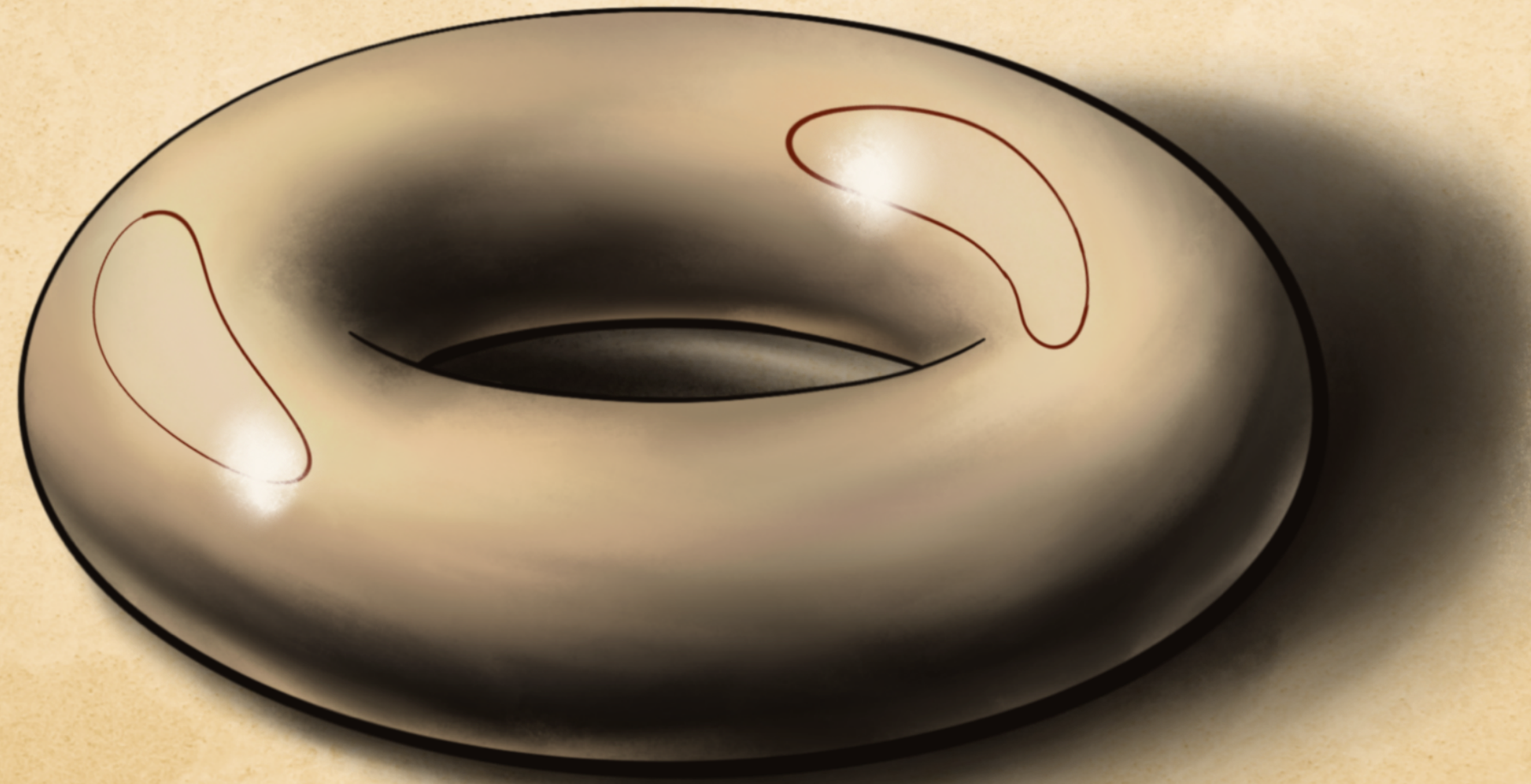
P.O.V:

Lightest shading

half angle

How to place highlights

1. find region of light shading
2. Place highight on boundary, pointing towards viewer



What shape are the highlights?

