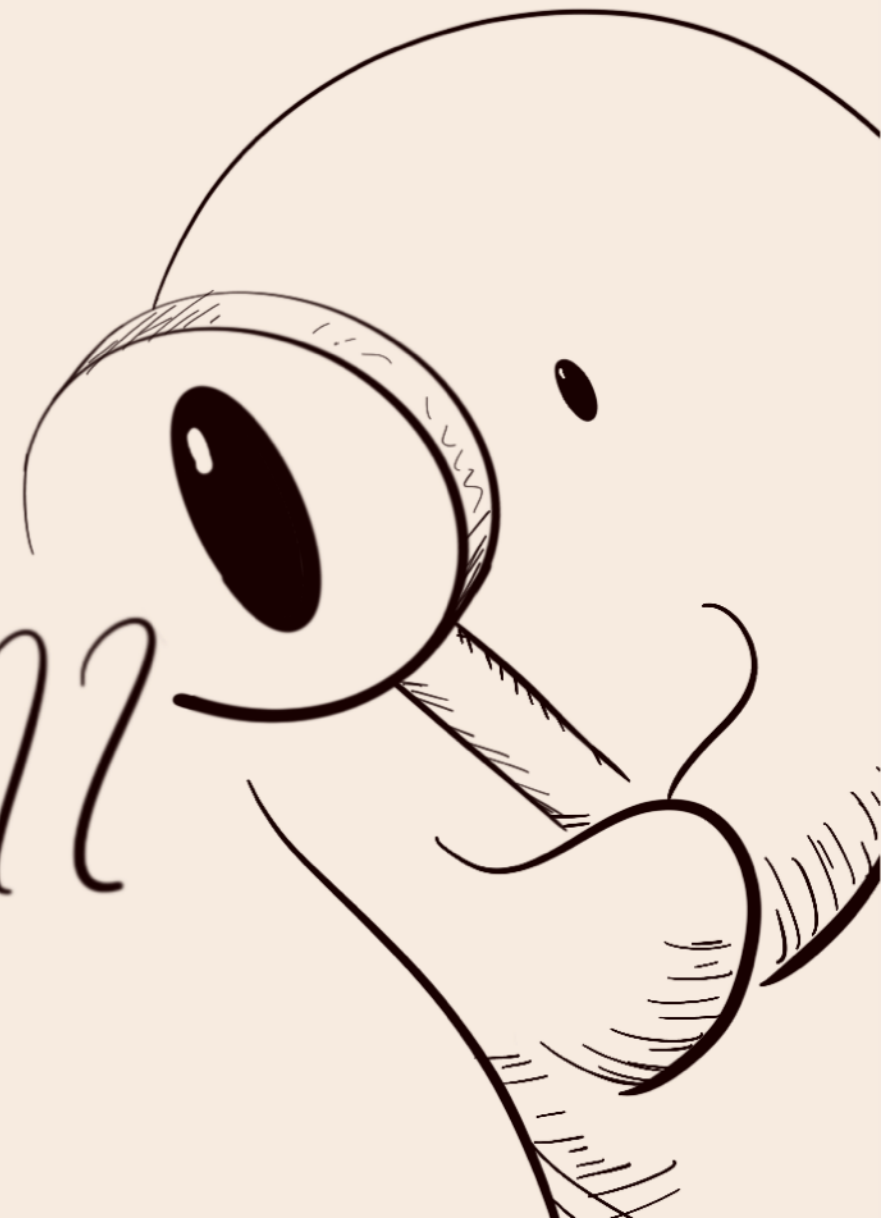


Is Math



OR

small??



Imagine a torus:

- How big is it?

- Why?

Anecdote 1:

