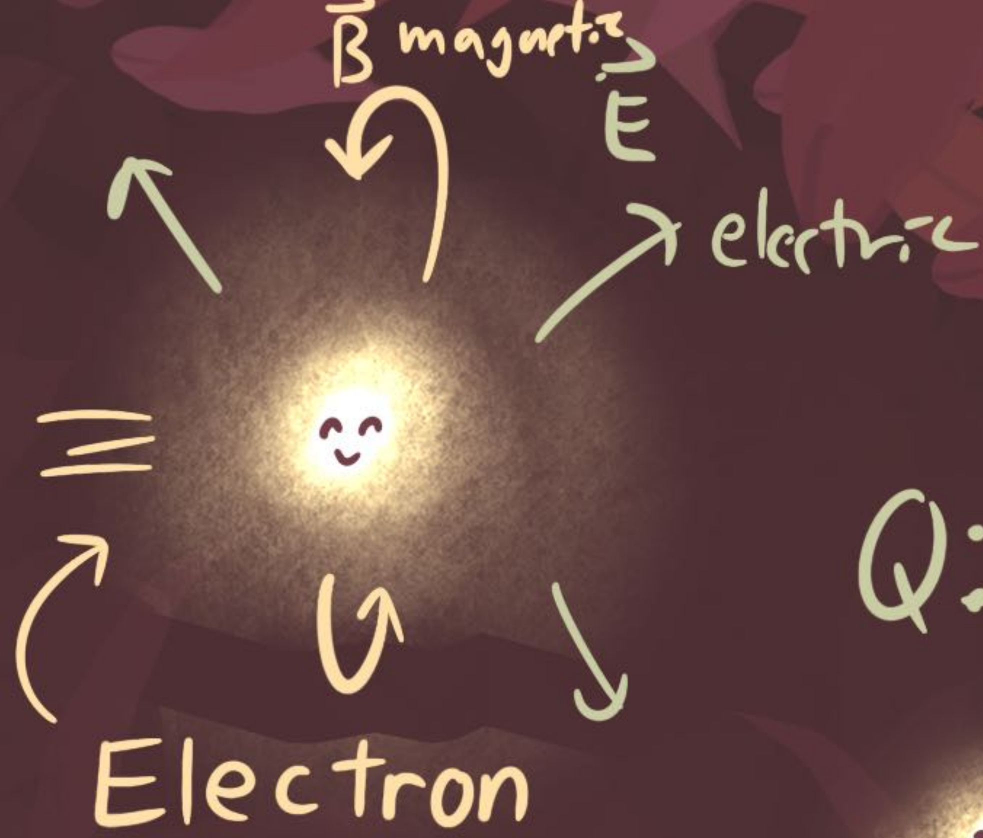
E & M on Riemann surfaces



$\vec{E}$ = force on electron

Q: Riemann surface?? $\vec{E}$? $\vec{B}$?

