

# Manifesting Mathematical worlds In digital art

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# How can you visualize things which don't fit on a page?

Here are some regimes:

## **Literal:**

How do you draw a high dimensional manifold?

## **Abstract:**

How can you put a face on a mathematical abstraction

## **Cartographic:**

How do you map the landscape of a field of math?

**TOP NINE** Tips and Tricks for  
drawing mathematical ideas with  
pedagogical and emotional content  
(number six will **SHOCK** you!)



# How can you visualize things which don't fit on a page?

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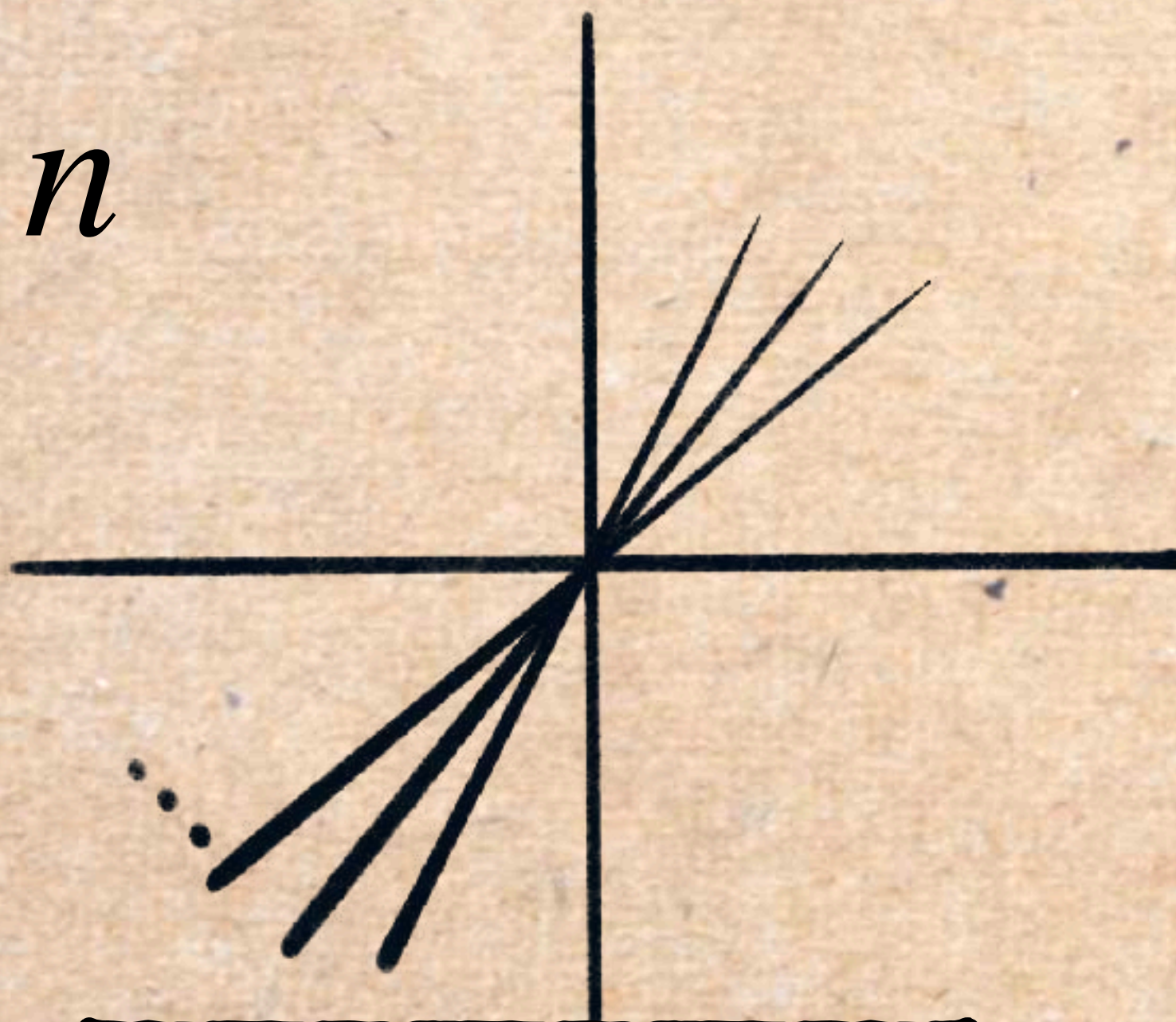
How can you put a face on a mathematical abstraction

## **Cartographic:**

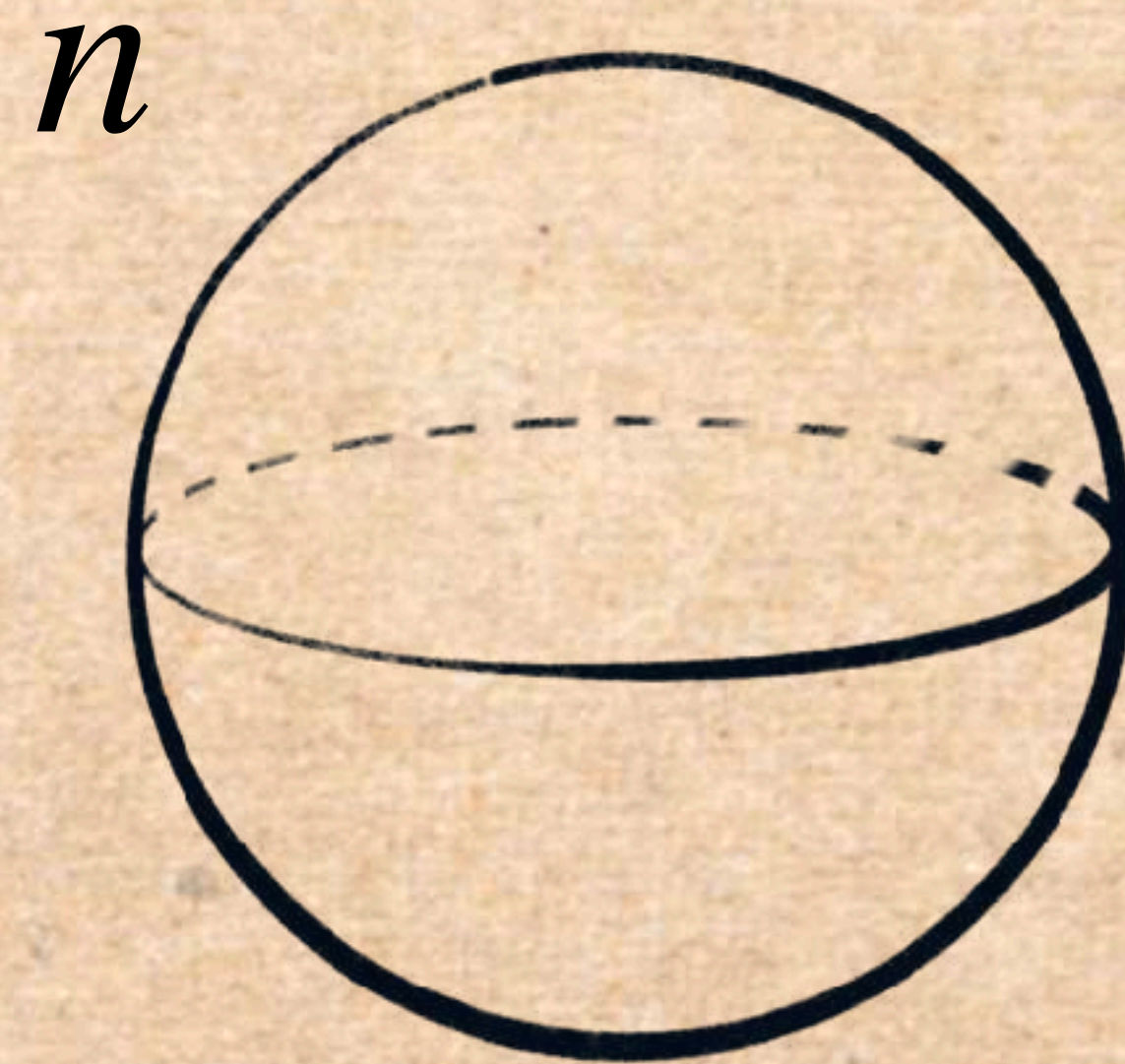
How do you map the landscape of a field of math?

# Which high-dimensional spaces can you draw?

These spaces already have shorthand:



Vector space  $\mathbb{R}^n$

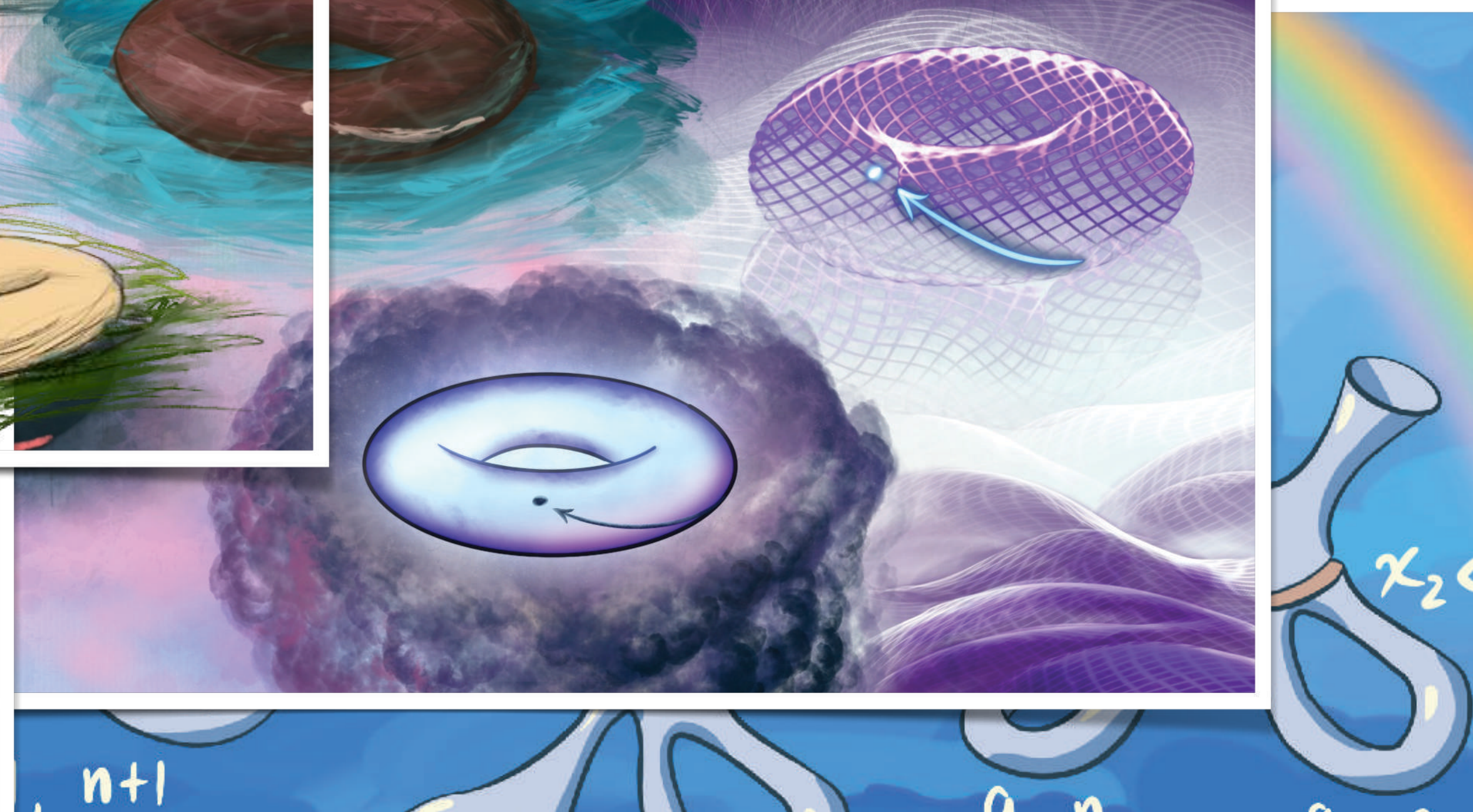
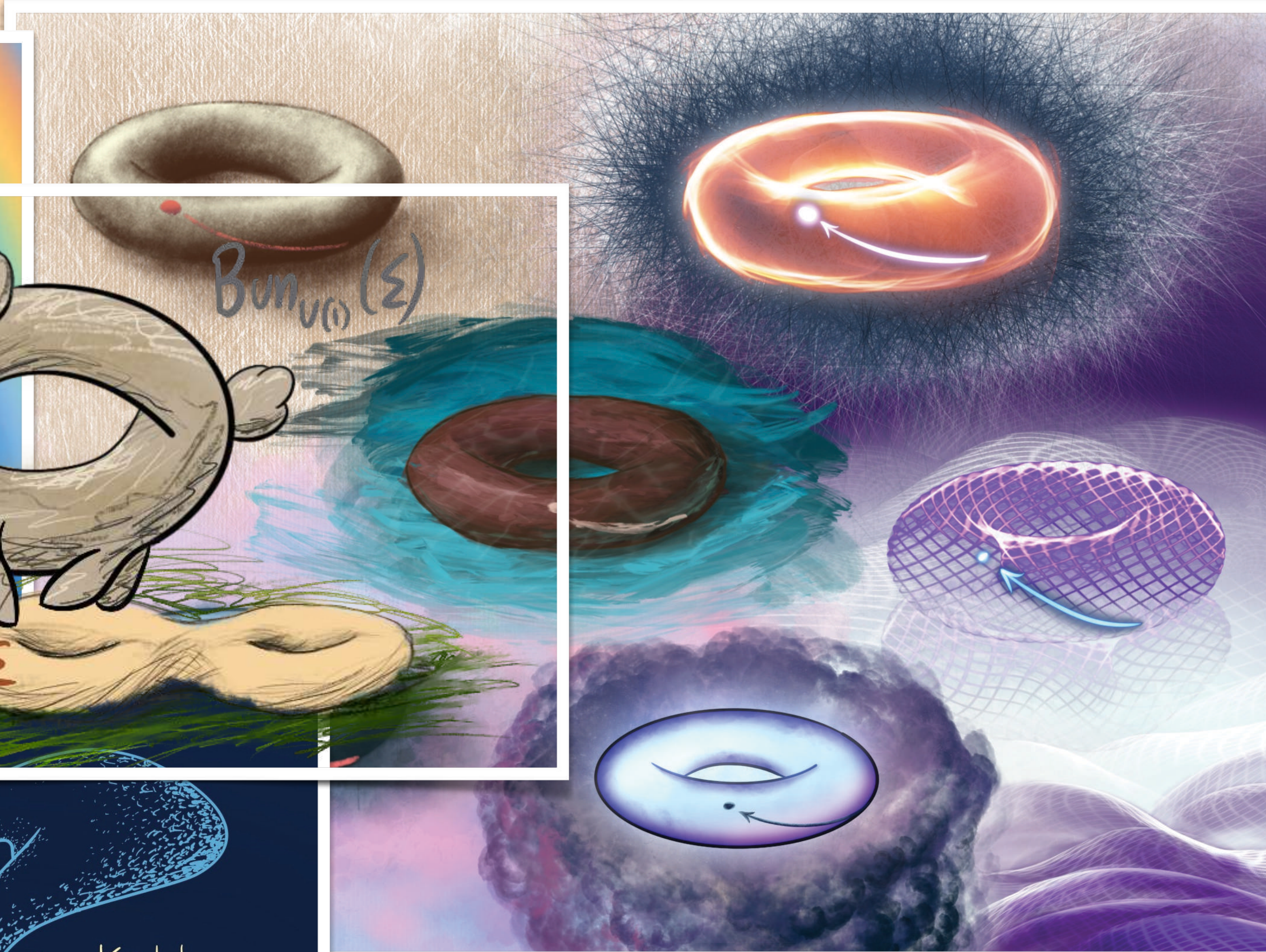
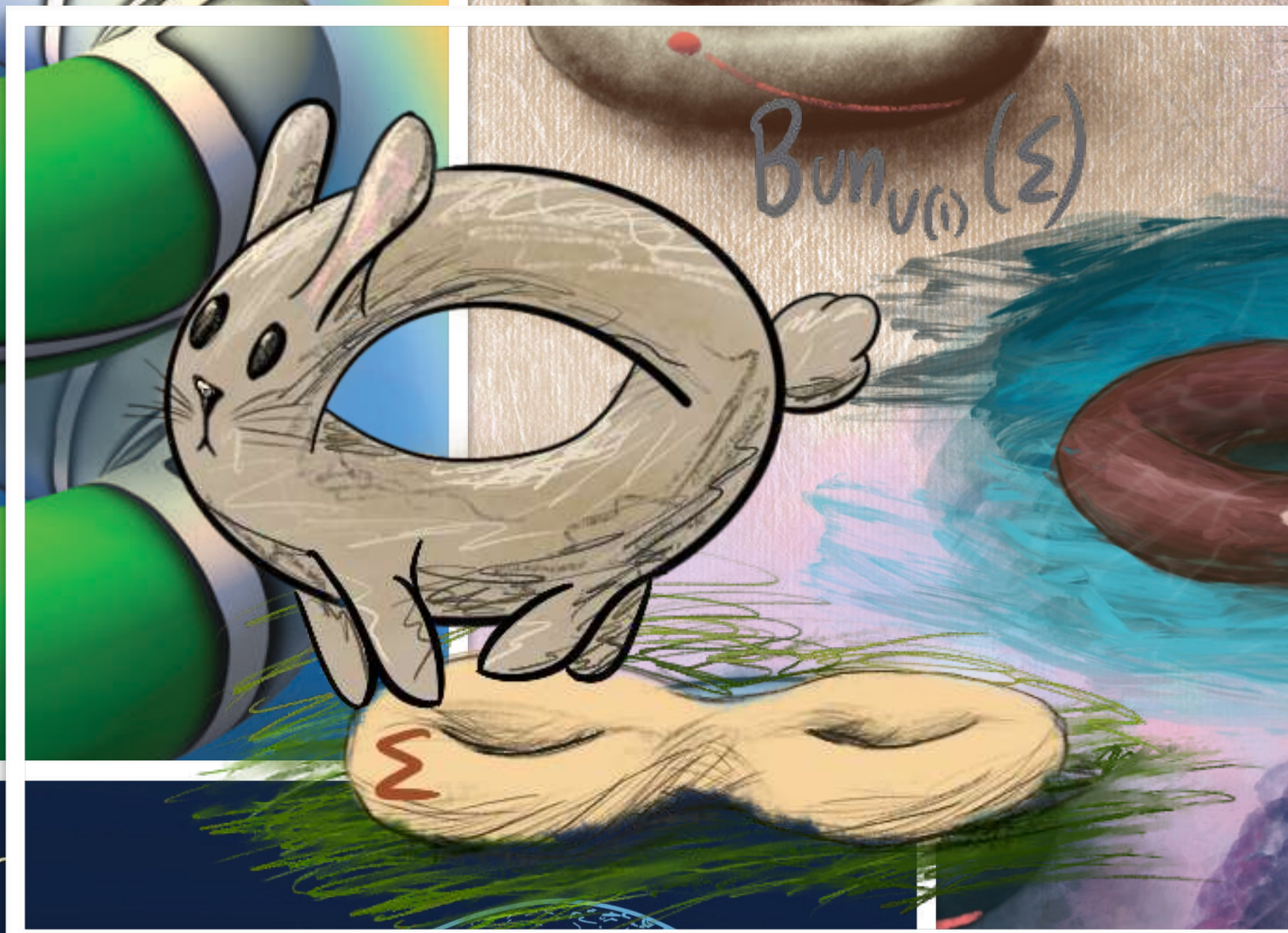
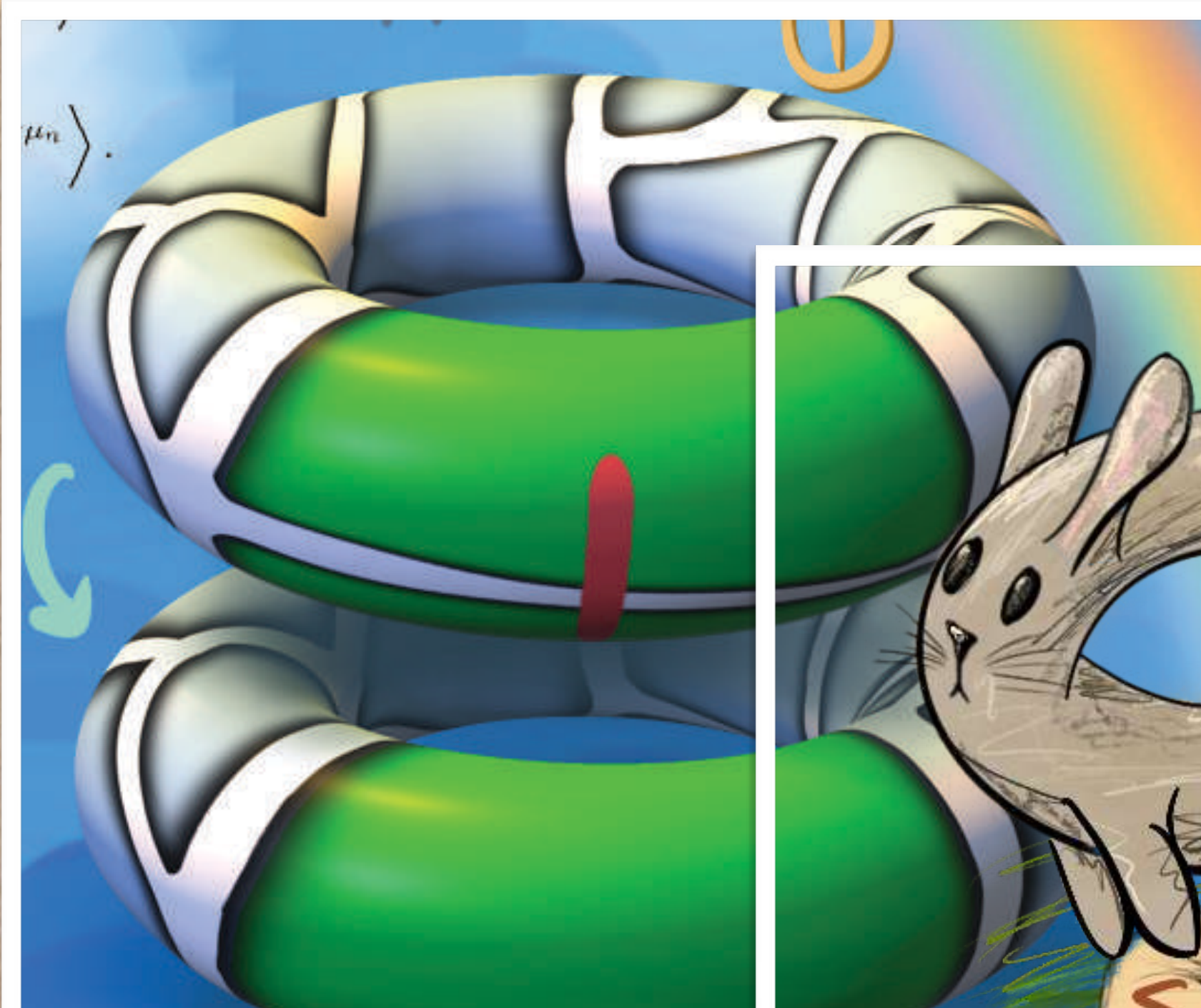


$n$ -sphere



$n$ -torus

# Tip 1: Learn to draw tori



# Tip 2: Slice things up

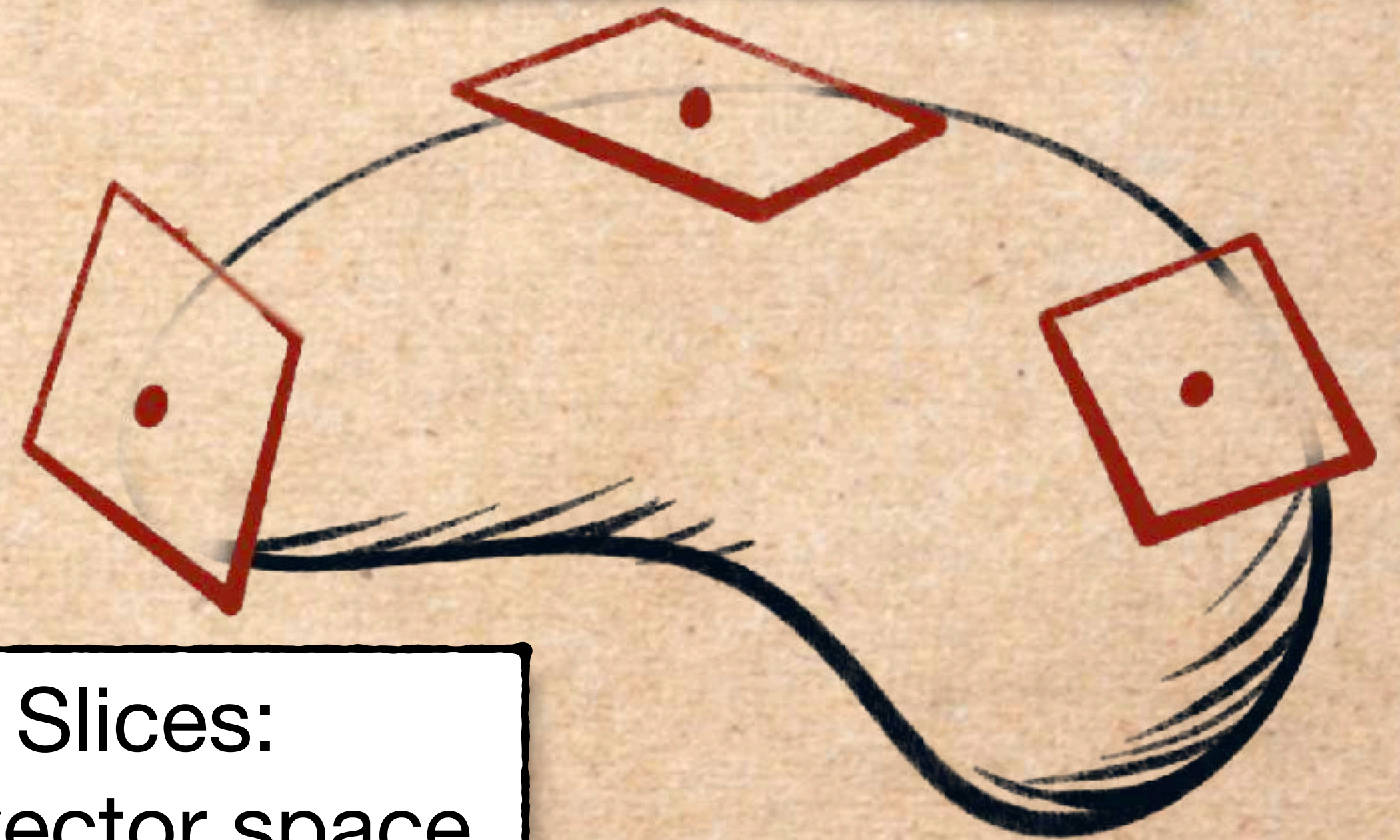
To visualize a high dimensional space:

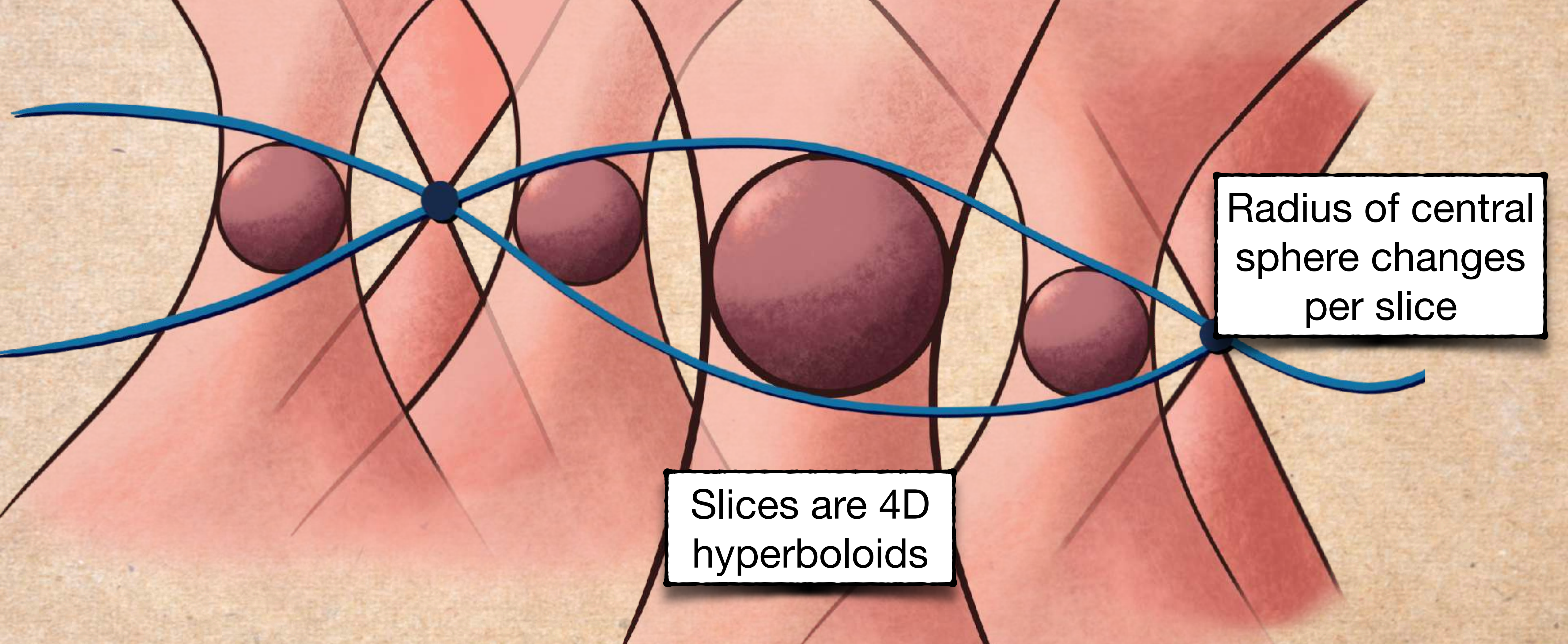
- Draw low-dimensional cross-sections (like an MRI)
- Show how they fit together