TRAIN

TRACKS



Evans Hall, Built 1971

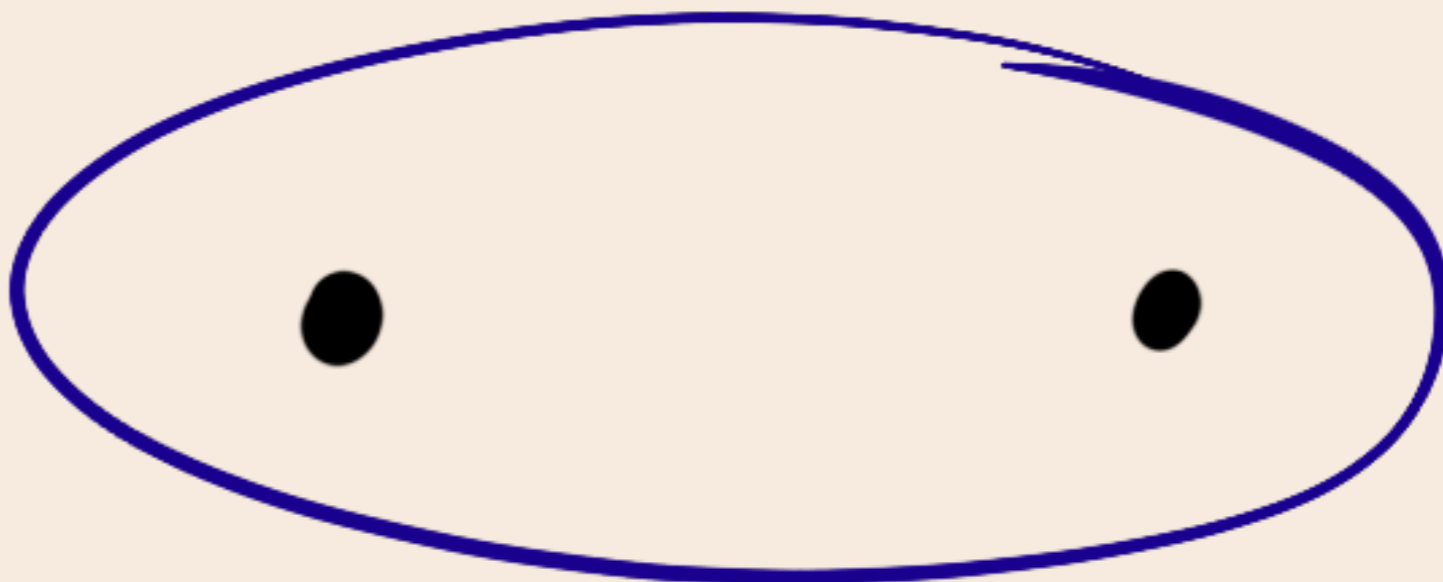


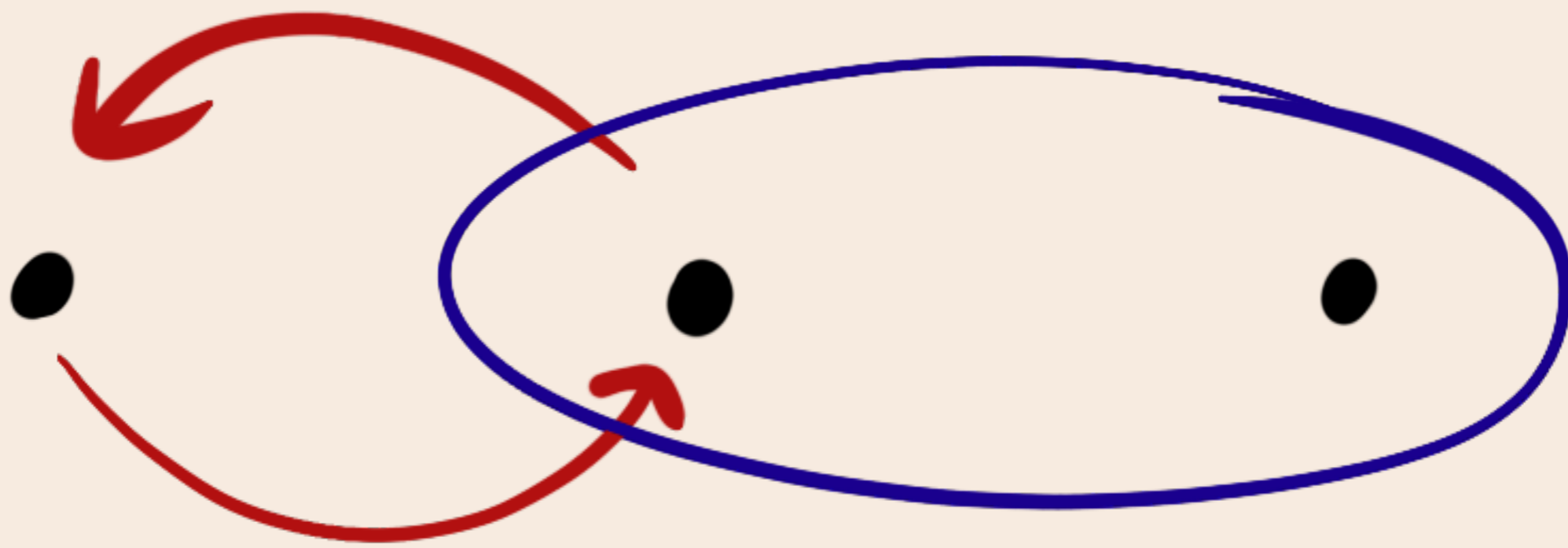


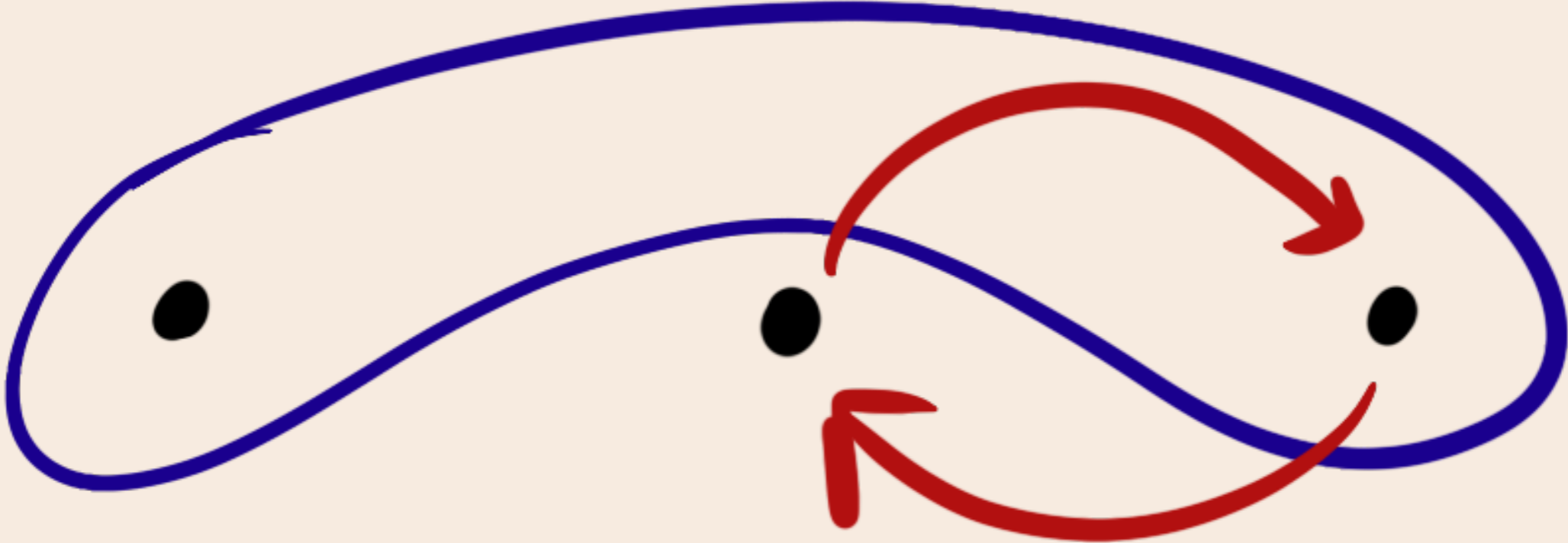
12/9/71
D.S.
B.T.
/n

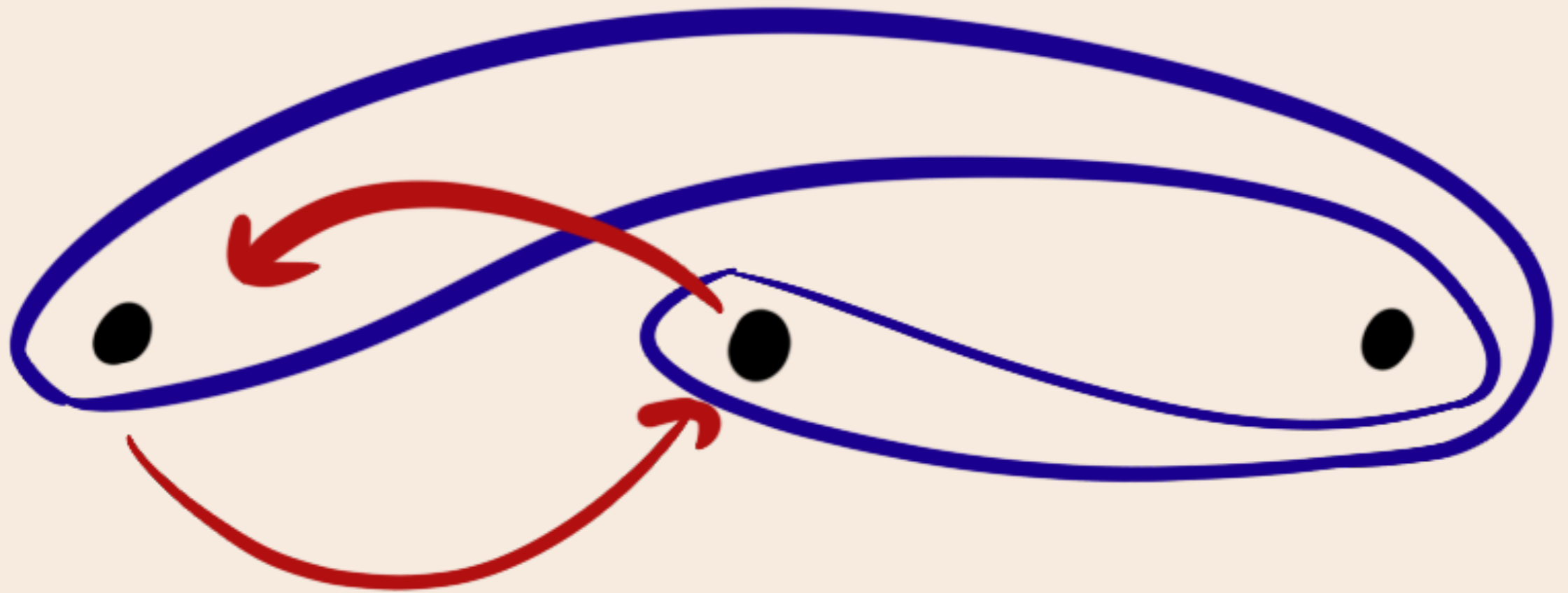
abc'ba'babcb'abc'ba'babcb'abc'ba'babcb'