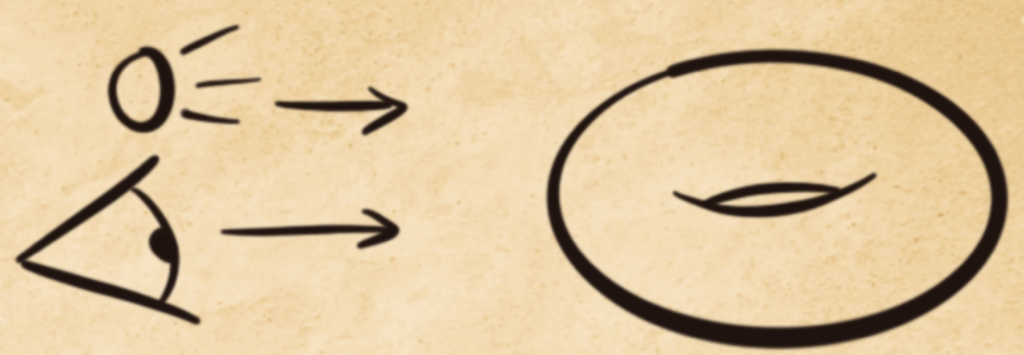
1.

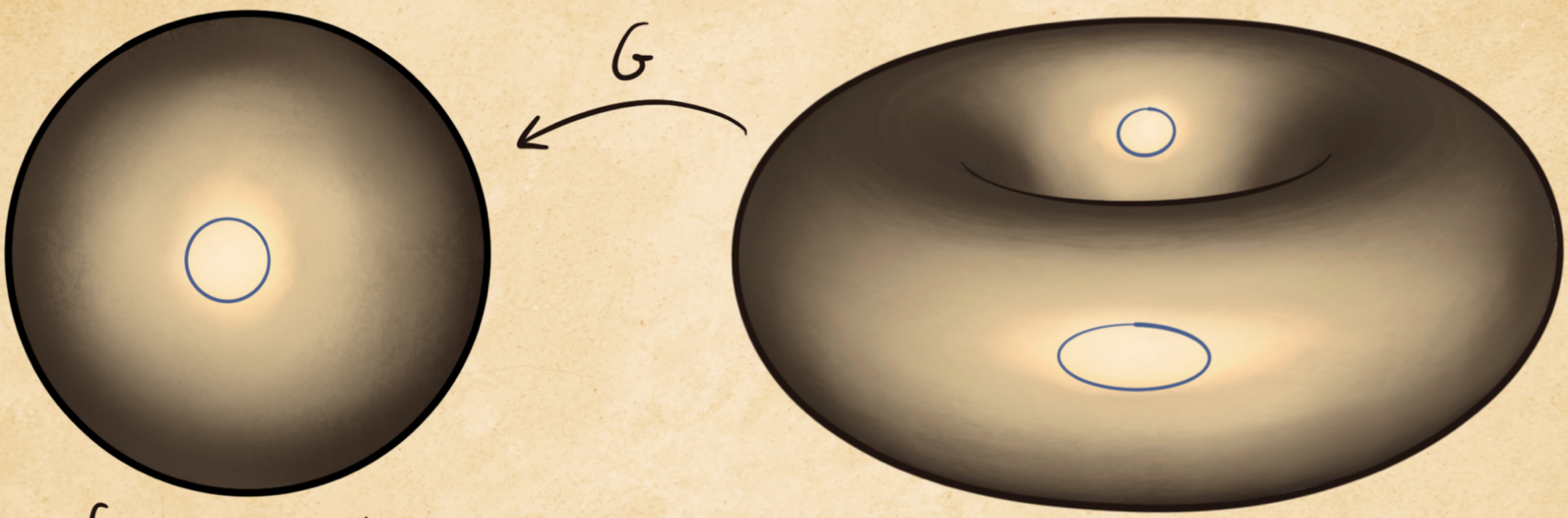
2.

computer-generated

✓ 3.