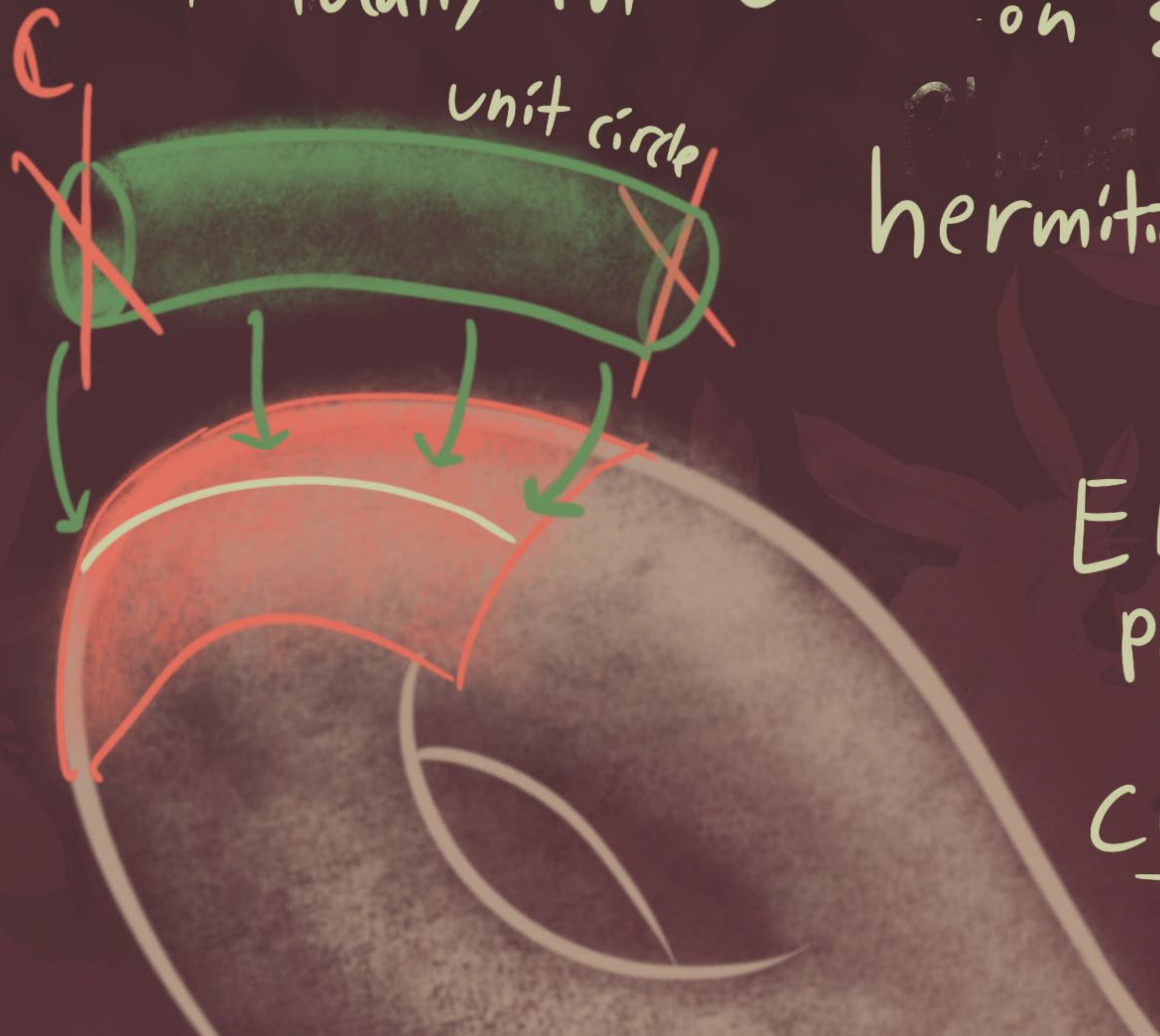
Riemann surface Σ

need Geometric formulation of EM fiel

fundamentally: EM describes <u>Phase</u>

Quantum: $\psi = a \underline{e^{i\theta}}$

ψ locally in $\mathbb{C}$: line bundle on Σ



hermitian metric $\Rightarrow$ $U(1)$ bundle

Phase

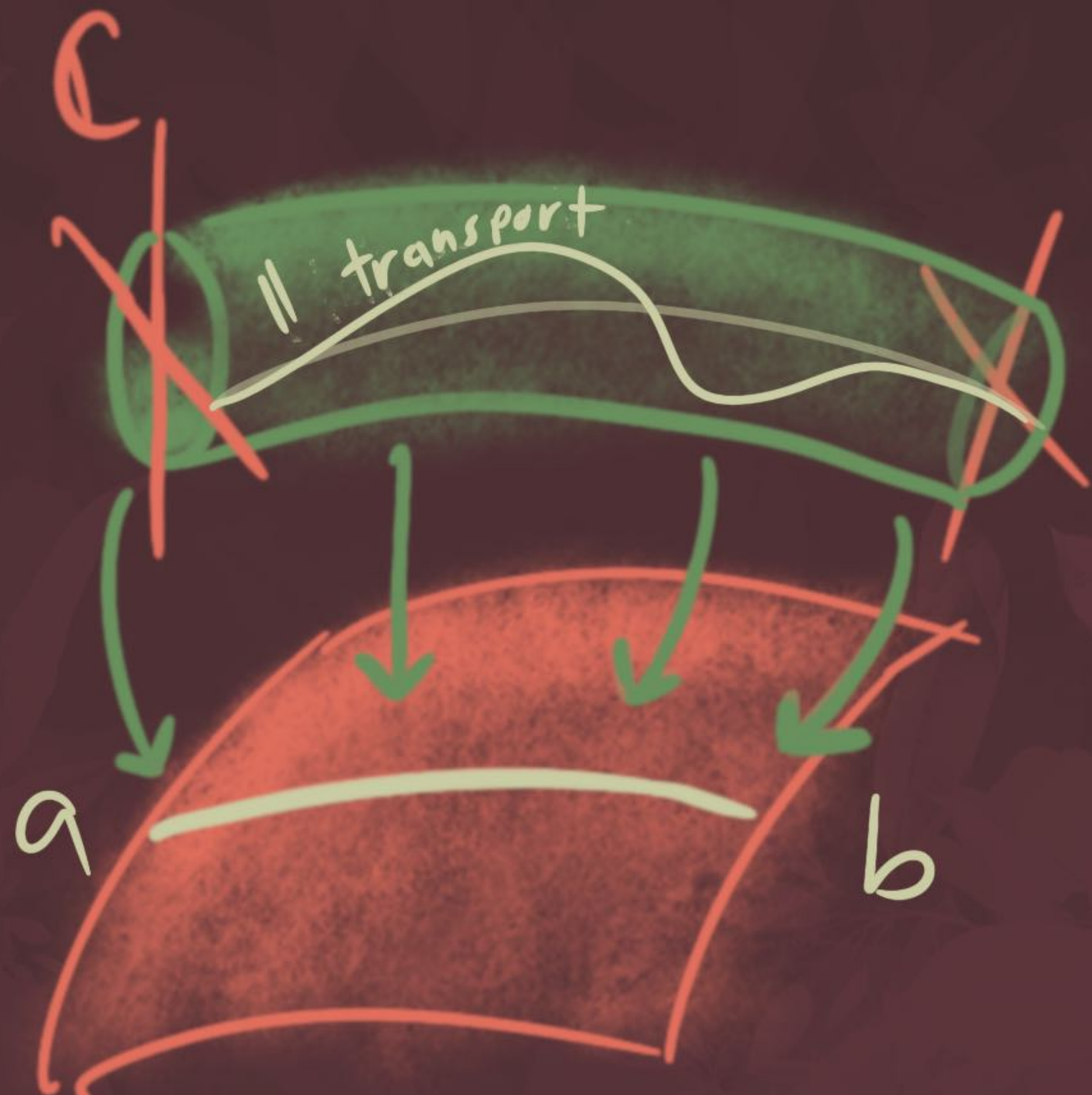