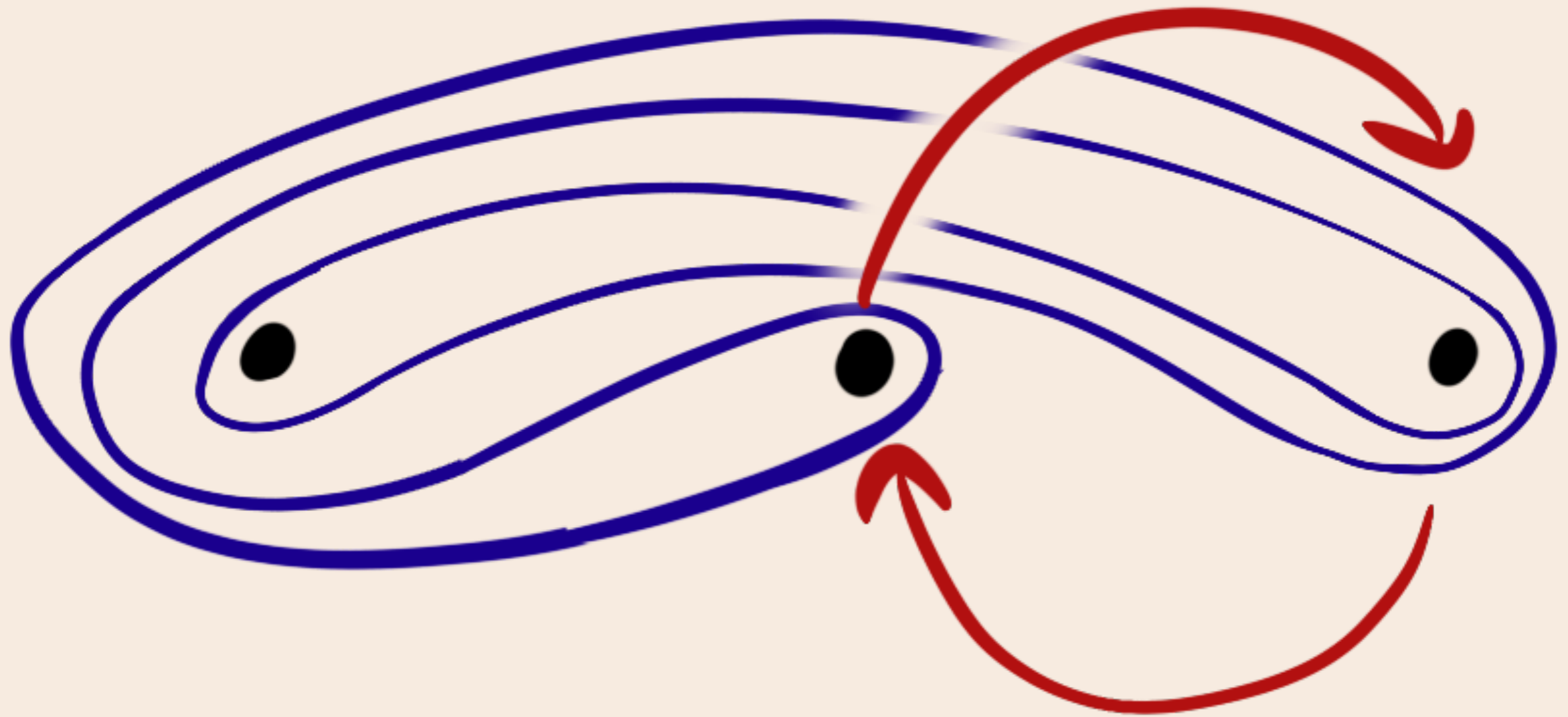
Bill Thurston & Dennis Sullivan, 1971

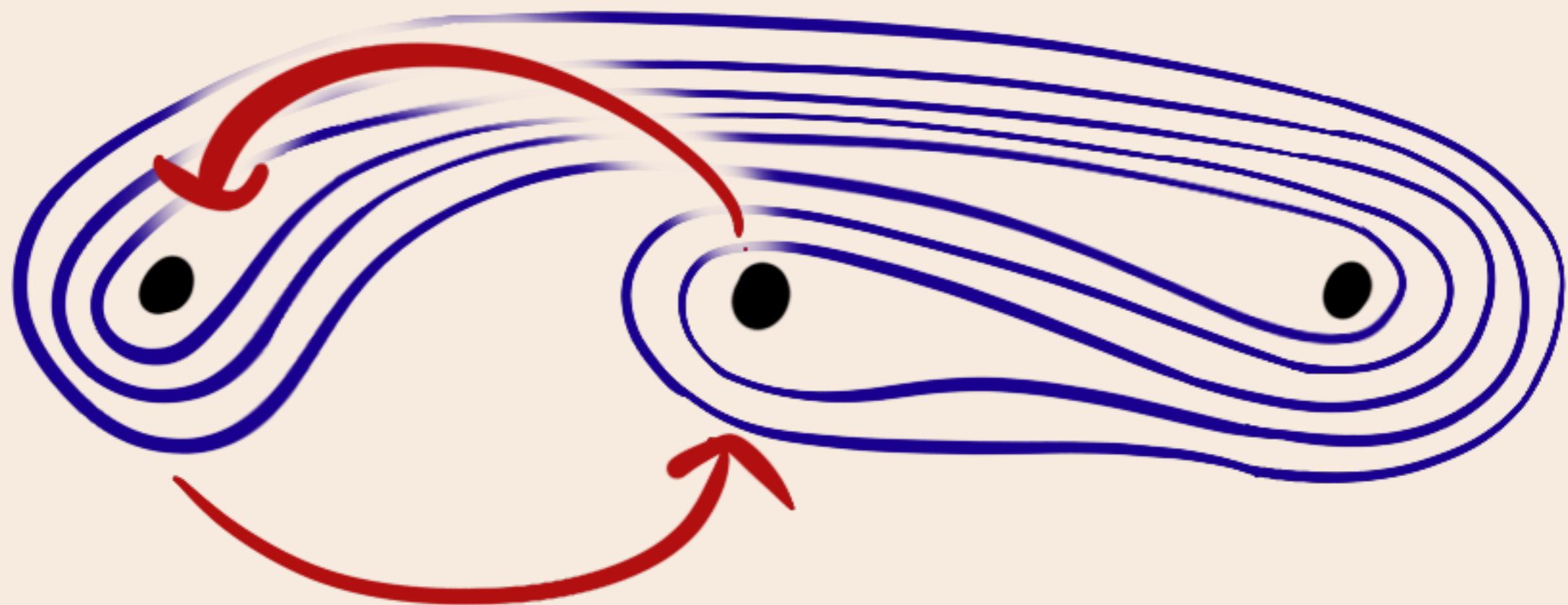


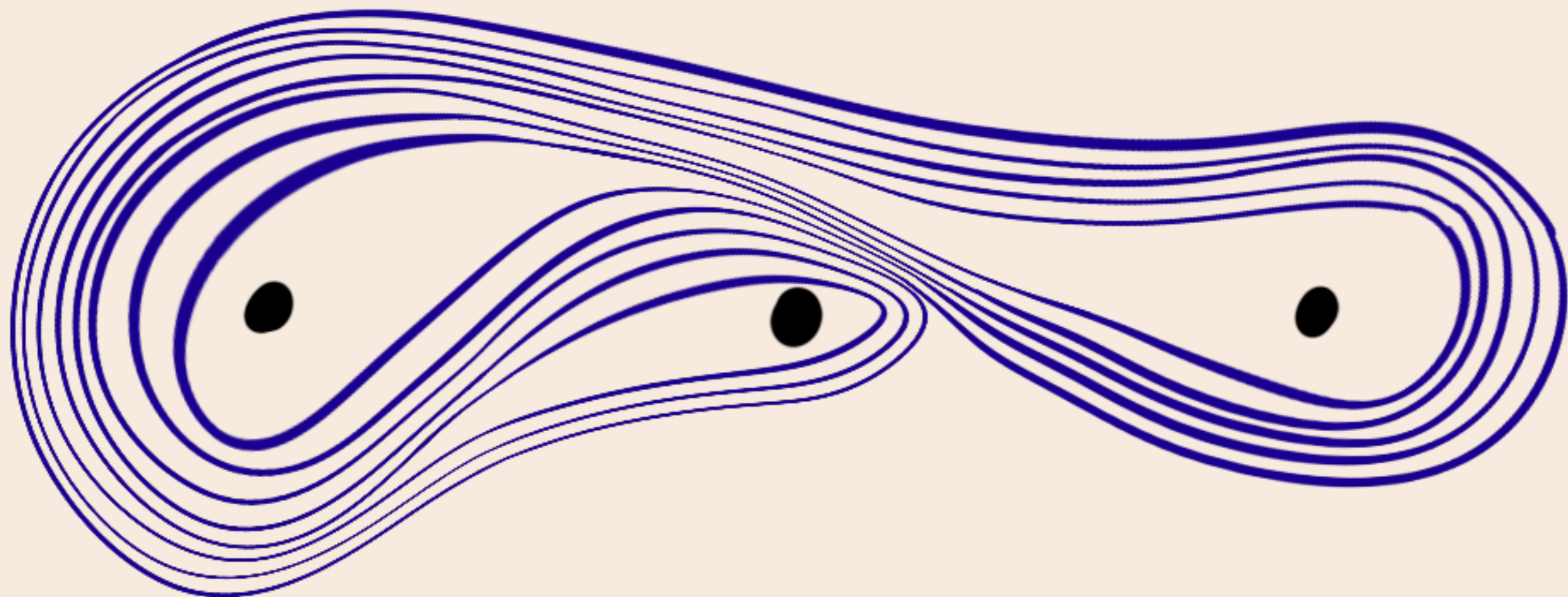