Headlight lighting:





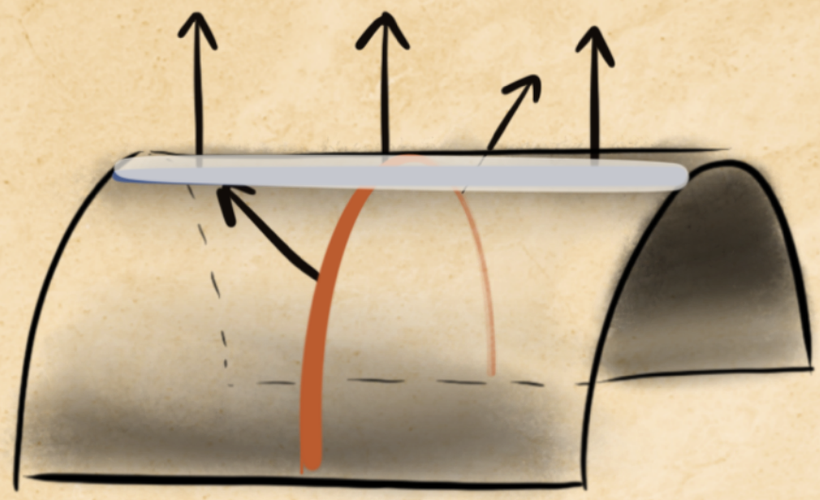
reference sphere

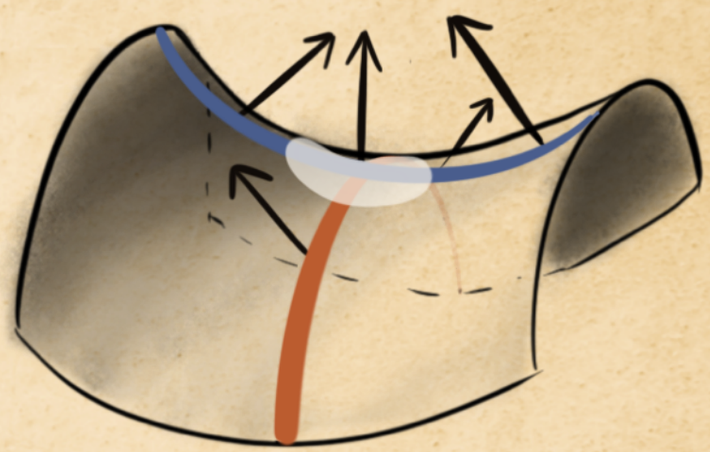
highlight is circle

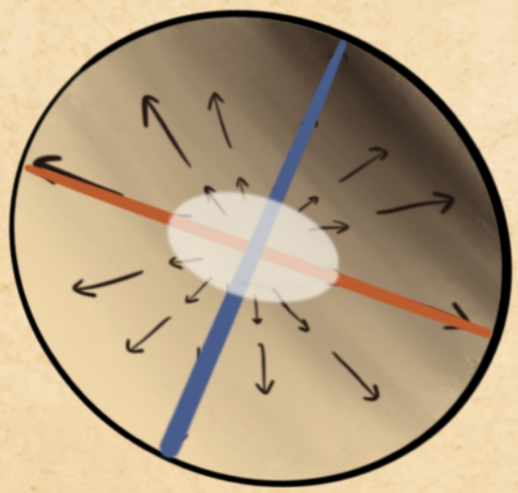
Shaded torus

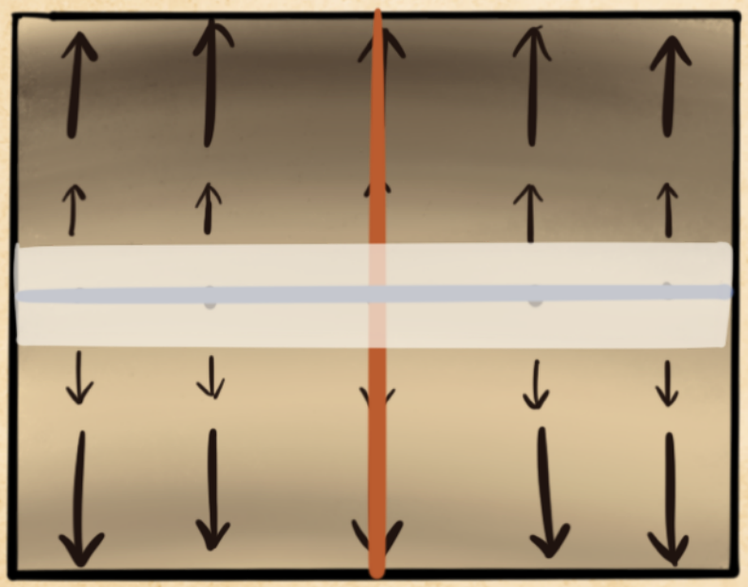