EM: relates phases @ diff pts

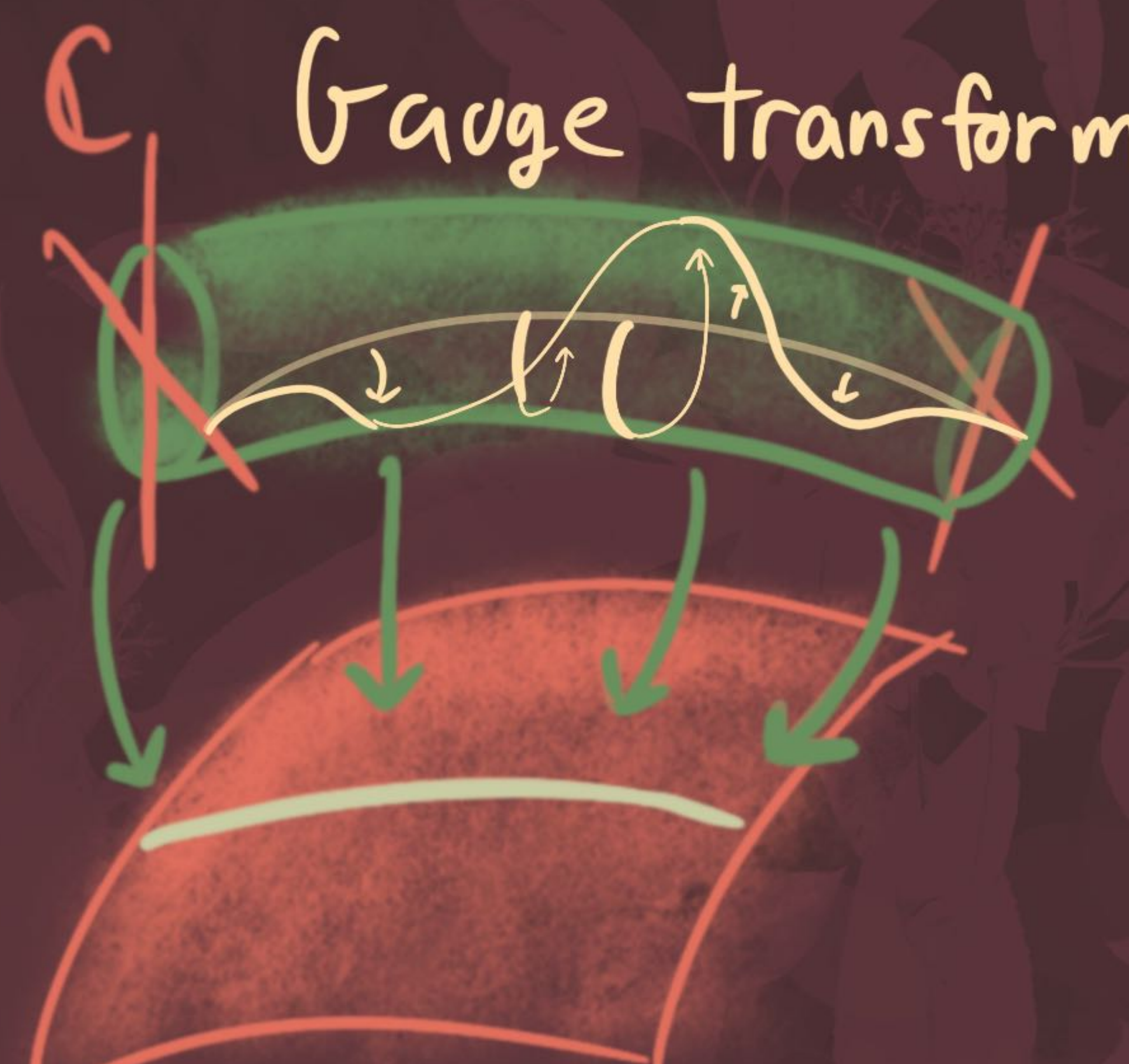
<u>connection</u> A

A : slope of parralel transport

A 1-form : phase $e^{\int_a^b A}$ "Vector potential

baseline ambigous $\Rightarrow$ $\int_a^b A$ ambigous if $a \neq b$





$$\oint_{\partial C} A = \int_C dA$$

$F_A = dA$ curvature

gives local monodromy

this is EM field!

2-form

Energy $E = \int_\Sigma \|F_A\|^2 = \int_\Sigma *F_A \wedge F_A$

only uses $*$: just Riem. surface structure!!

Topology: $\int_\Sigma F_A = c_1(L)$

Physics: minimal $E \Rightarrow$

$*F_A = \text{const}$!!

$\Rightarrow \quad d*dA = 0$

maxwells eqs

Solutions: $F_A = 0 \Rightarrow dA = 0$, A closed

$\Omega^1(\Sigma, \mathbb{C})$?