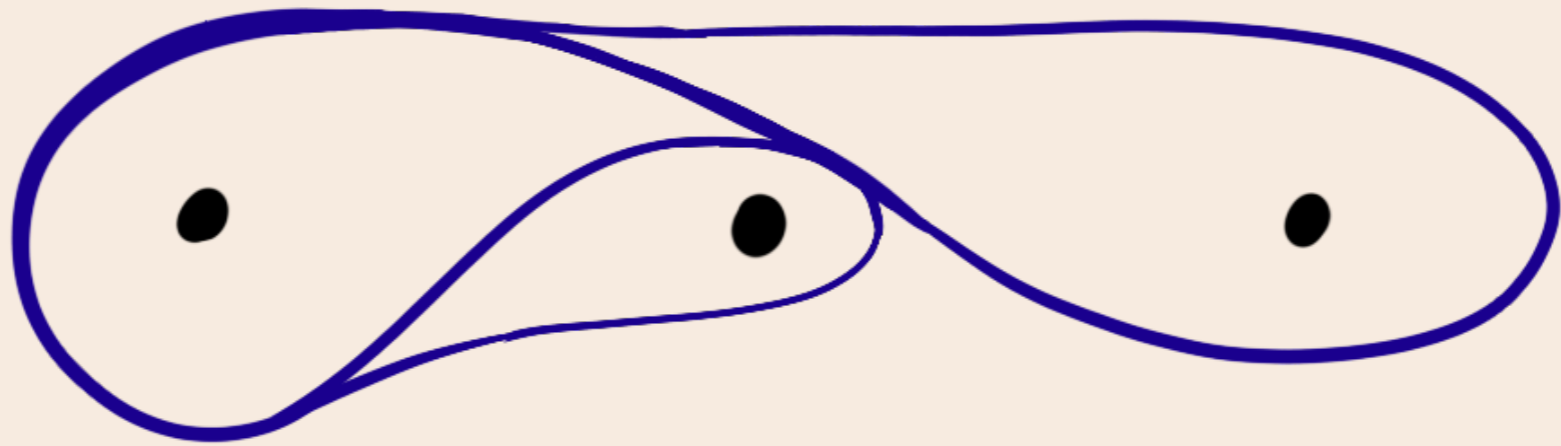
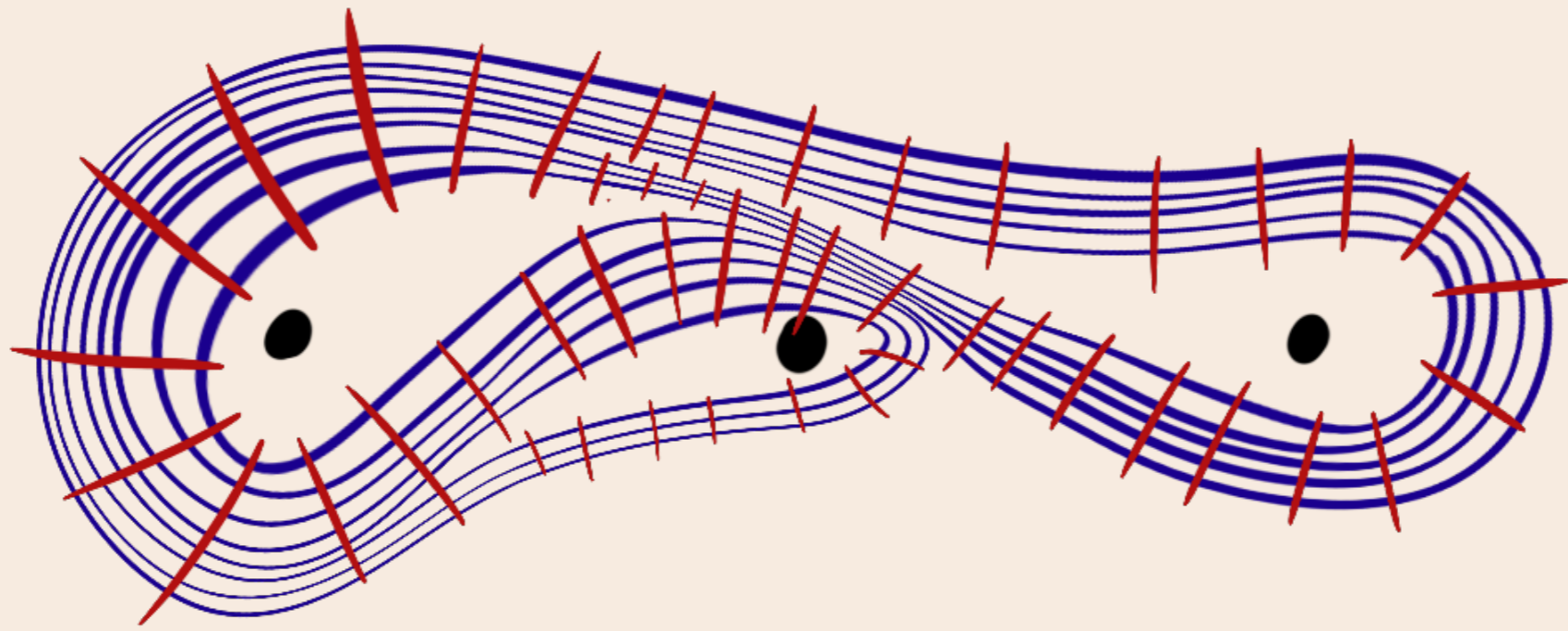






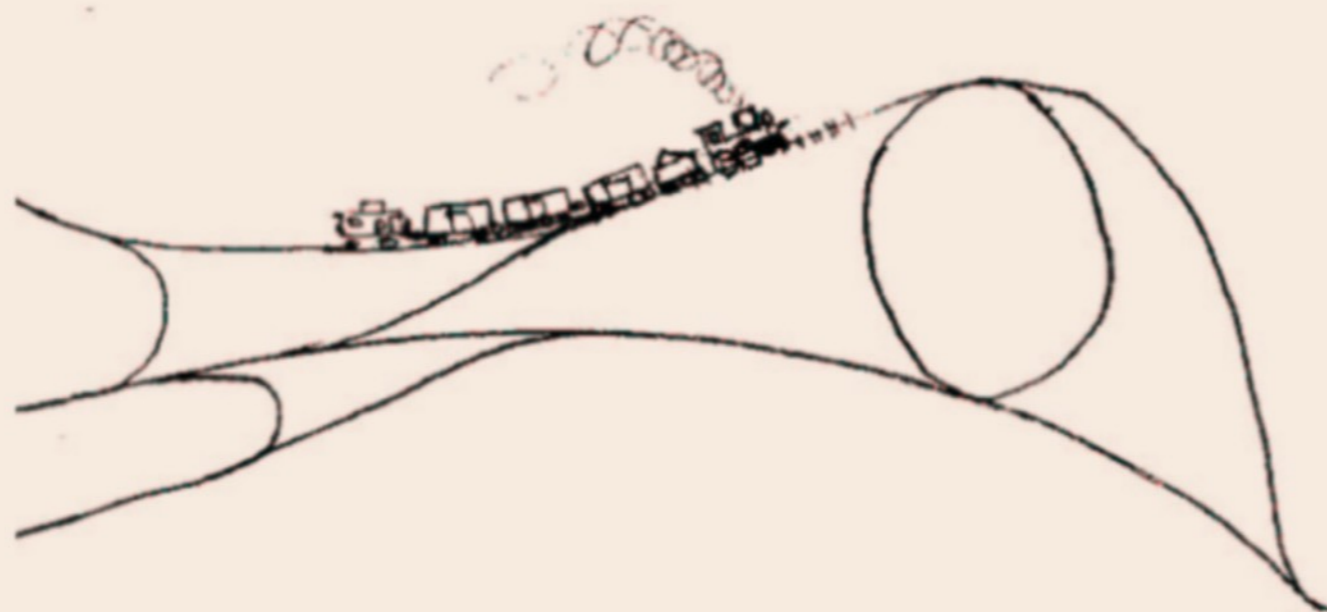
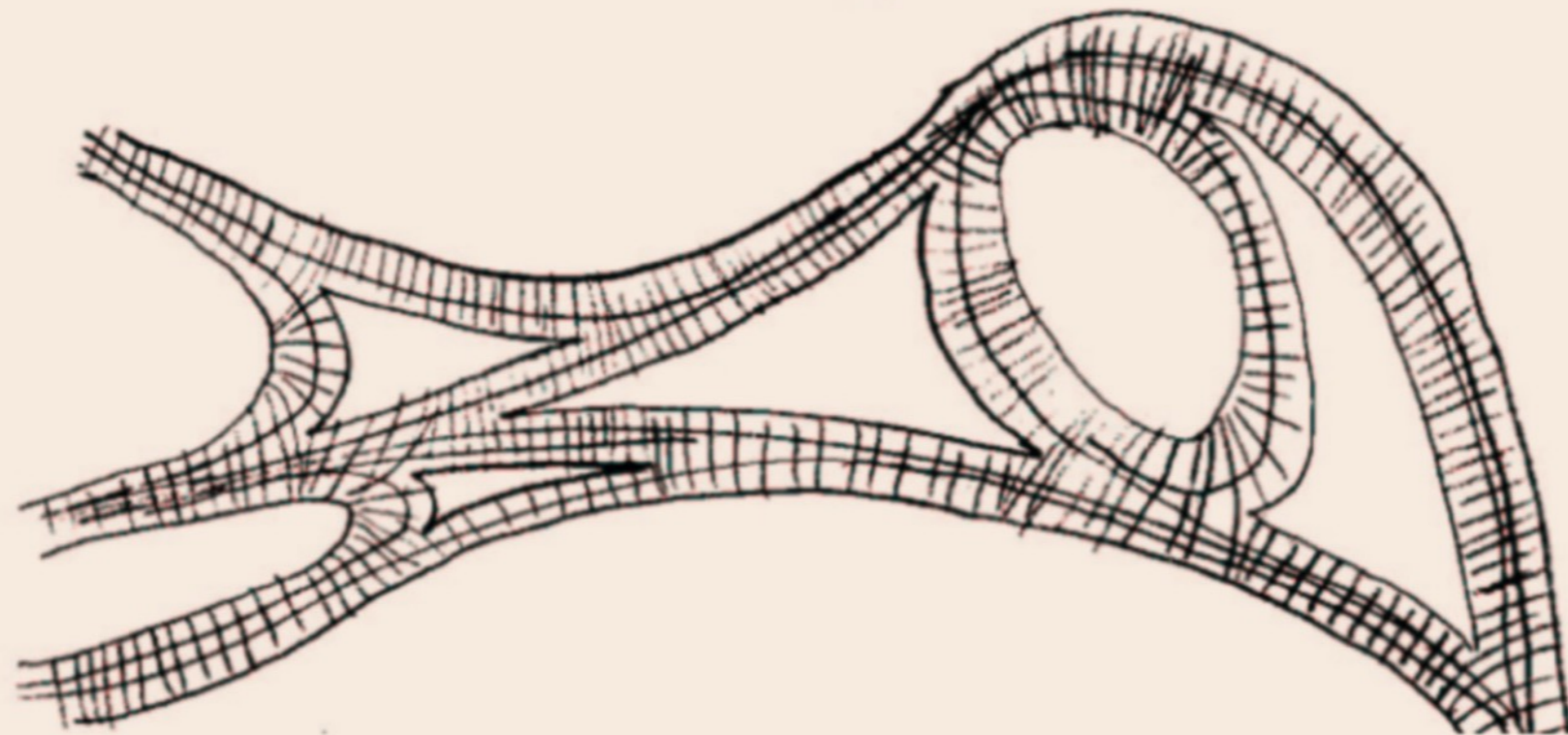