highlight is preimage of circle
under gauss map

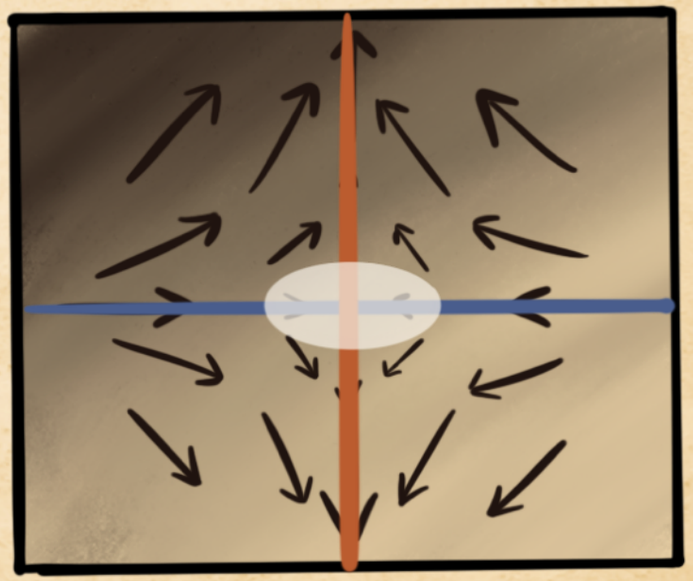
Local structure of gauss map











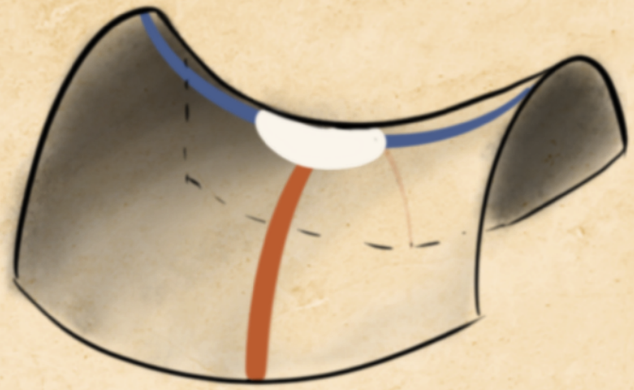
Preimage of circle $\xrightarrow{G}$ stretches according to rate of change of the normal vector