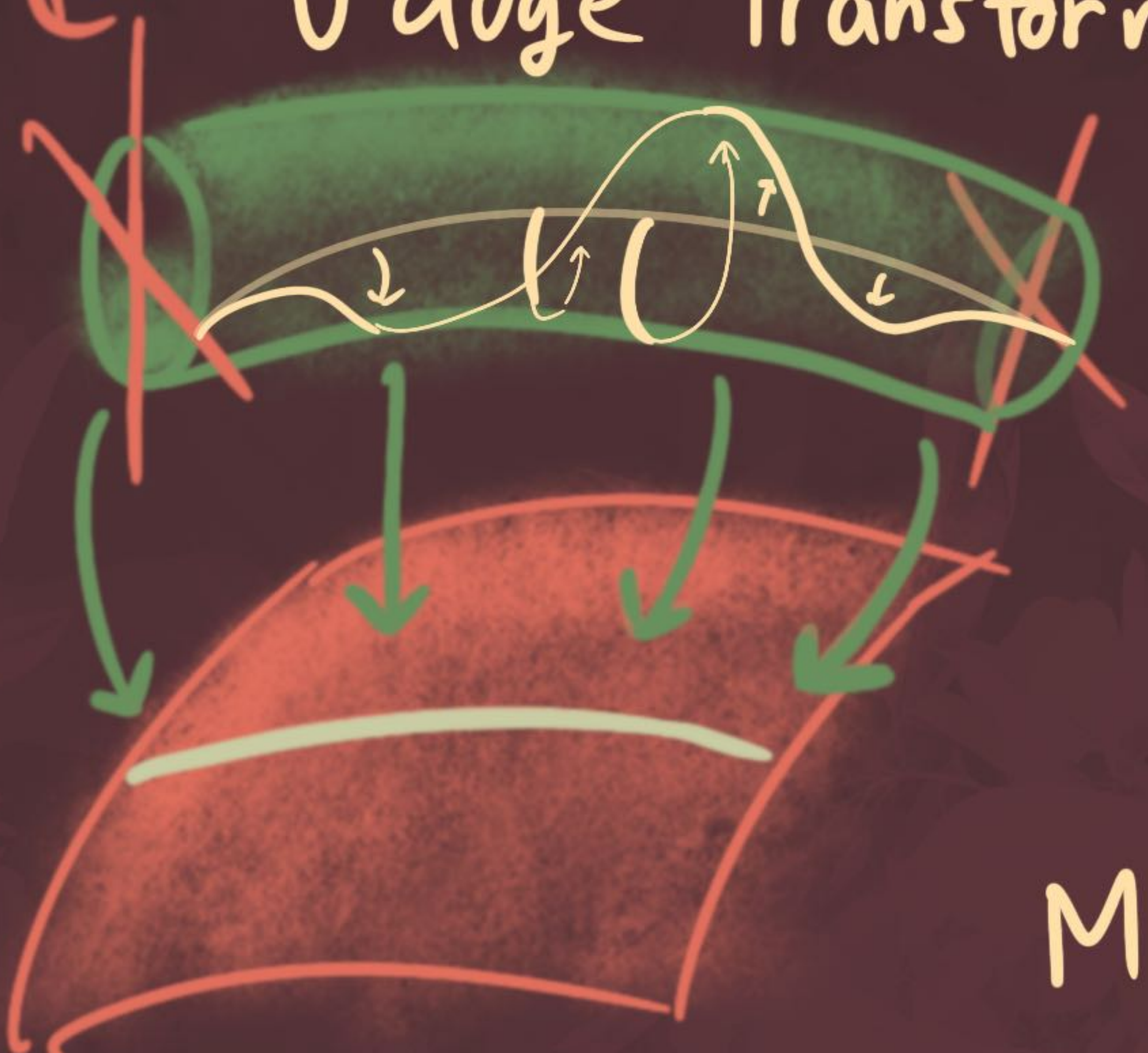
$A \mapsto A + d\phi$

Gauge ambiguity

A defined up to exact

$\mathbb{C}$ Gauge transform

$$\Omega^1(\Sigma, \mathbb{C}) / d\Omega^0(\Sigma, \mathbb{C}) = H^1(\Sigma, \mathbb{C})?$$