Train Tracks



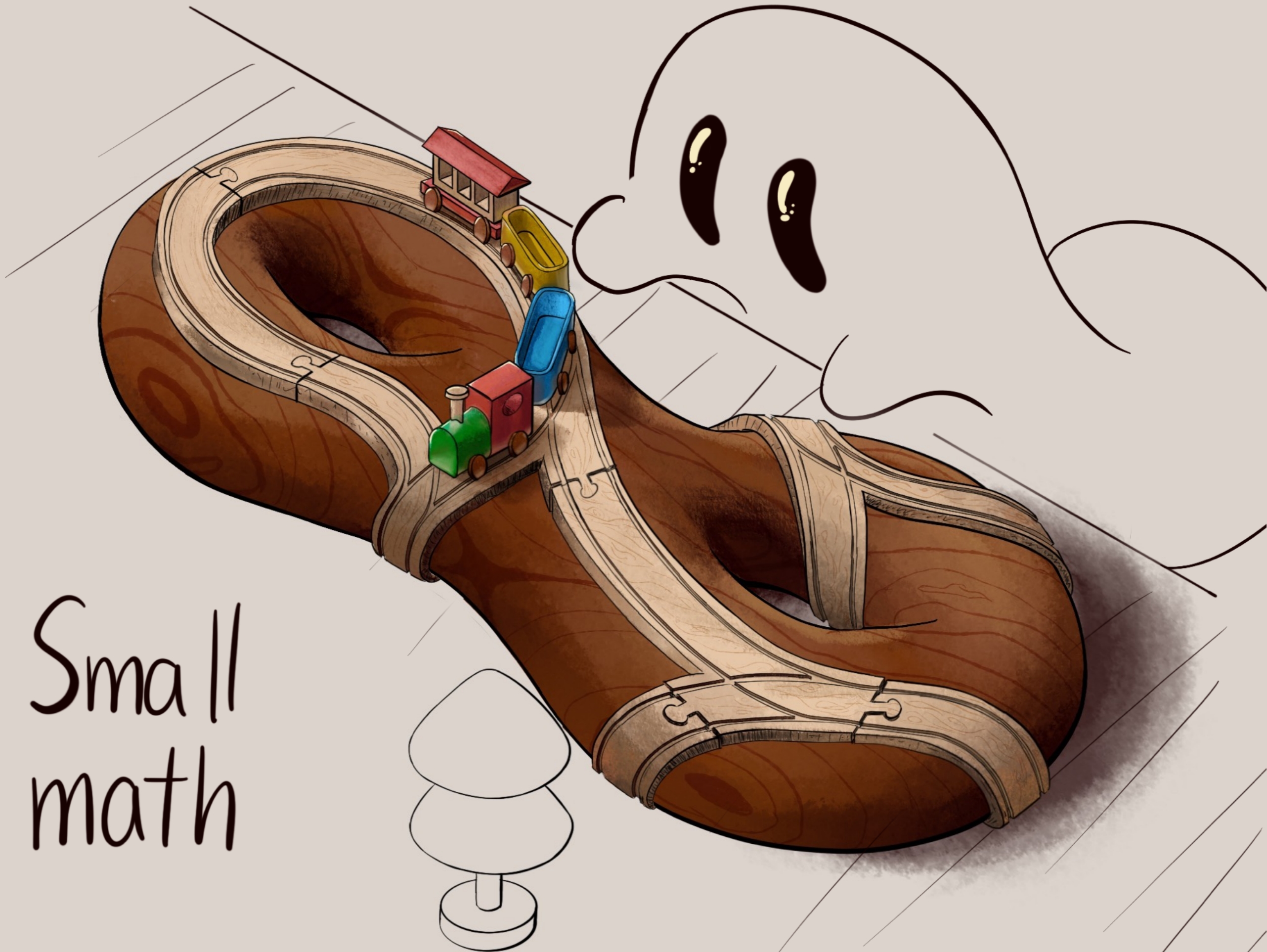


Thurston, "The geometry & topology of 3-manifolds"