Tangent bundle (4D)

Slices:  
2D vector space

Base:  
2D manifold

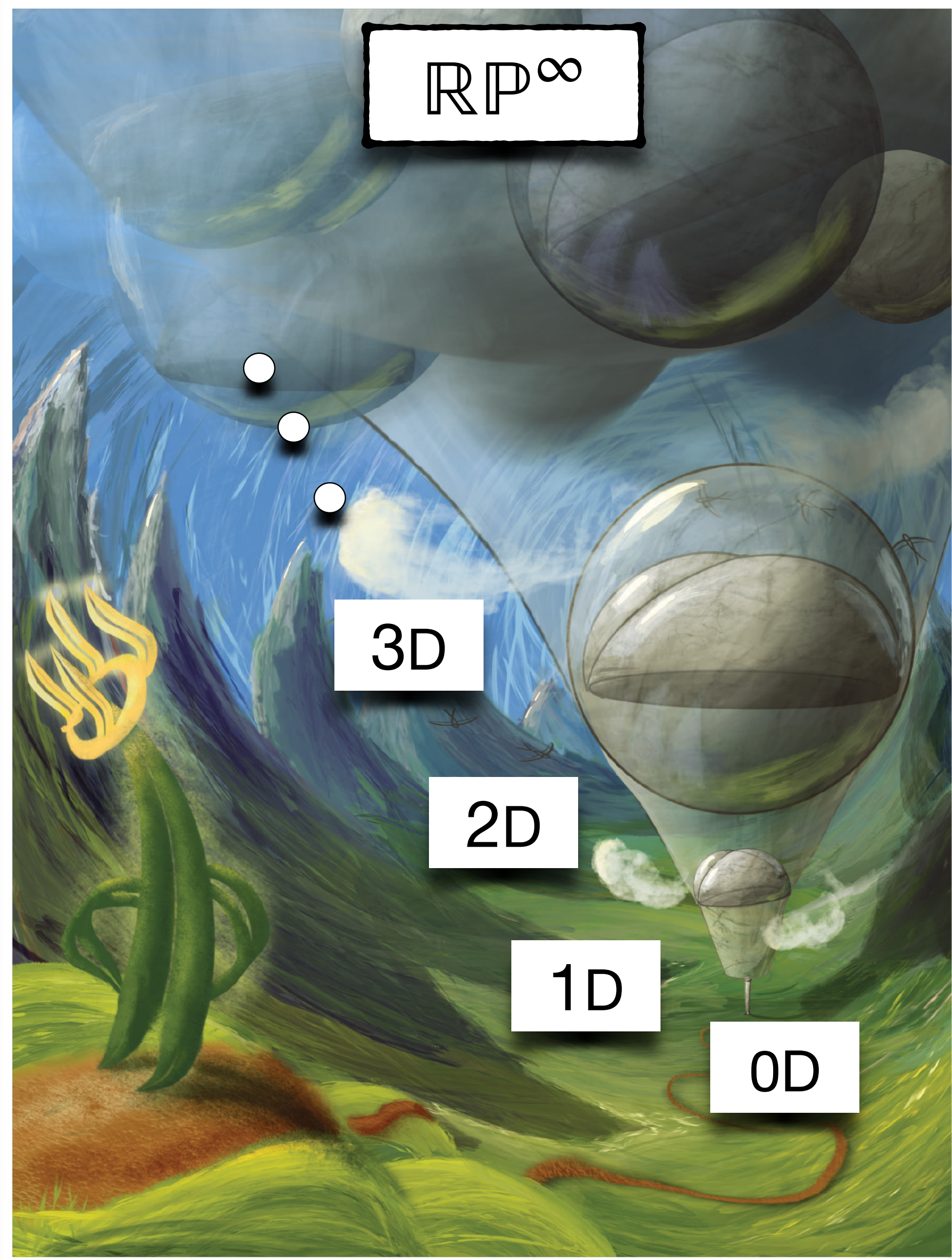




Radius of central sphere changes per slice

Slices are 4D hyperboloids

Slicing is *mathematically significant*,  
Mathematics and visualizations both seek to understand



Interpretive  
Disappears into infinite  
dimensions



Literal slicing  
(*Cell decomposition*)

# Tip 3: Slice things semantically

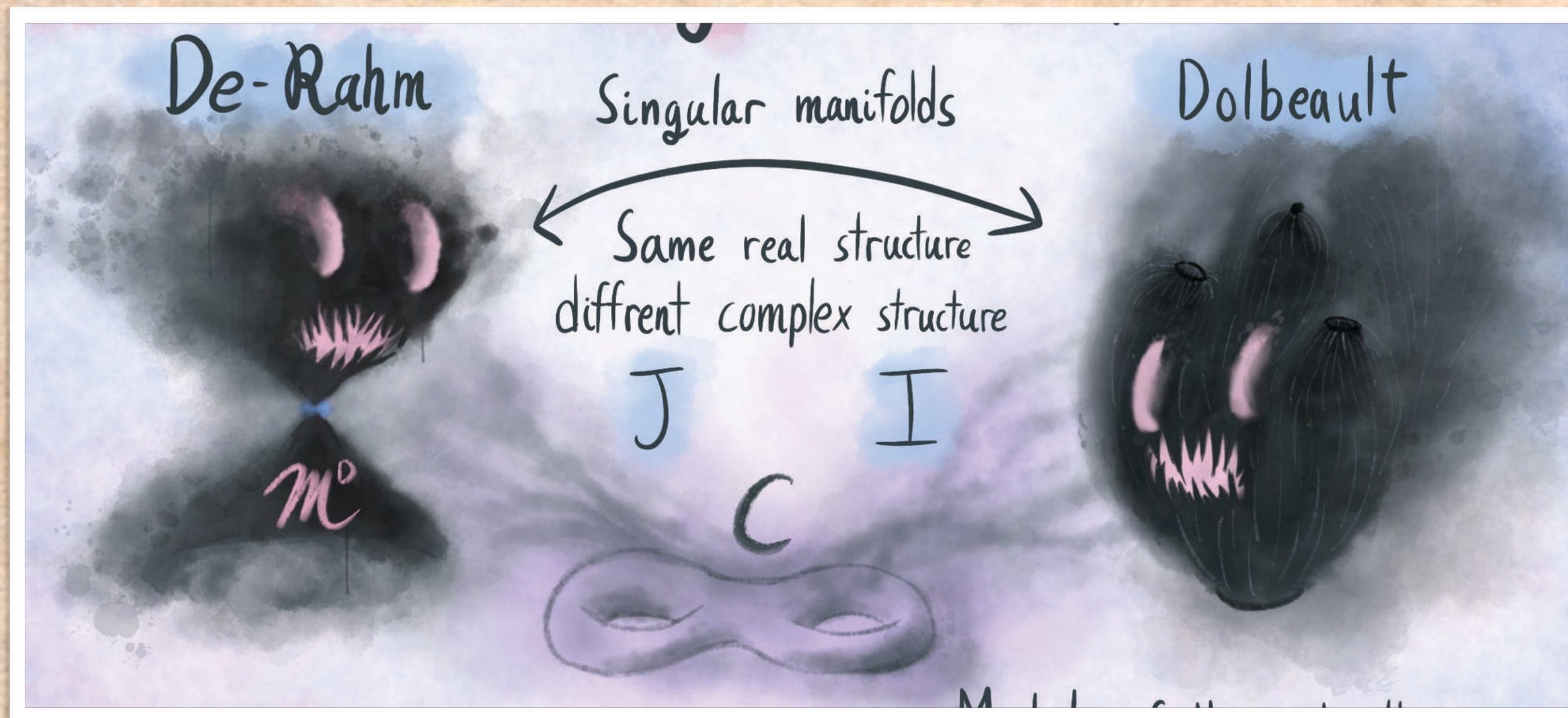
Hitchin moduli space

- Mathematical objects contain multitudes of information
- You can legibly represent 1 or 2 things at once
- **Have different pictures to visualize different data**



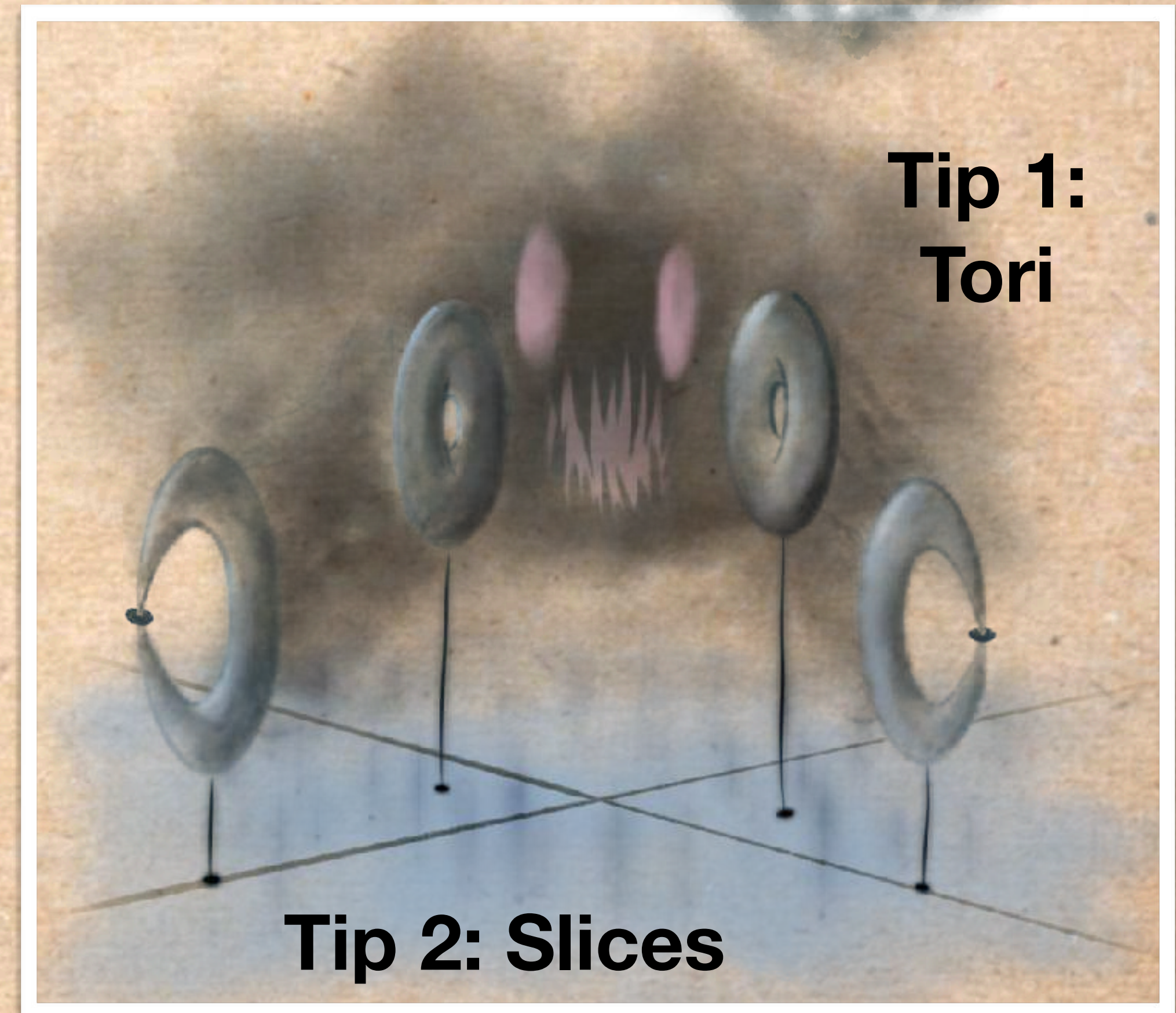
# Hitchin moduli space

She is a shapeshifter, taking many forms...