Small highlight ⟷ Large curvature

axes of ellipse

length of axis

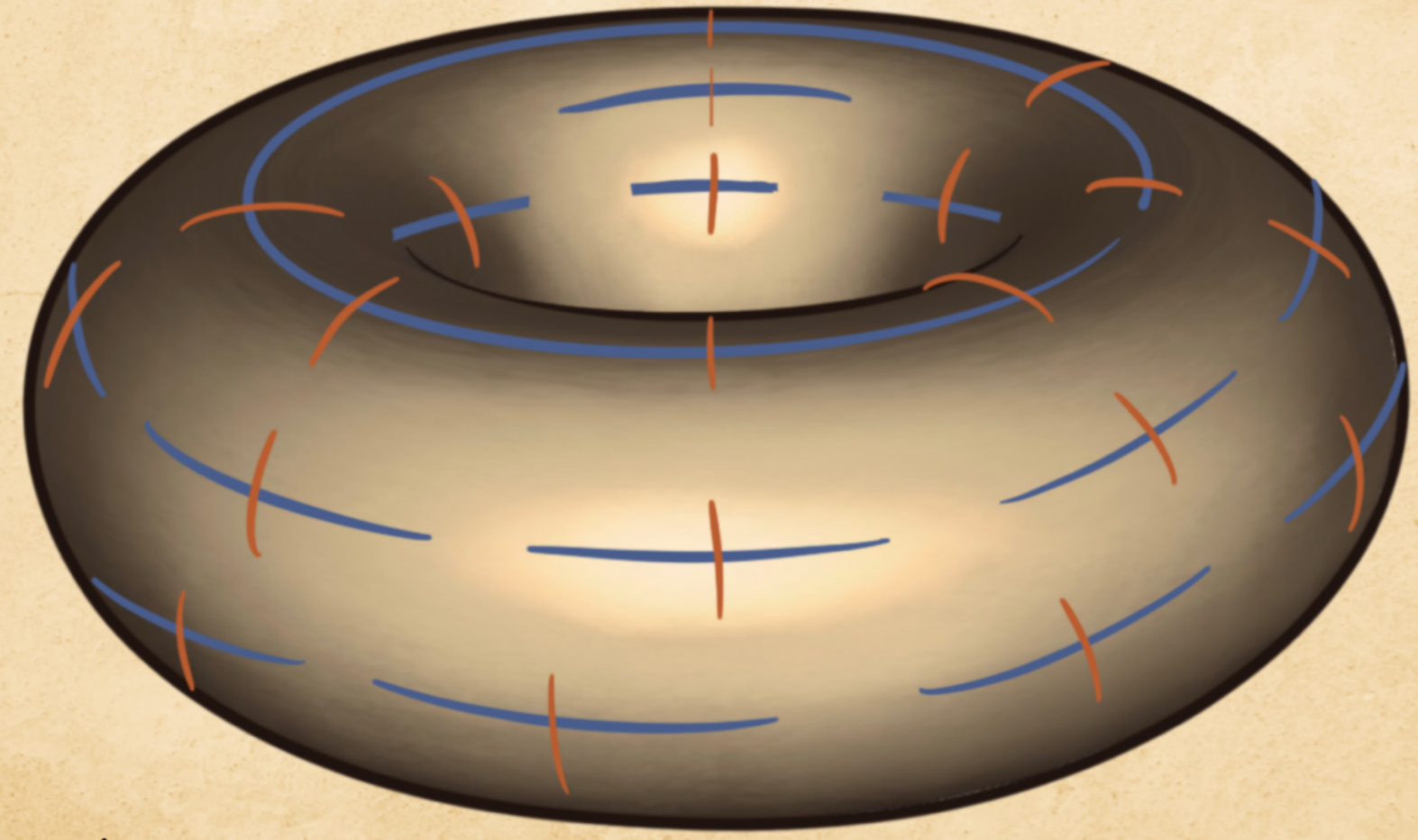
Area of ellipse



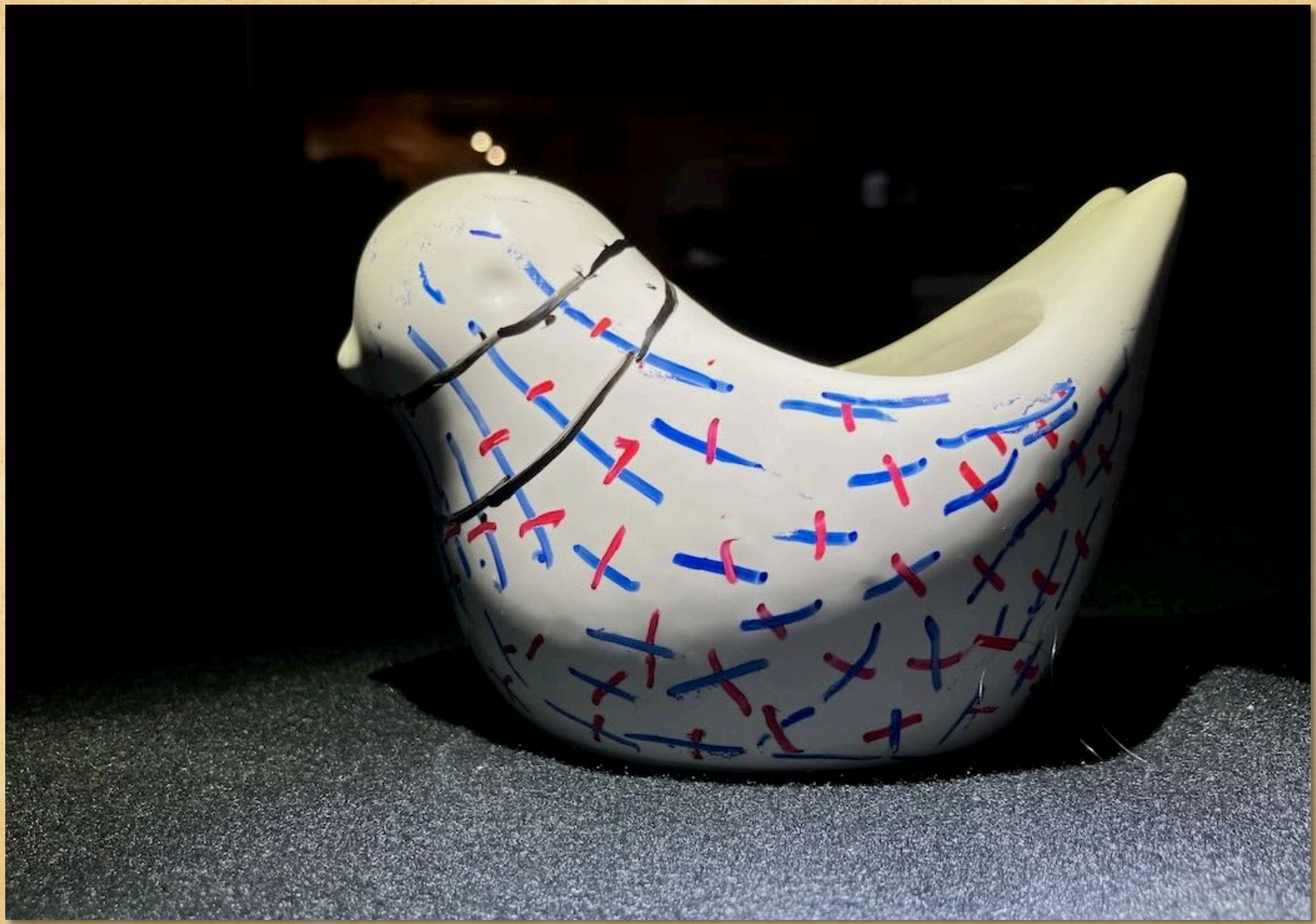
directions of maximal/minimal curvature

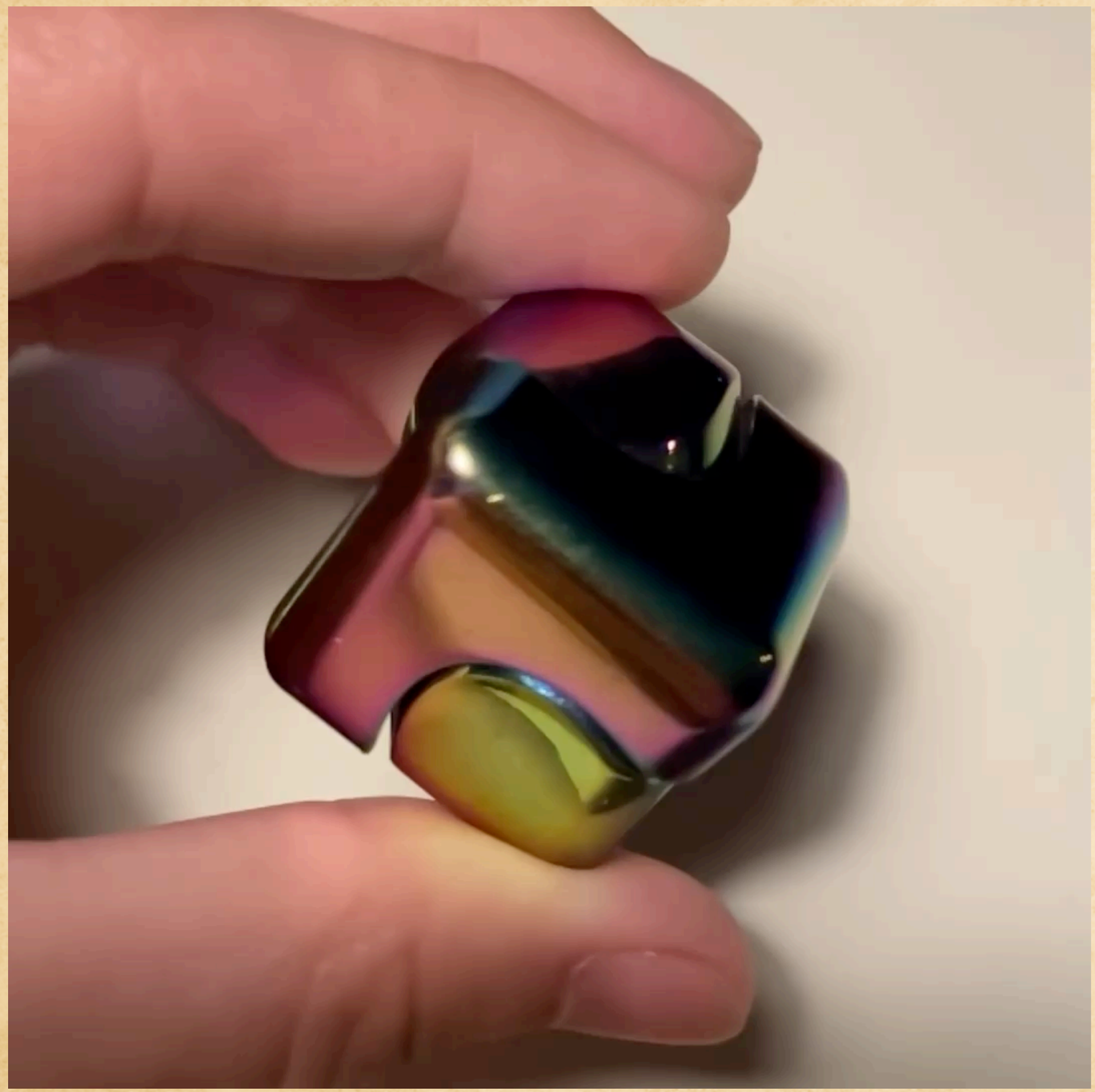
1/principal curvature

1/Gaussian curvature



lines of curvature on torus





Highlight attracted to regions of high curvature

Highlights yearn to be small

Gauss-Bonnet theorem:

the gauss map of a genus g surface is degree $1-g$

$\Rightarrow$ each point on unit sphere has at least $g+1$ pre images

$\Rightarrow$ there are at least $g+1$ highlights!

genus 3 $\Rightarrow$ At least 4 highlights

Mathematical figures

Topology

Geometry

Shading is Geometry

Why I Draw A Torus