Big Math



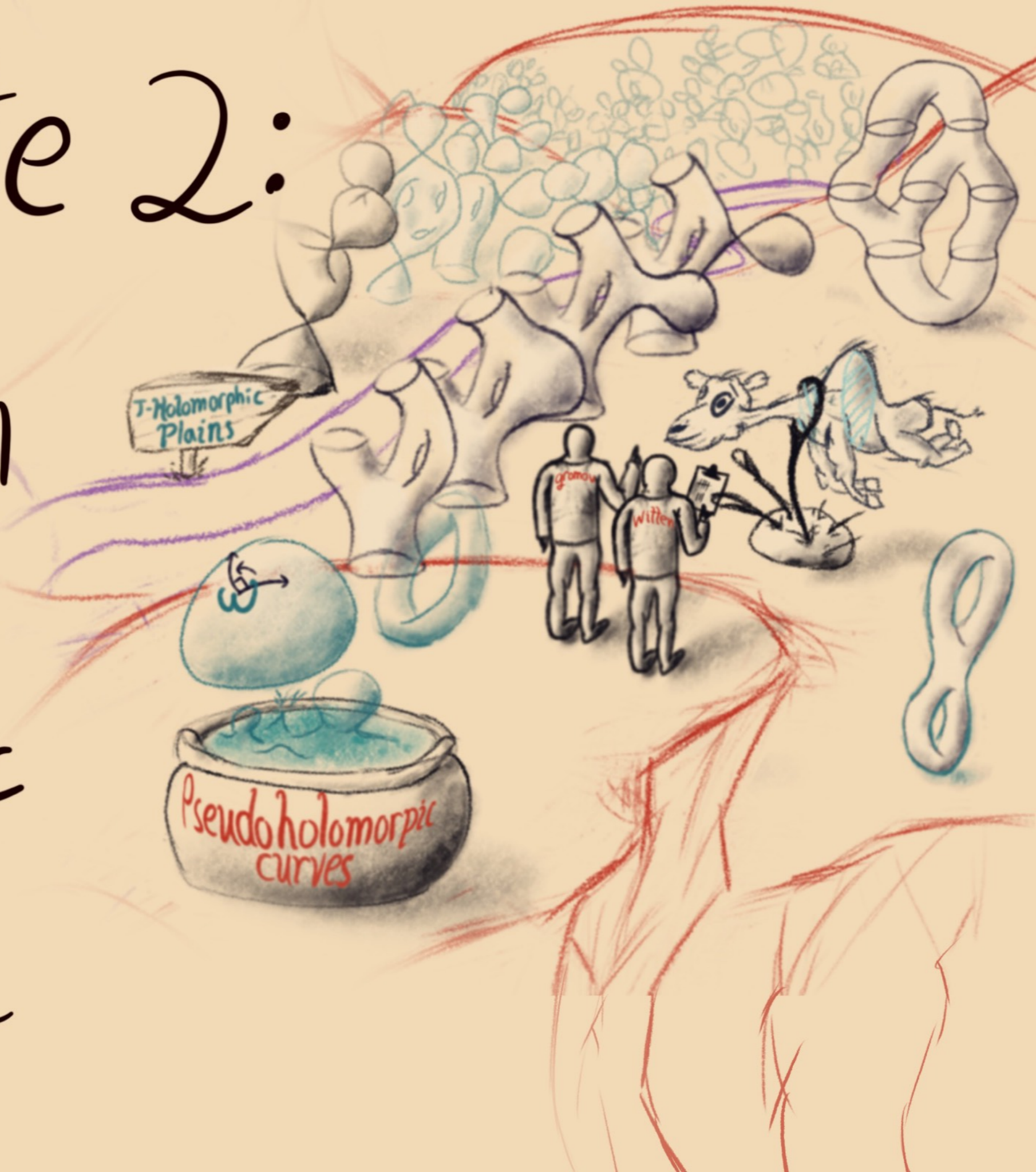
Conan Wu



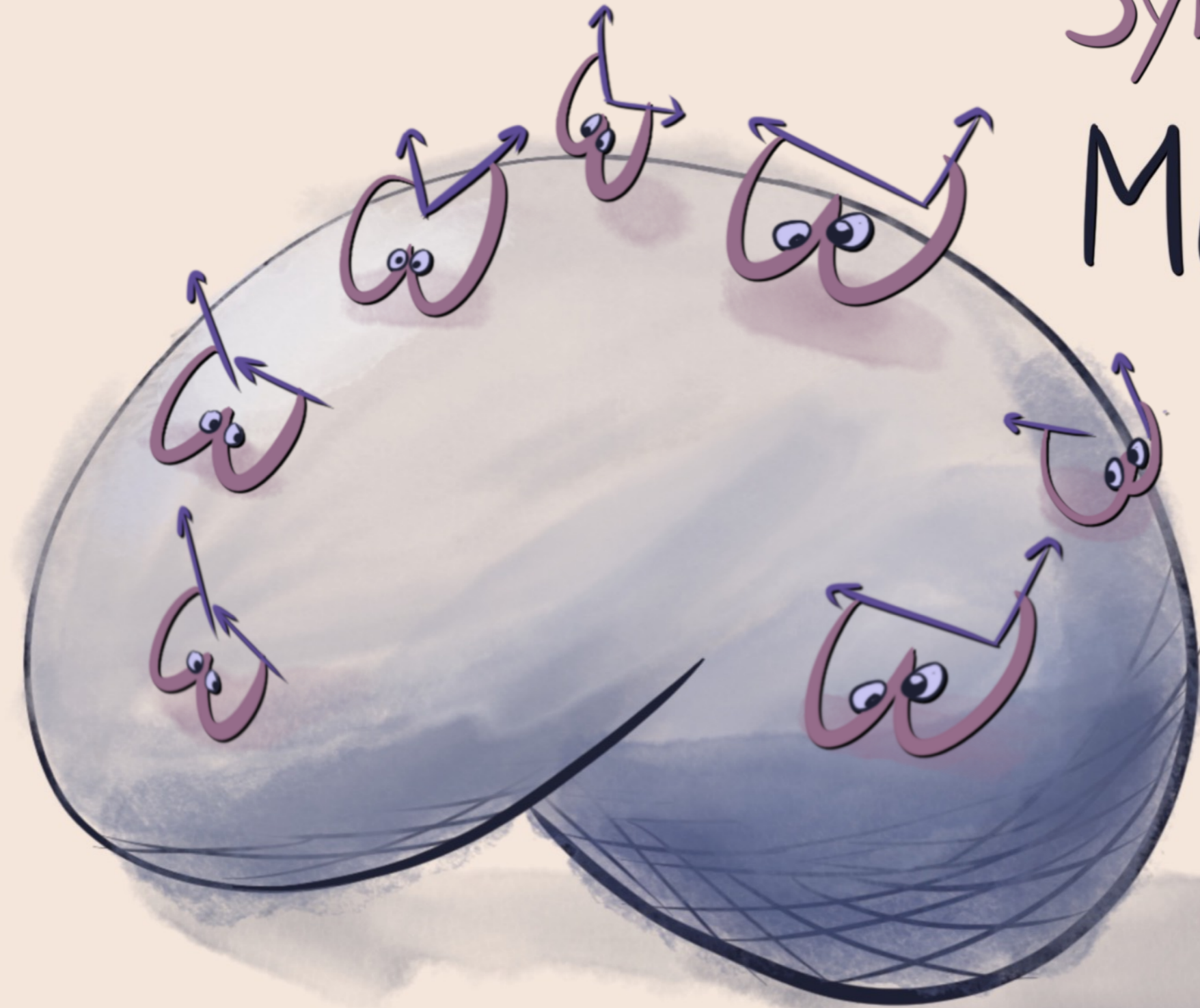
Small
math

Anecdote 2:

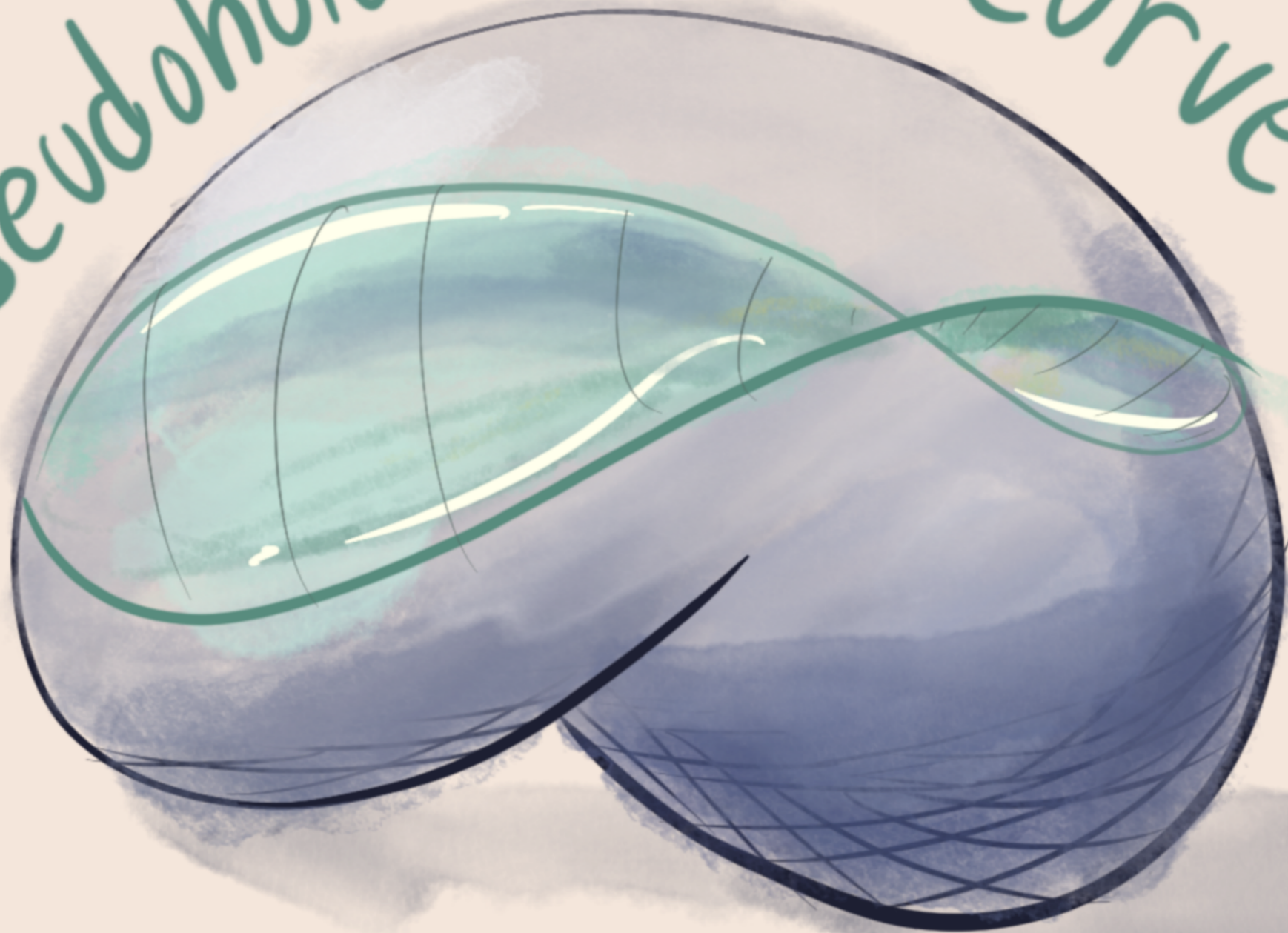
Big Math
in
symplectic
geometry



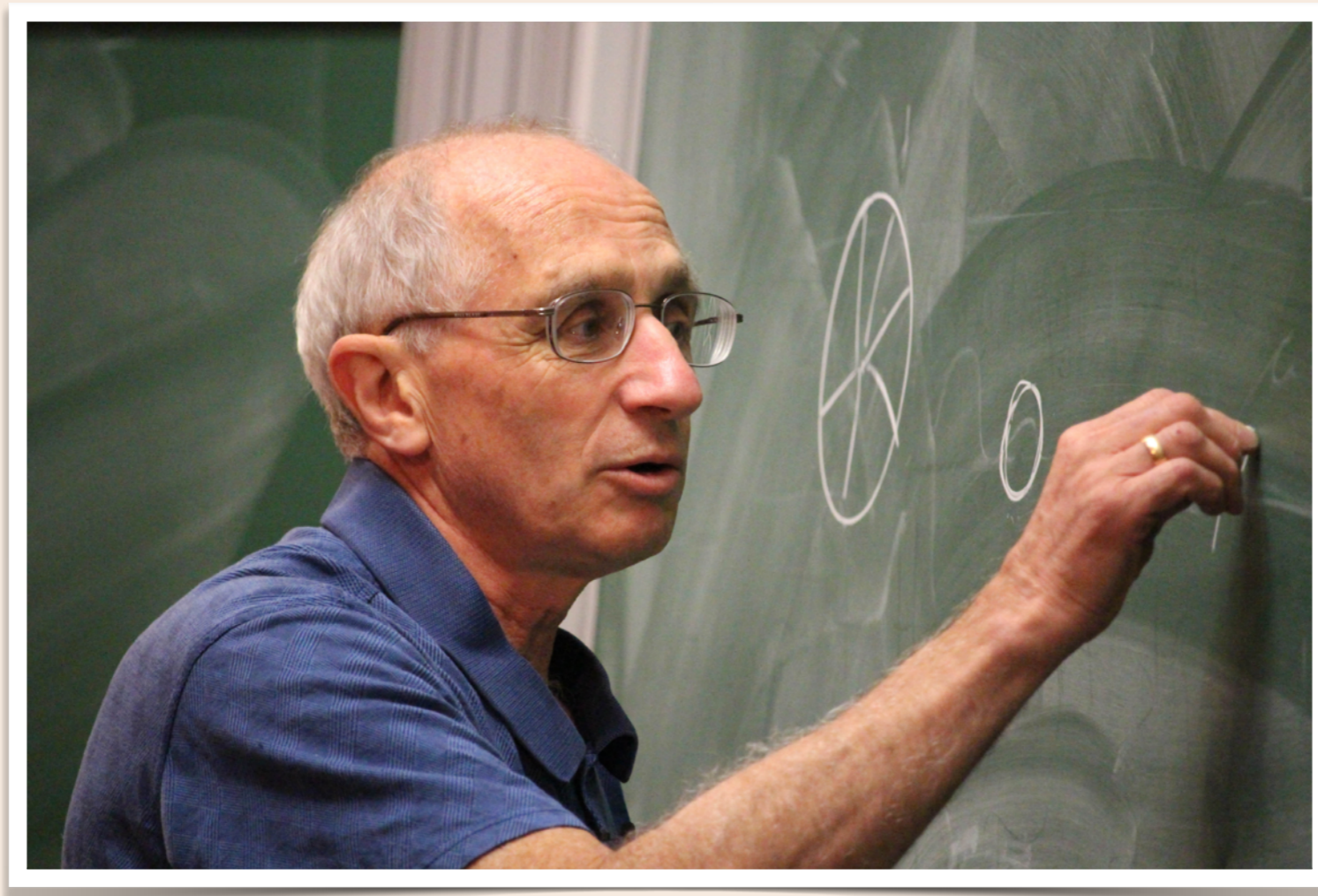
Symplectic Manifold



Pseudoholomorphic curve



Yasha Eliashberg



"Yasha's vision of mathematics is so great because he is very small"

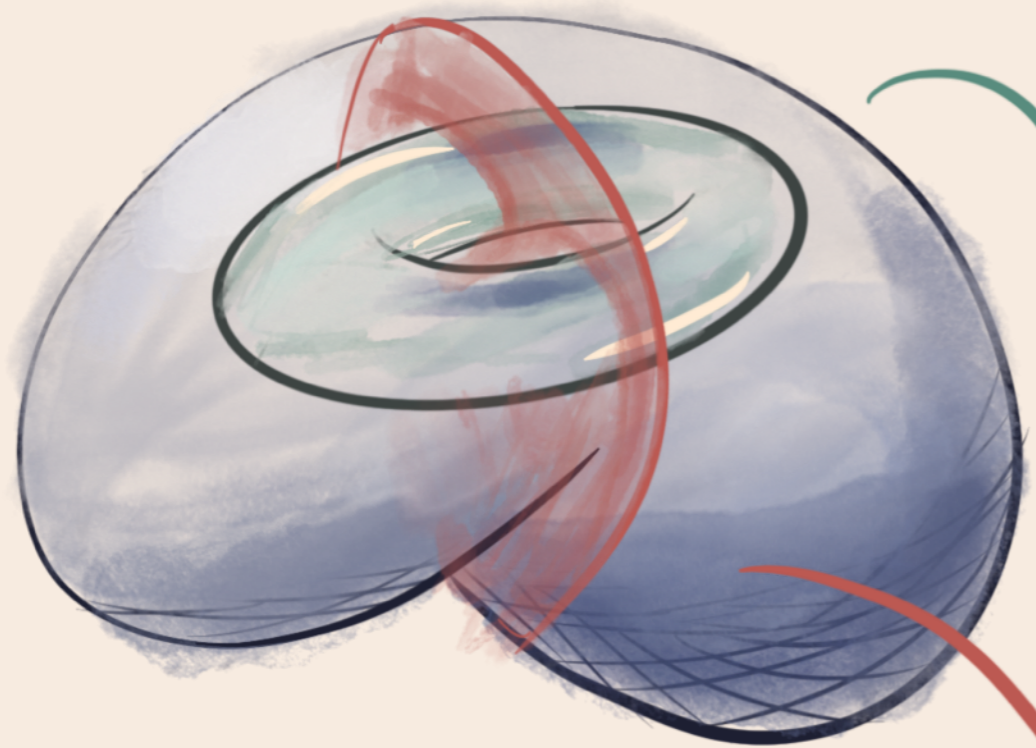
-Yashafest 2007

"... Yasha imagines mathematical objects as being much larger than himself, so that he can visualize all their details and develop a good intuition.