An algebraic variety

An analytic manifold



A fibration

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# Tip 4: Add eyes

- Hijacks human emotional and social understanding
- Provides layer of emotional interpretation
- Lookit those eyes how could you say no



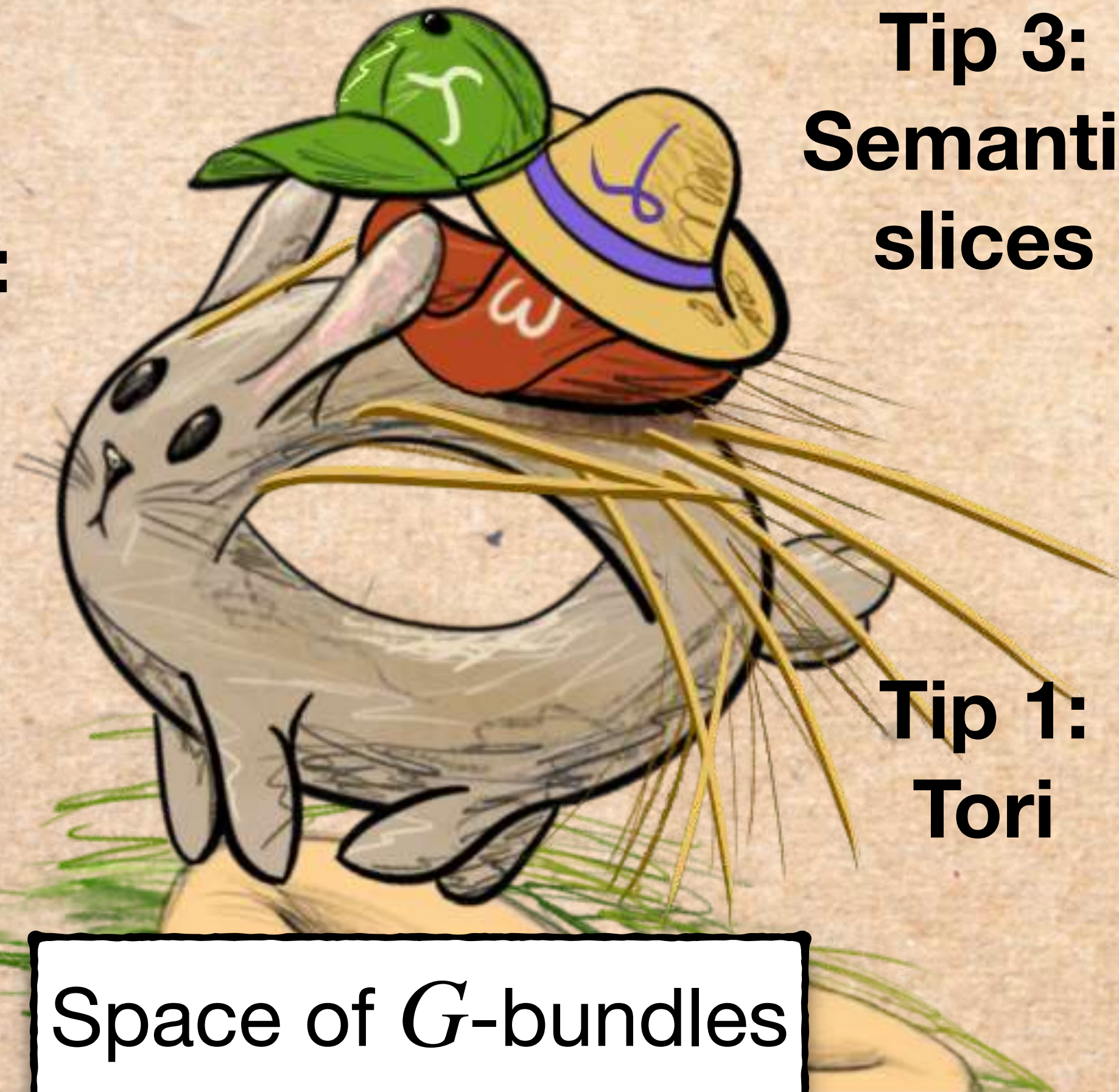
**Hitchin moduli space**



**Desingularized  
Hitchin moduli space**

# Tip 5: Puns!

Tip 4:  
Eyes



Tip 3:  
Semantic  
slices

Tip 1:  
Tori

Space of  $G$ -bundles  
 $\text{Bun}_G$  the Bunny



Tip 4:  
Eyes

Symplectic structure  
(Often denoted  $\omega$ )

# Spectral Curve



Defined by  $F_A + [\Phi, \Phi^\dagger] = 0$



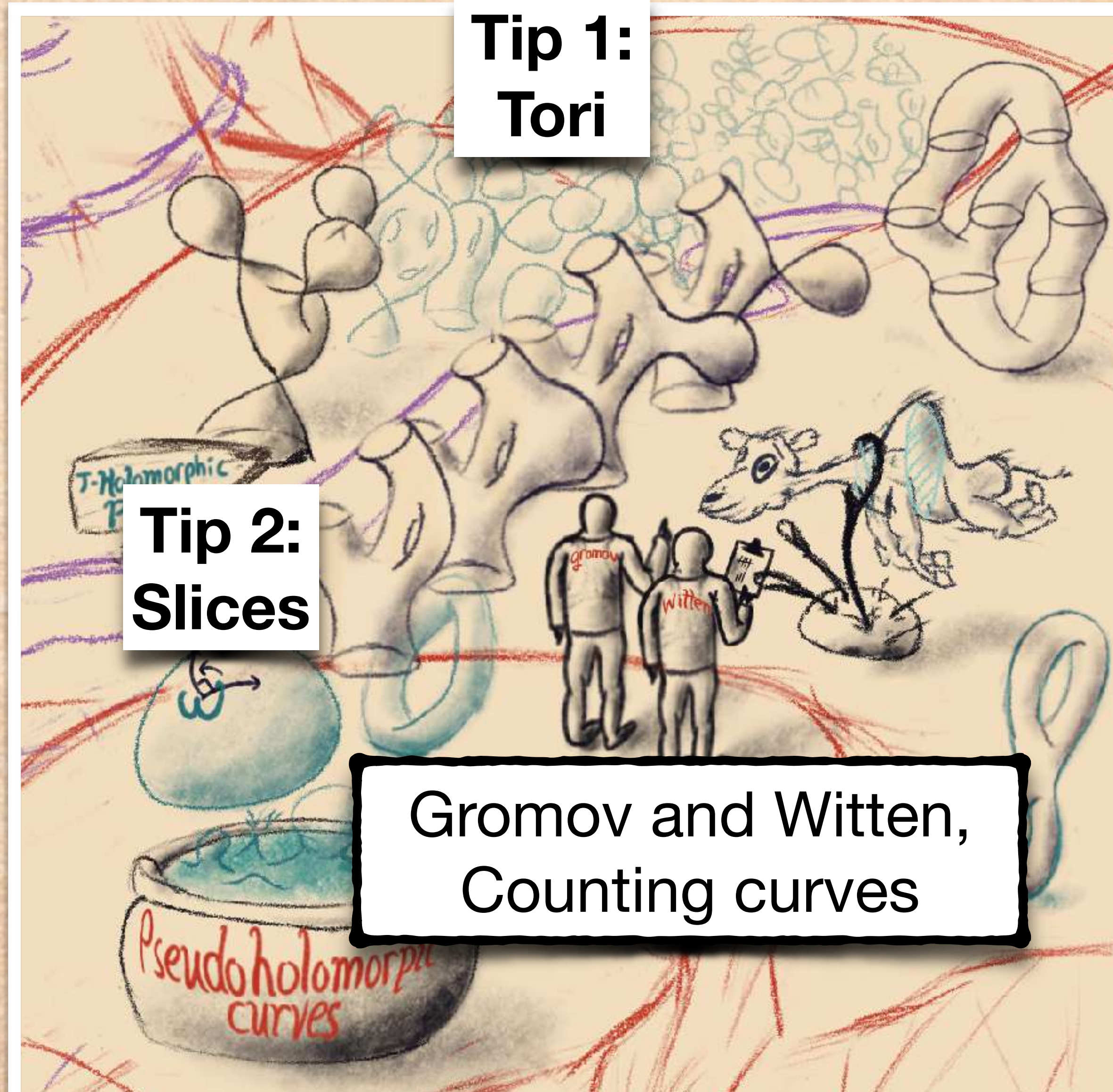
SYZ mirror symmetry

# Gromov-Witten invariants:

**Tip 1:  
Tori**

**Tip 2:  
Slices**

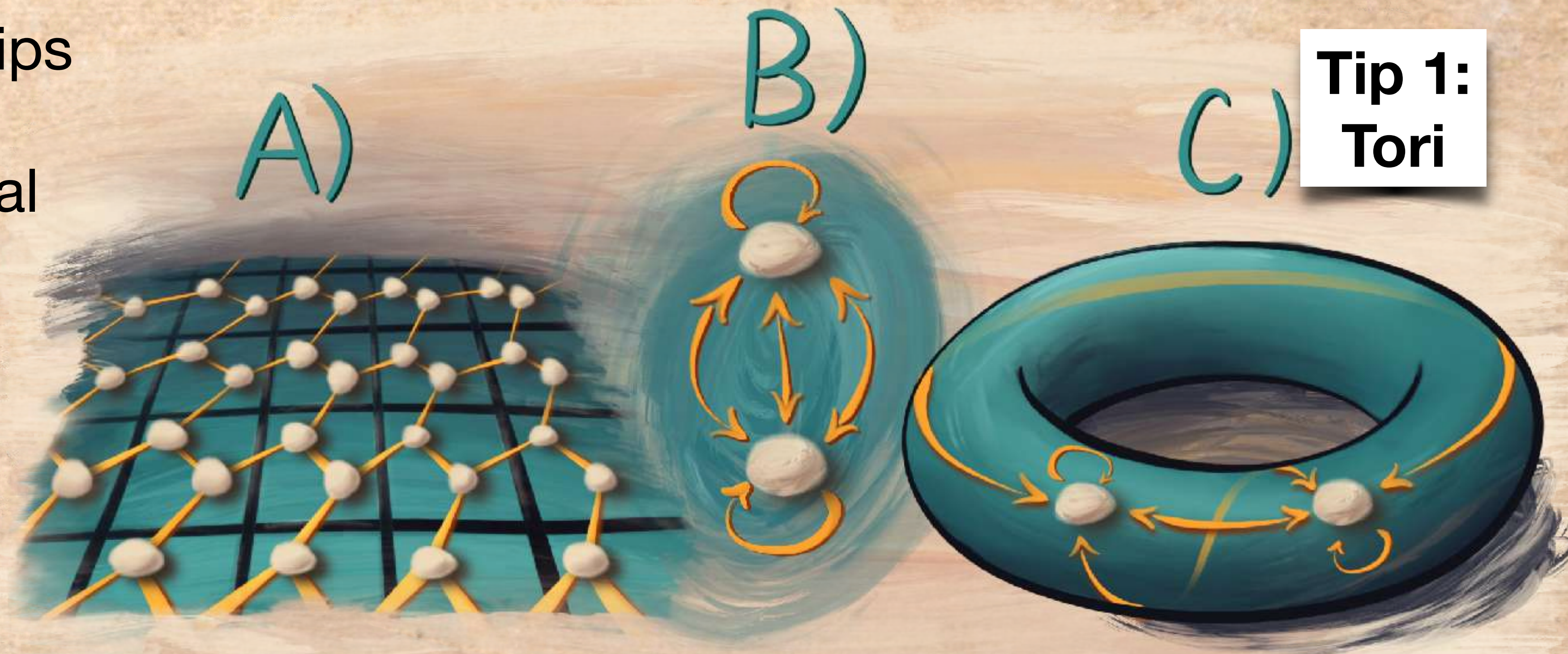
Gromov and Witten,  
Counting curves

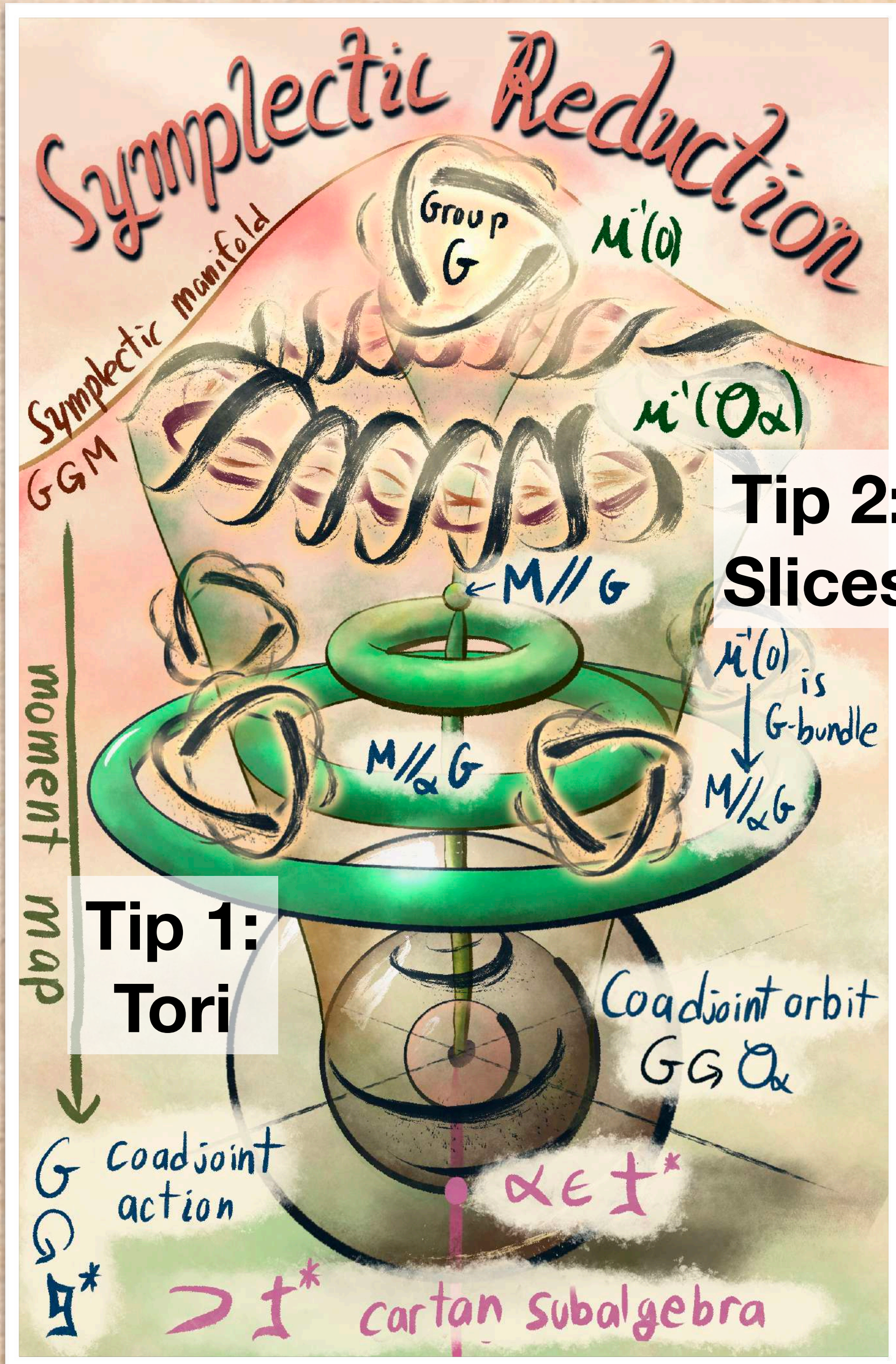


# Tip 6: Build visual rules

- Readable and aesthetically consistent
- Visual similarities communicate semantic relationships
- Hijacks human visual understanding