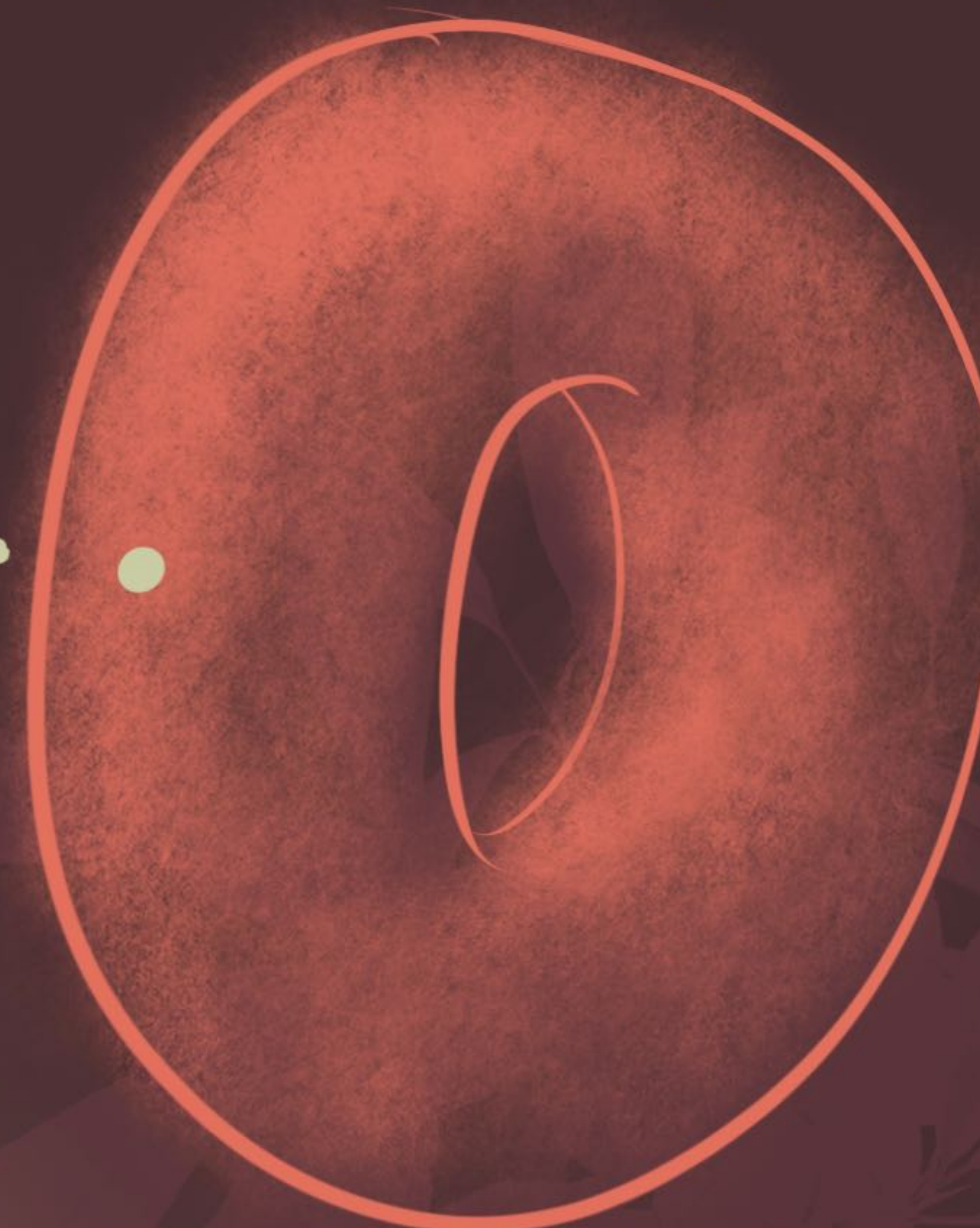
go all the way round?

# of times about $U(1) \in \mathbb{Z}$

Classified by $H^1(\Sigma, \mathbb{Z})$

$$\text{Moduli of solutions} = \frac{H^1(\Sigma, \mathbb{C})}{H^1(\Sigma, \mathbb{Z})}$$

$$\frac{H^1(\Sigma, \mathbb{C})}{H^1(\Sigma, \mathbb{Z})} = \mathrm{Jac}(\Sigma)!!$$