Example:  
Color coding



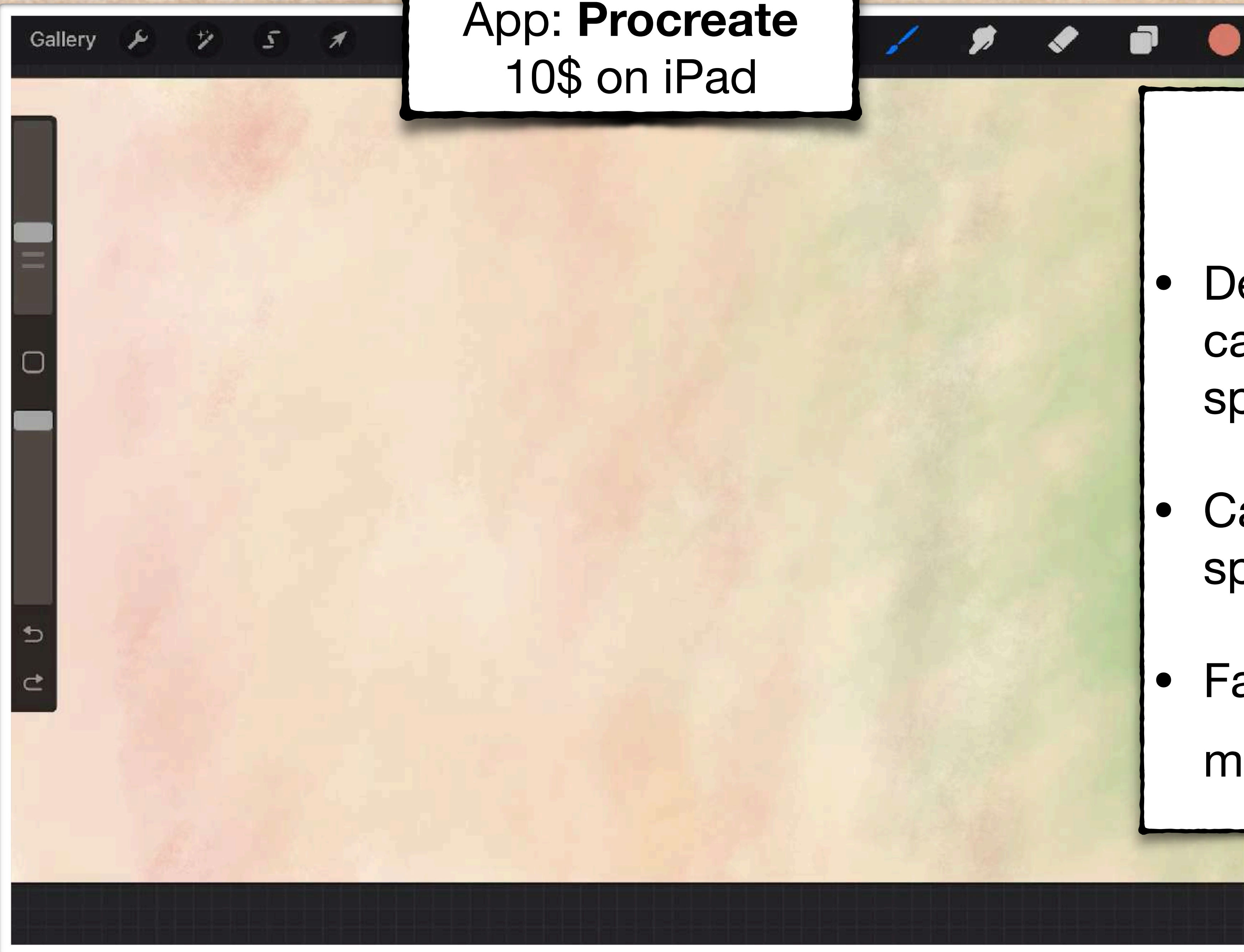


**Brush design:**  
 Black Ink represents group actions



The ink brush is kinetic, suggesting motion along the group action

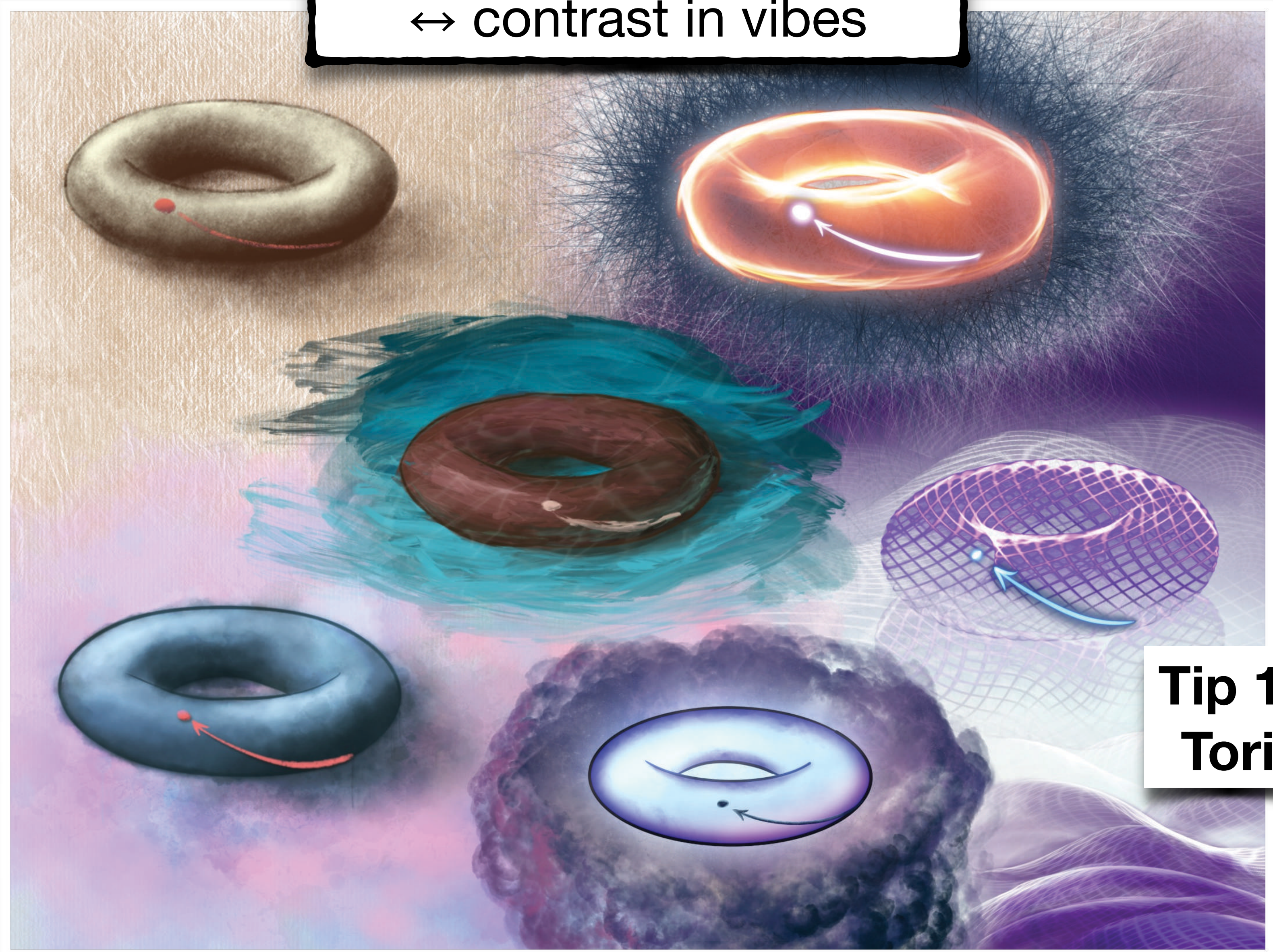
App: **Procreate**  
10\$ on iPad



## Ink brush

- Designed by combining calligraphy brush with splotch brush
- Calligraphy follows pen, splotches lag behind
- Faster pen motion ↔ more splotch separation

Contrast in visual design  
↔ contrast in vibes



**Tip 1:**  
**Tori**

# Tip 7: Use Cliches

The Principle of the symplectic camel

- Cultural shorthand quickly communicates lots of data
- Hijacks human cultural understanding



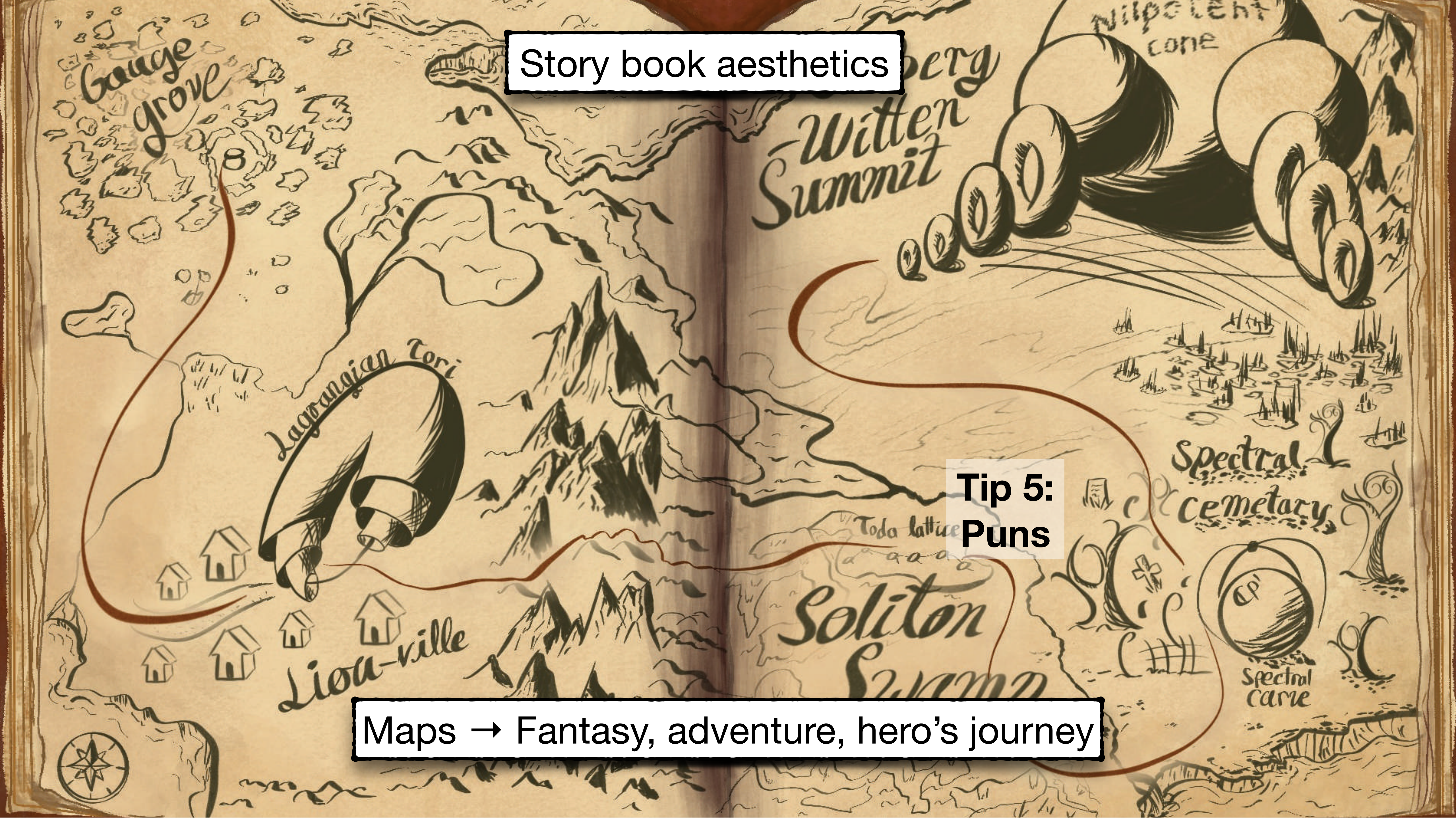
**Tip 4:  
Eyes**

“it is easier for a camel to go through the eye of A needle than for a rich person to enter the Kingdom of God” - Matthew 19:24

Story book aesthetics

Maps → Fantasy, adventure, hero's journey

Tip 5:  
Puns



— Nilpotent cone —  
Castle on dark and stormy night

**Tip 1:**  
**Tori, Spheres, and vector spaces**

Cultural shorthand metatextually  
frames mathematics

**Tip 2:**  
**Slices**

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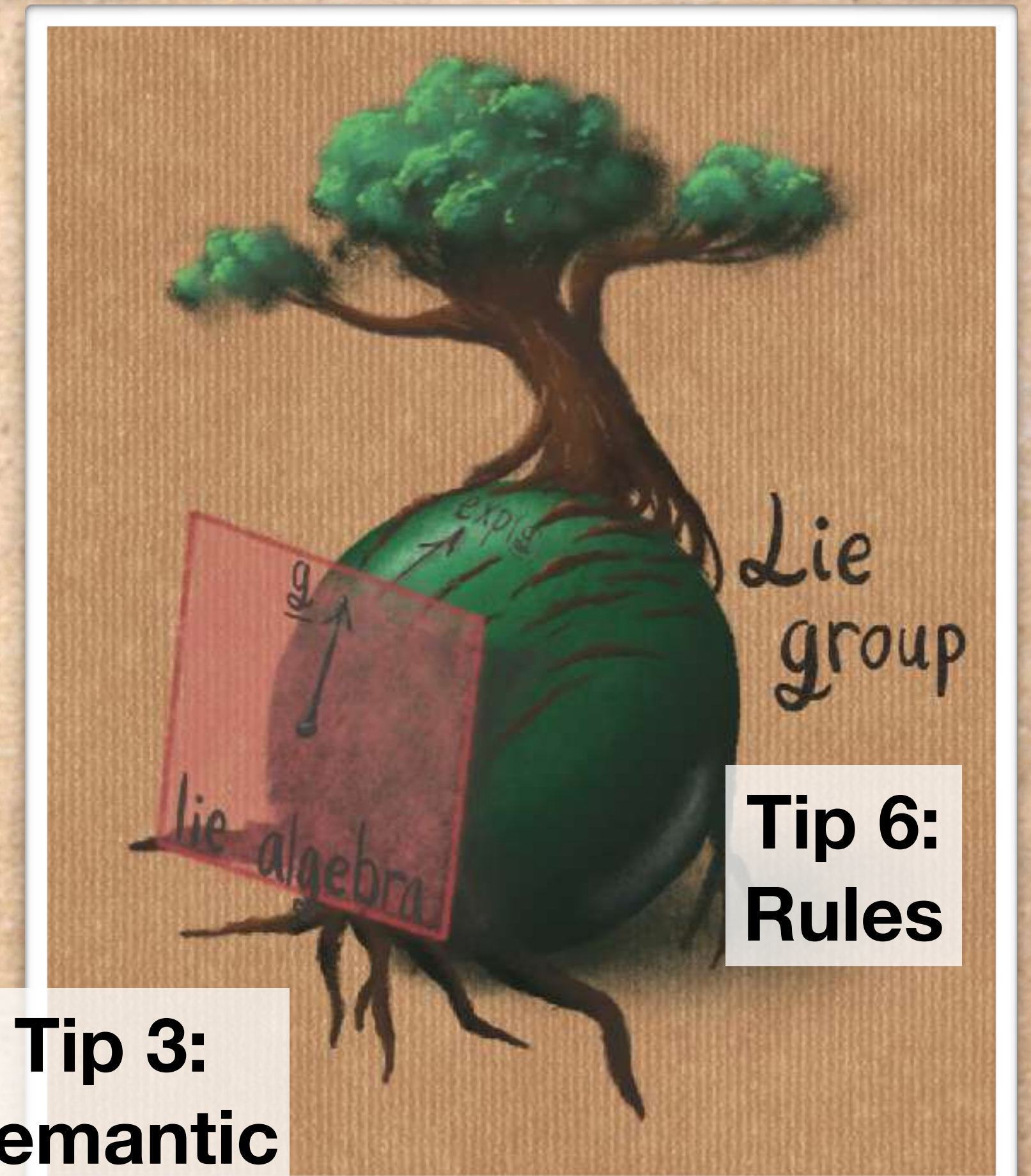
## **Cartographic:**

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# Tip 8: Use real world hierarchies

- Establish relationship between math objects with real world analogies
- Hijacks human worldly understanding

The group is the *Tree*  
The algebra is the *Roots*



**Tip 3:**  
**Semantic**  
**Slices**

**Tip 6:**  
**Rules**

Symplectic forms  
**Live on** Manifold

Symplectic  
geometry (4D)

**Tip 4:**  
**Eyes**

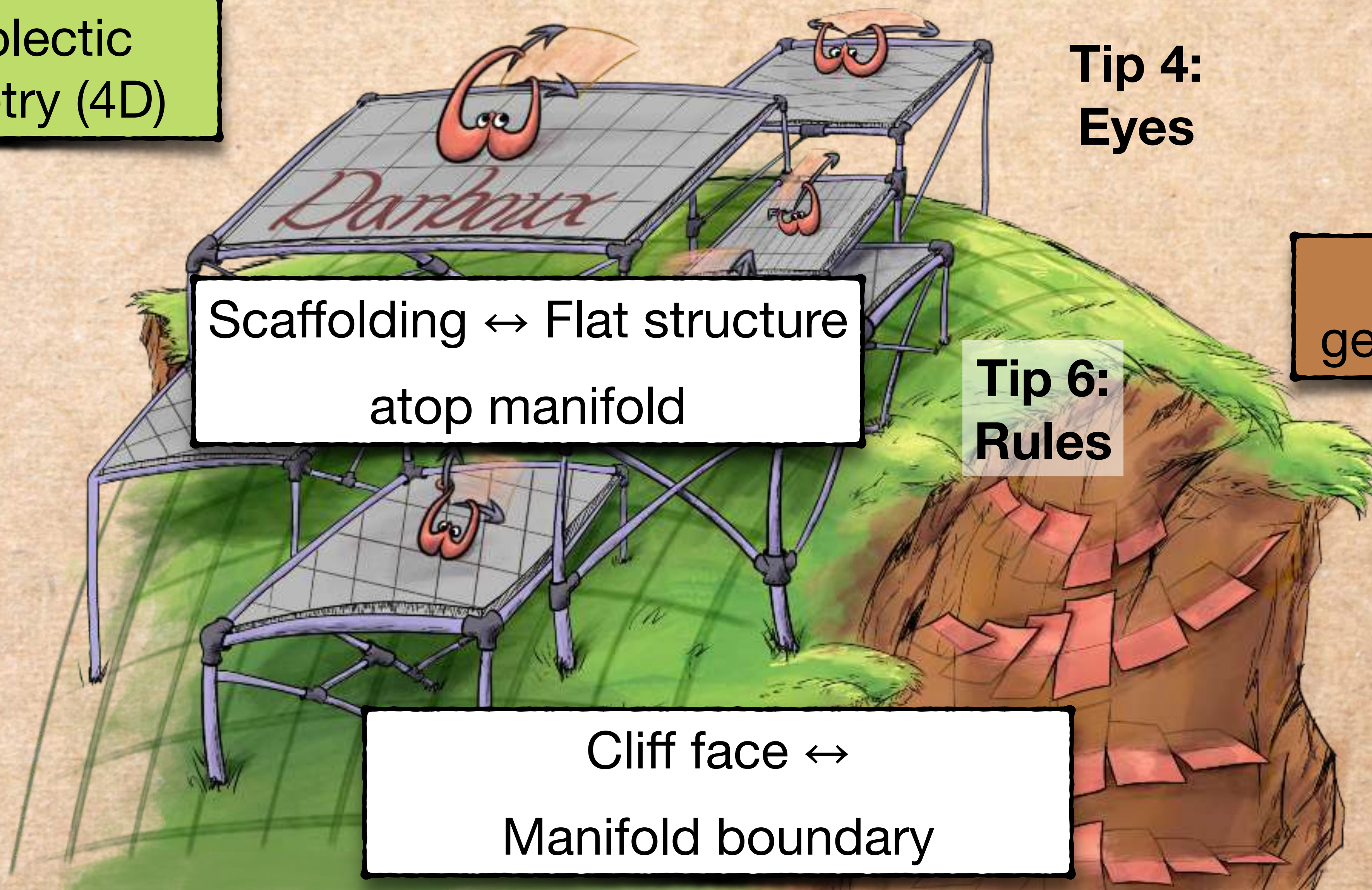
Scaffolding  $\leftrightarrow$  Flat structure  
atop manifold

Contact  
geometry (3D)

**Tip 6:**  
**Rules**

Cliff face  $\leftrightarrow$   
Manifold boundary

**Tip 2:**  
**Slices**



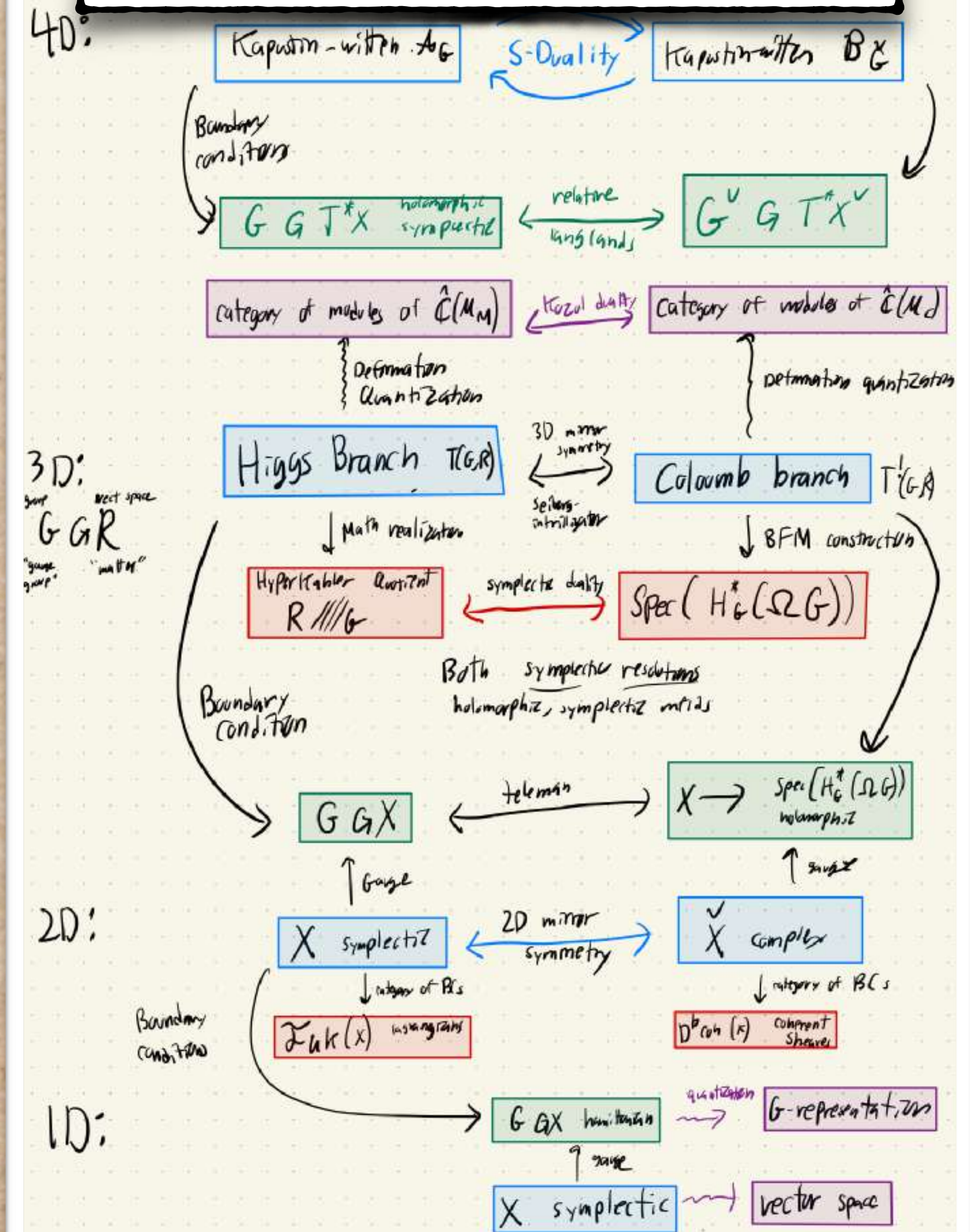
# Case study: The mirror symme-tree



# Original flowchart

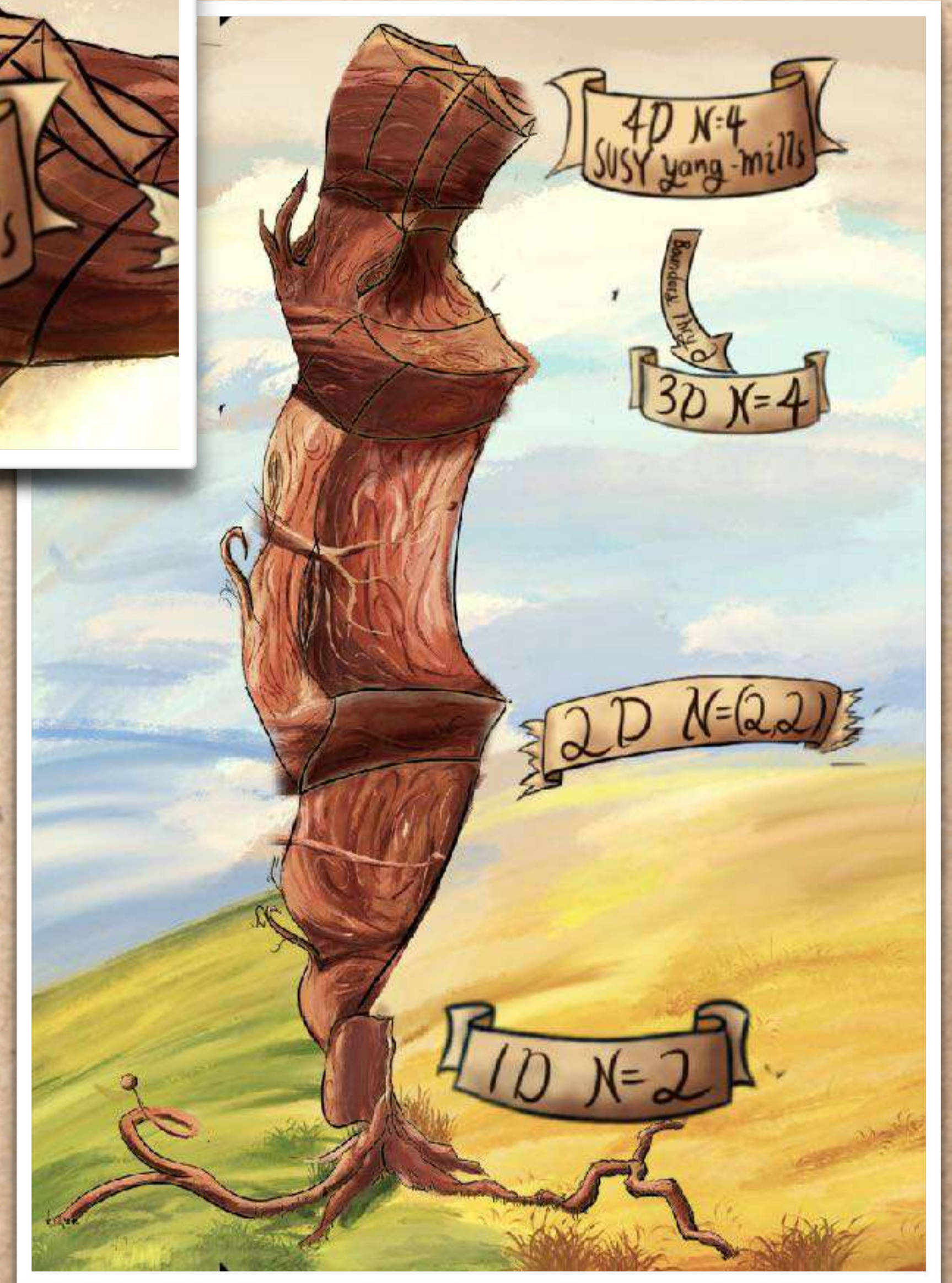
- Started with a flowchart:
- Explains the structures of various types of supersymmetric quantum field theory
- All related by mirror symmetry, envisioned by a the *mirror Symme-tree*

Tip 5: Puns!