$F_A = 0 \Rightarrow$

$\nabla_A = d + A$ flat connection

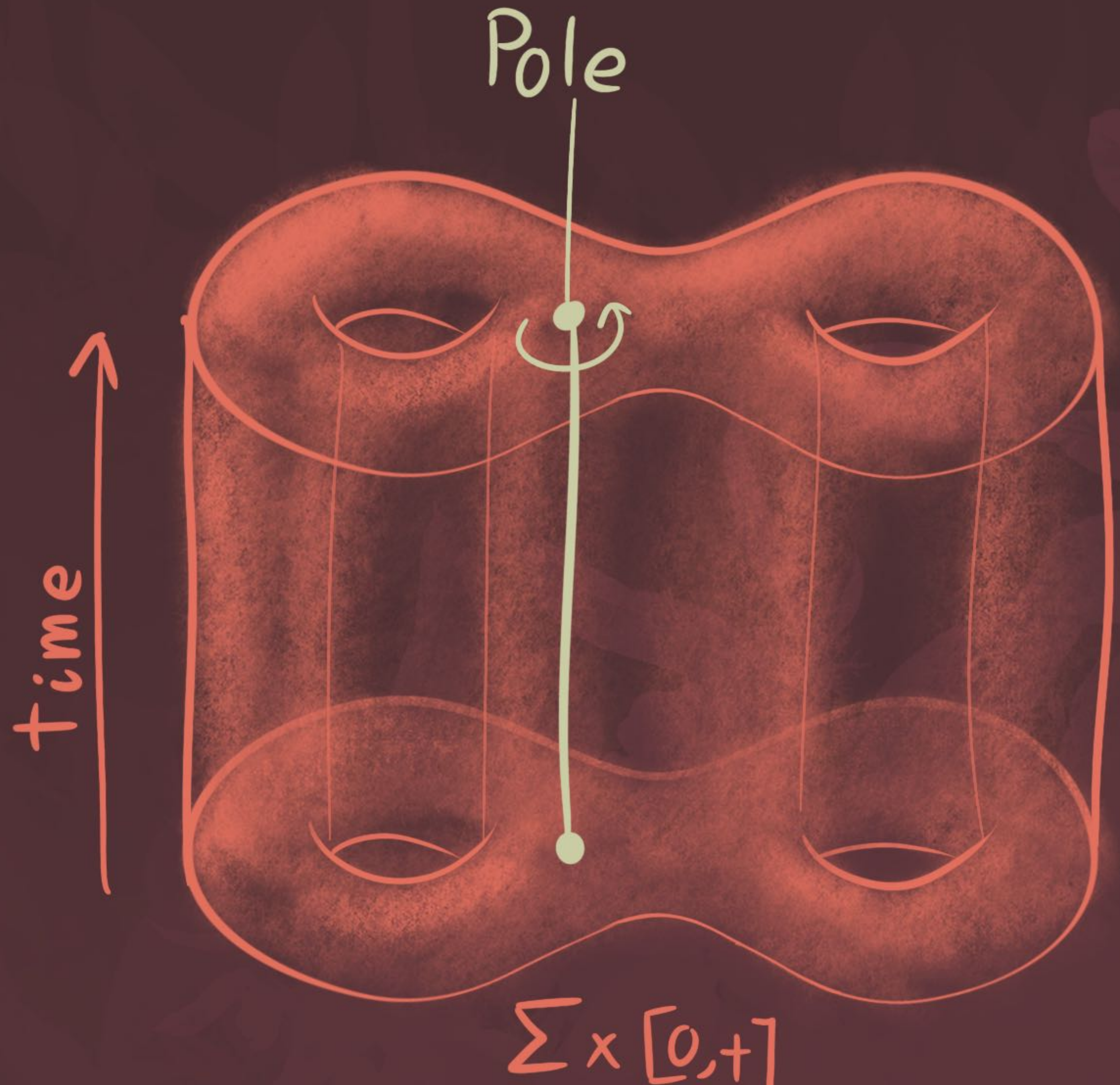
$\nabla_A = \bar{\partial} + \partial_h$

1-1 corresponds

$\mathrm{Jac}(\Sigma)$ = moduli space of solutions to maxwells eqs.

What about $\mathrm{Jac}(\Sigma) = \dfrac{\mathrm{div}(\Sigma)}{\mathrm{Cl}(\Sigma)}$?

Pole $\Leftrightarrow$ magnetic Monopole!