## Tip 8: Hierarchies

- Tree is supported by its trunk
- At top, 4D theory on 4D trunk
- Each layer, the trunk decreases in dimension at the boundary
- Each theory is induced from the one above as a *boundary theory*





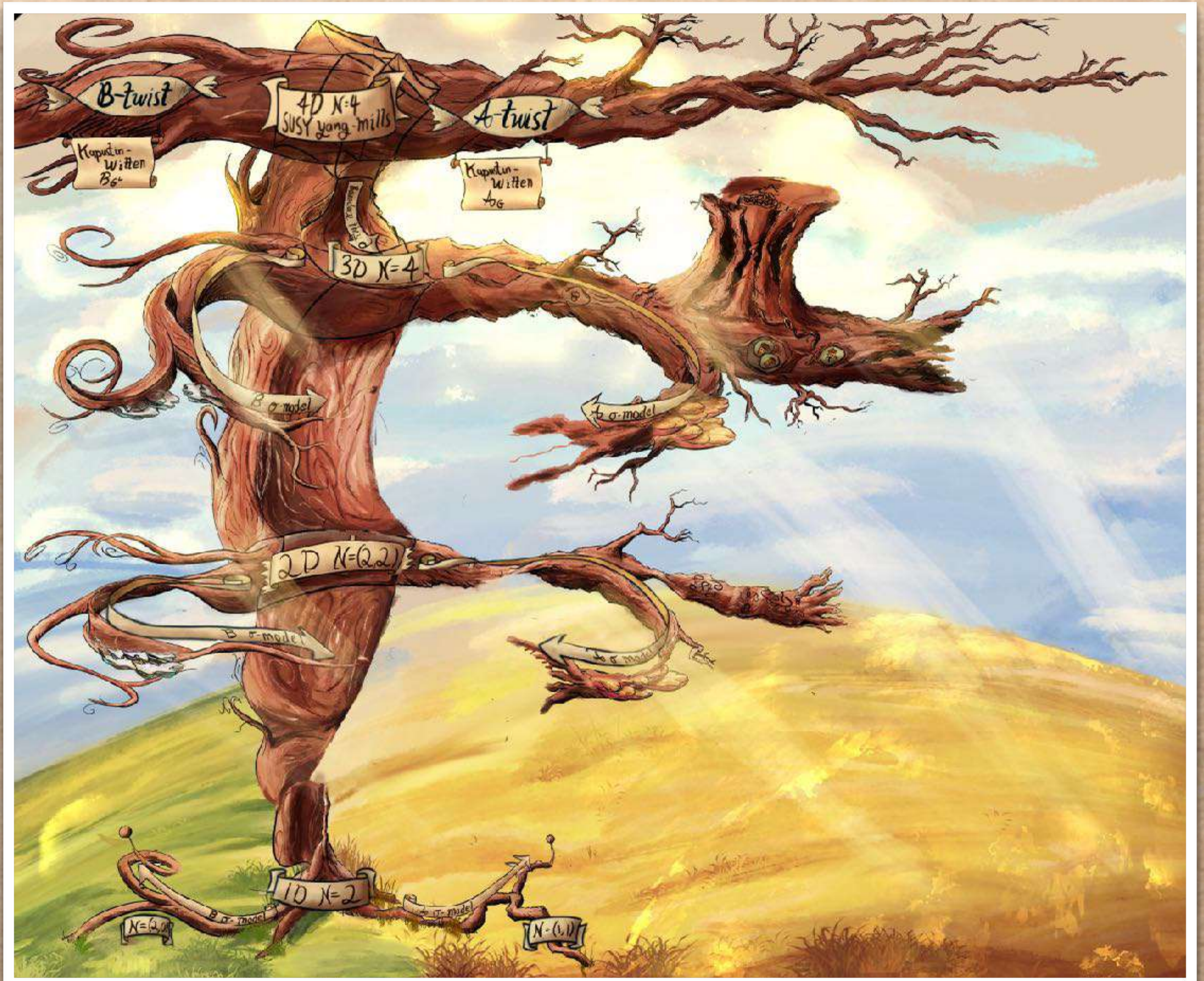
- The 4D theory spawns two “twisted” theories
- Related by mirror symmetry
- Each side has different aesthetics

Tip 6: Build  
Visual Rules

Every lower theory has two branches, related by mirror symmetry

In 3D: Coulomb branch and Higgs branch

Tip 5: Puns!



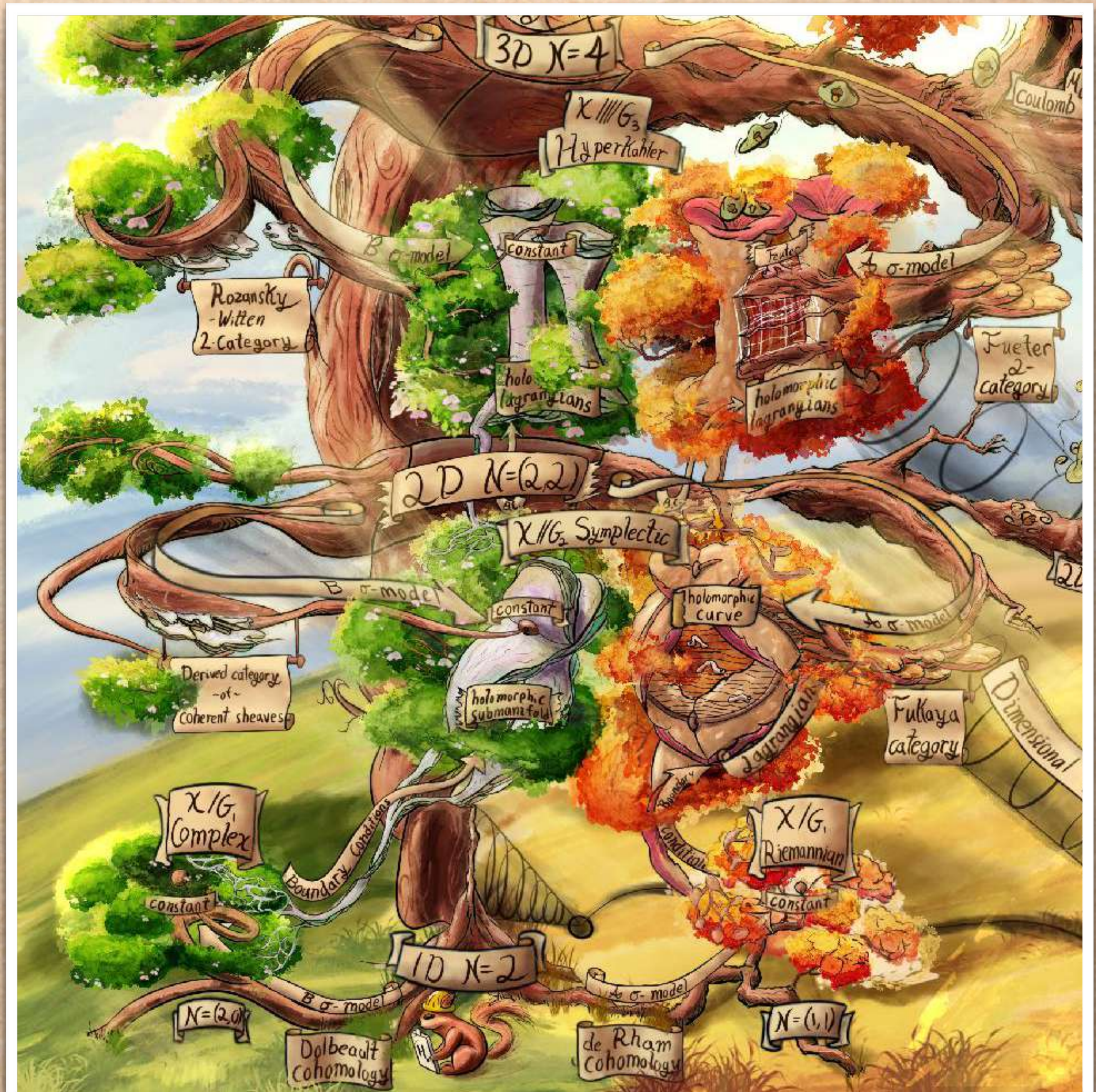
- **Math:** Each twist carries spacetime into a different target manifold
- **Tree:** Each branch carries the trunk into the leaves
- “Spring” side is mirror to “autumn” side

Tip 7: Use cliches

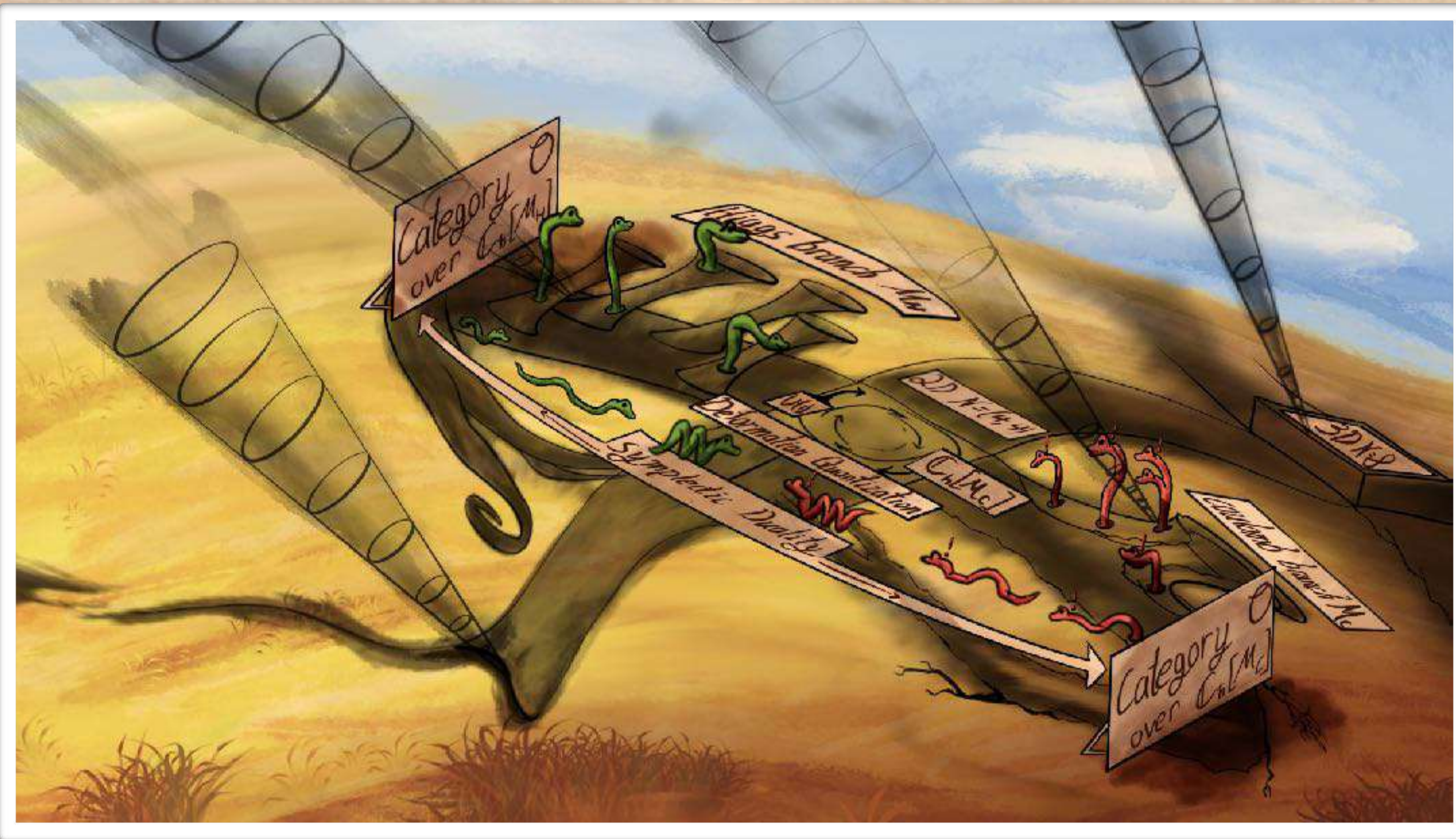


- **Math:** Each target maps into the one above as a “boundary condition”
- **Tree:** Fungi map each leaf blob to the one above
- Target manifests differently in different layers

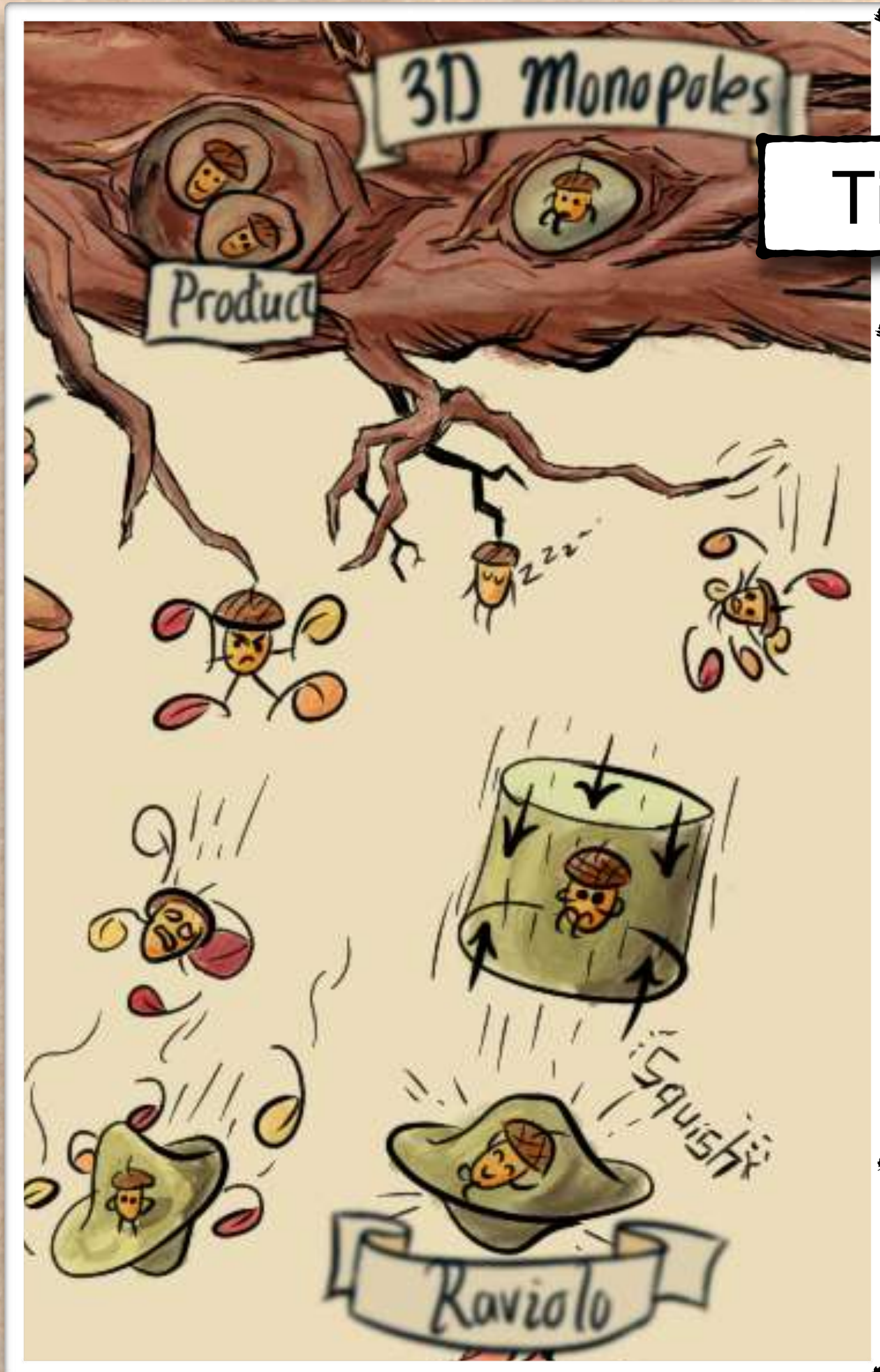
Tip 3: Semantic slices



## Tip 8: Hierarchies



- **Math:** “dimensional reduction” projects theories to lower dimensions
- **Tree:** dimensional reduction is the shadow



Tip 4: Eyes



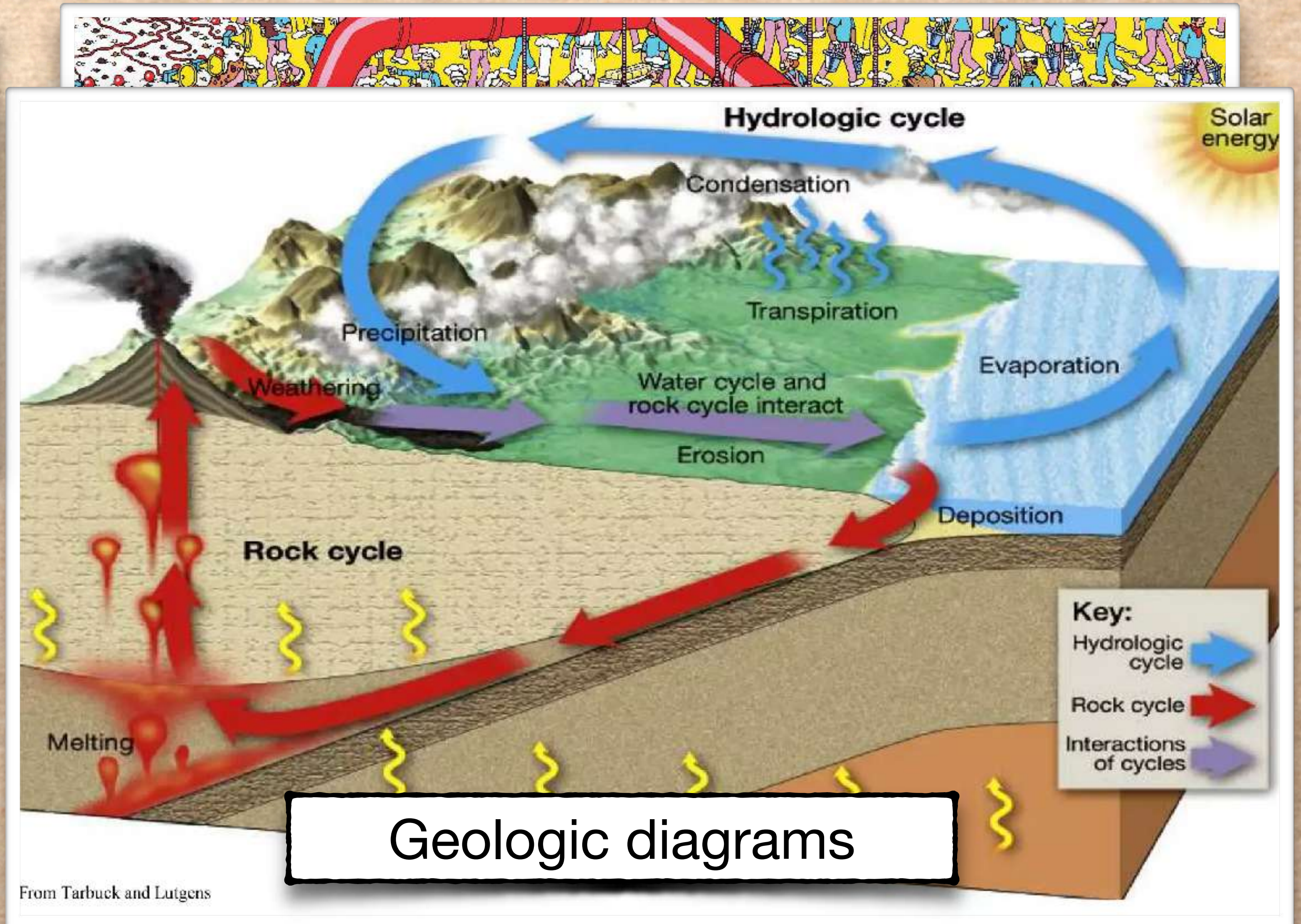
Tip 2: Slices



# Tip 9: Give it life

“Wimmelbilder”  
Large images teeming  
with figures

Math is an world  
And I get to *manifest it!*

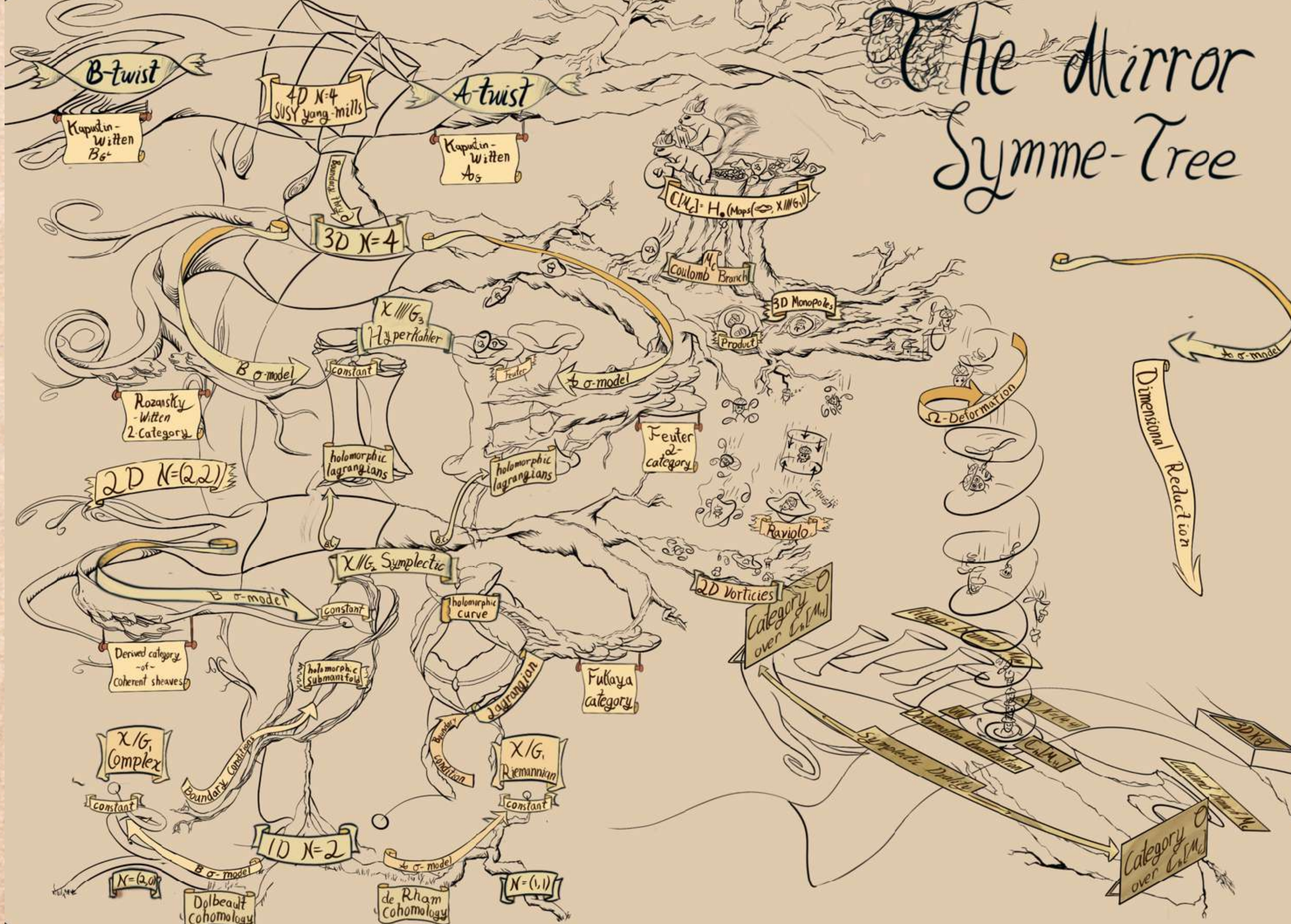


Geologic diagrams

# The Mirror Symmetry Tree

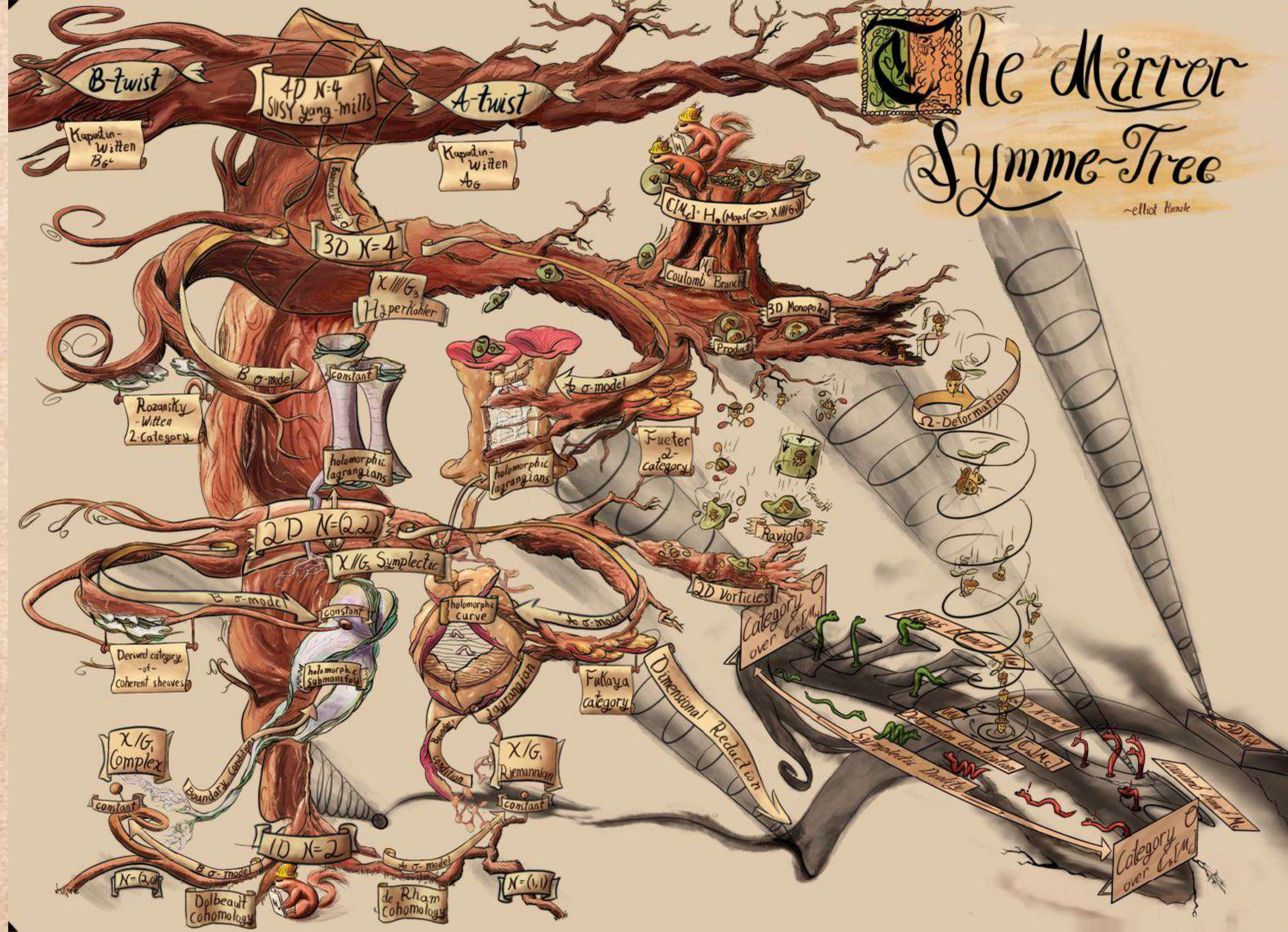


# The Mirror Symmetry Tree



# The Mirror Symmetry Tree

~elliot horton





# The Mirror Symmetry Tree

— Elliot Hore



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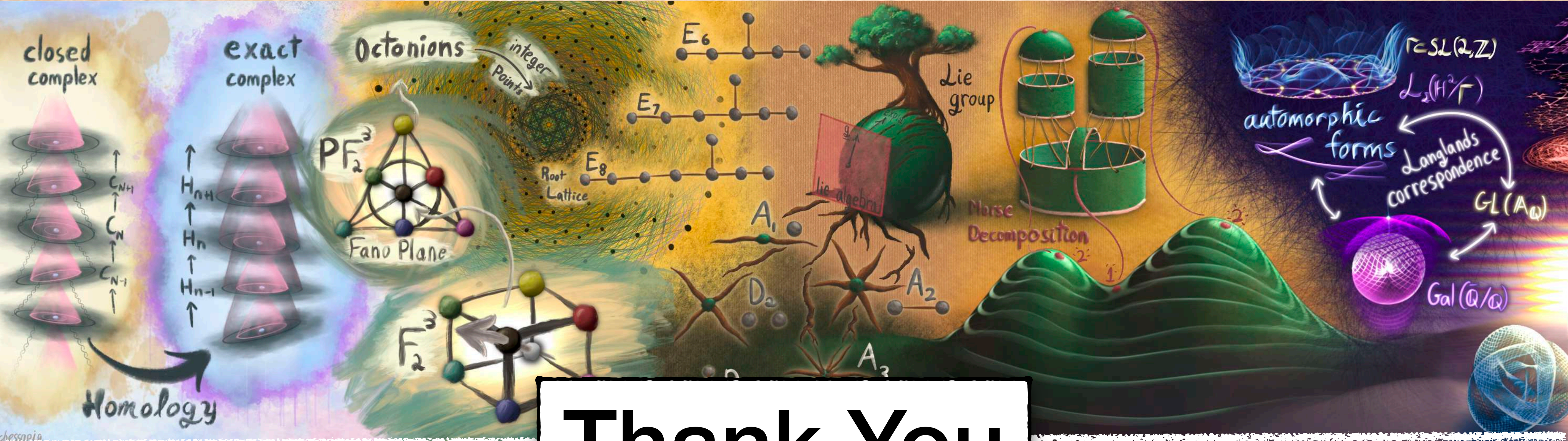


# The Mirror Symmetry Tree

-elliot horton



To me,  
Math glows



**Thank You**