$F_A = \delta_{[\mathrm{Div}\, L]}$

i.e magnetic field concentrated at a point replace $U(1)$ w/ $U(n)$: $L \mapsto V$ vector bundle

has $\underline{u}(n)$-1-form connection A, curvature F_A

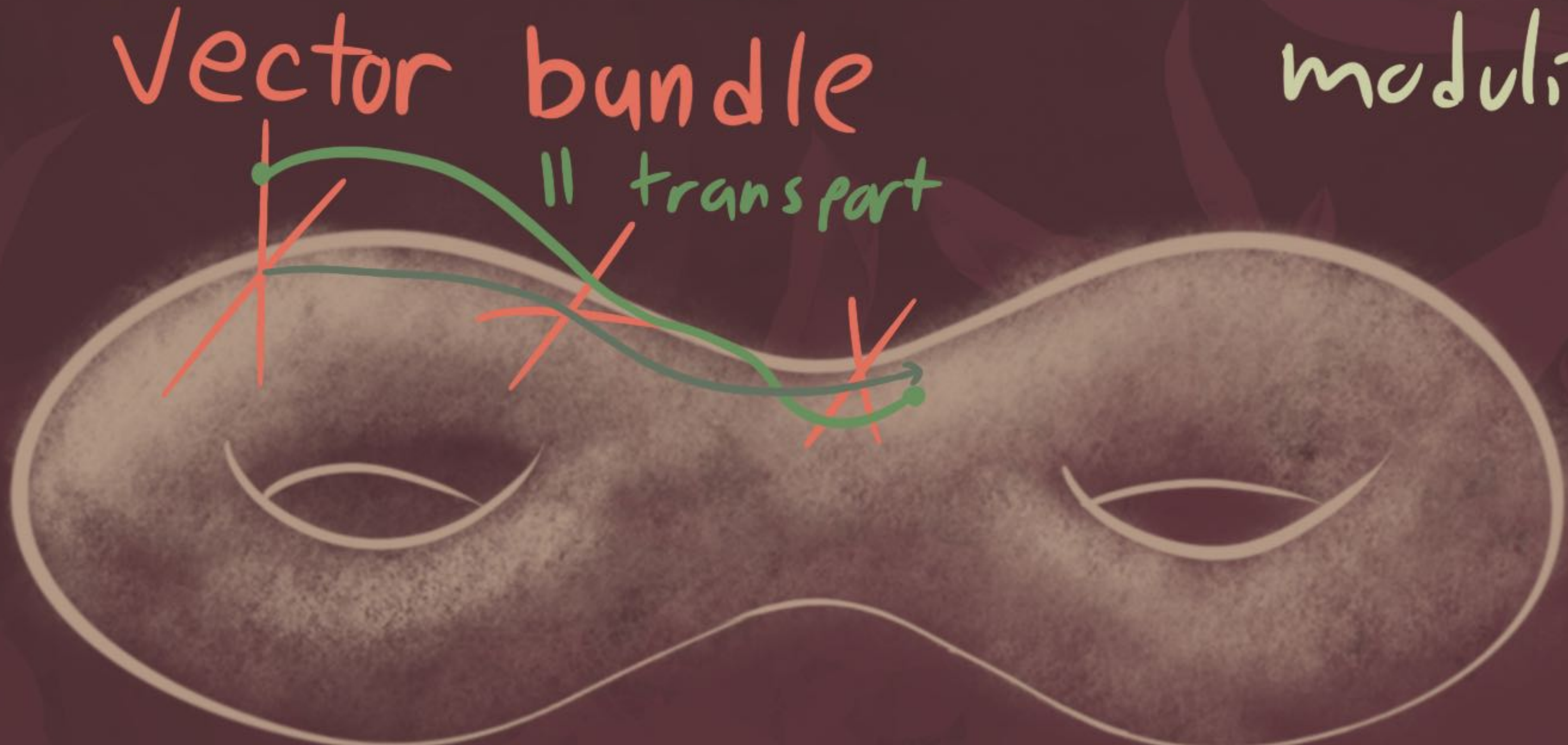
$\int \|\mathrm{Tr}\, F_A\|^2$ minimal $\Rightarrow$ $* F_A$ constant

$d_A * F_A = 0$

$F_A = dA + \underline{A \wedge A}$ nonlinear! $\Rightarrow$ Moduli solutions nonlinear

$\parallel$

moduli of (stable) v.bs



Kobiyashi - Hitchin: holds on any Kahler mfld