This is obvious for anyone who has had the chance to discuss mathematics with Yasha on a sidewalk terrace, where it would be typical for him to describe a huge pseudoholomorphic curve, say, coming from the other side of the street"

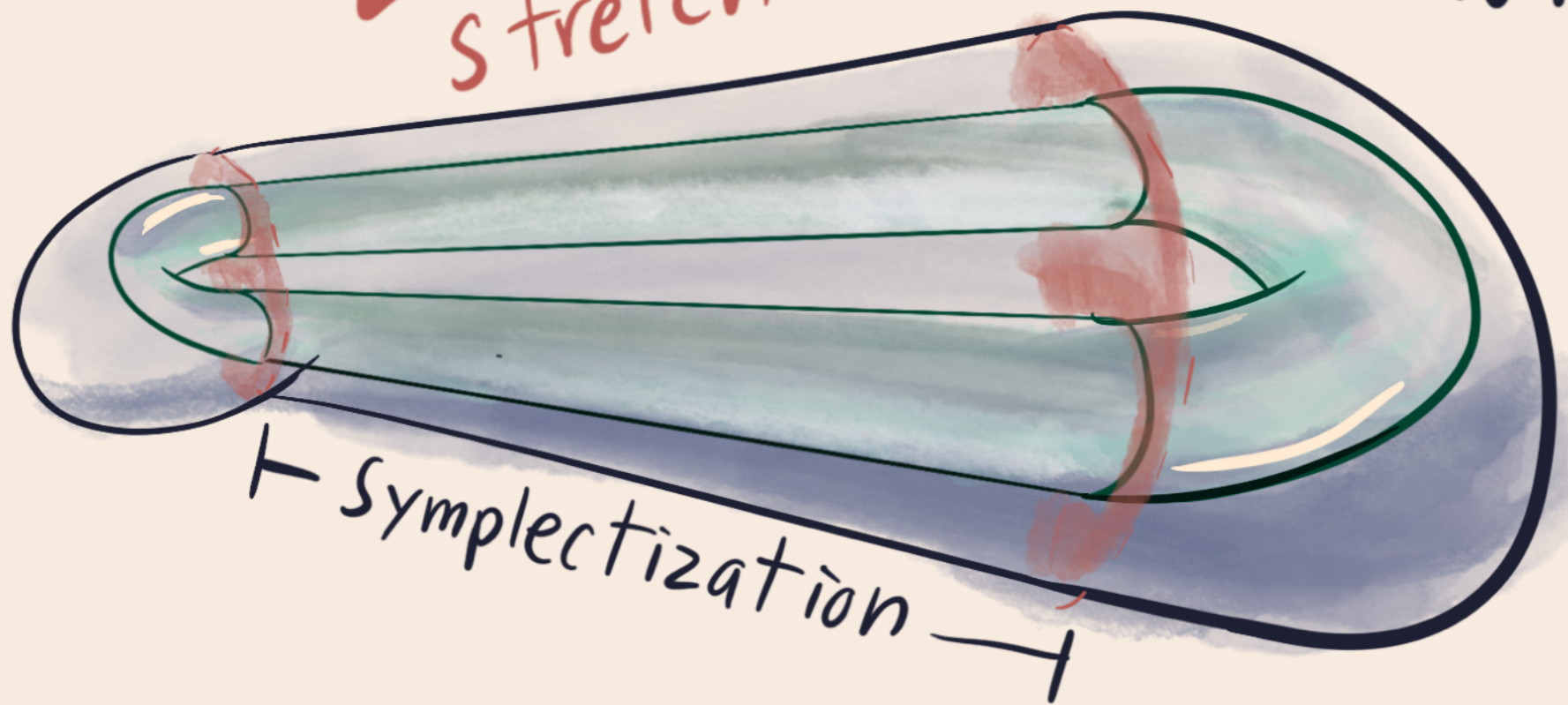
- Frédéric Bourgeois





Idea: Count
Pseudoholomorphic curves
By decomposing manifold

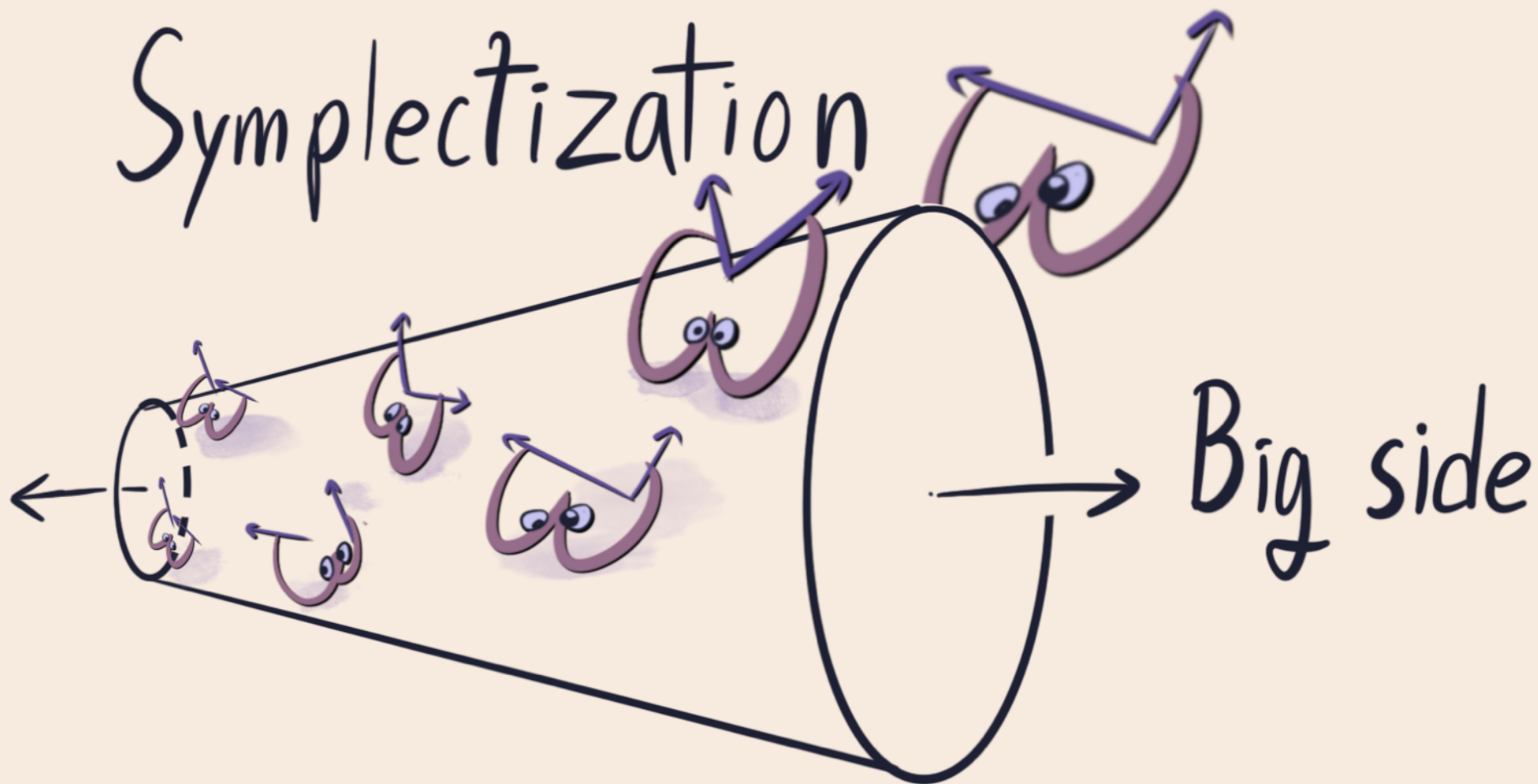
stretch



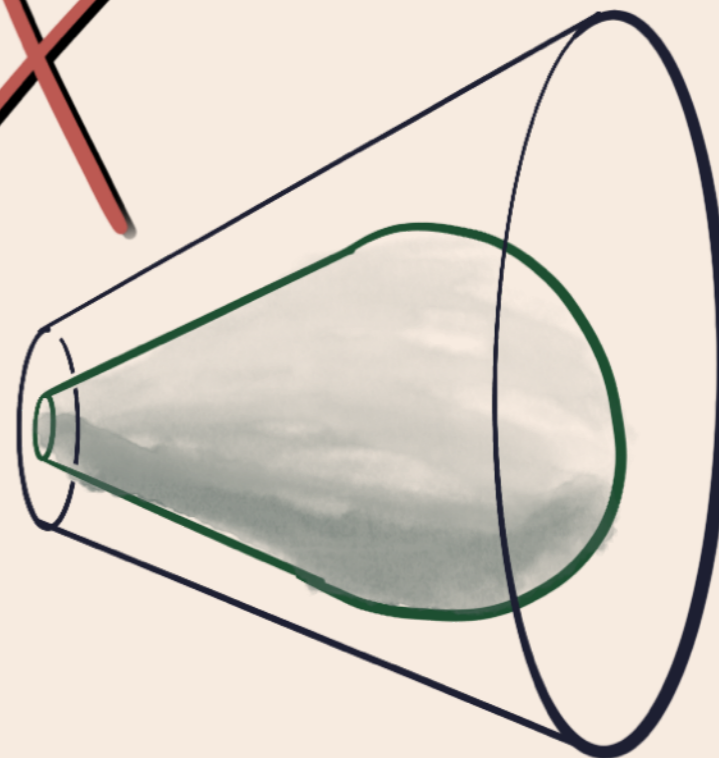
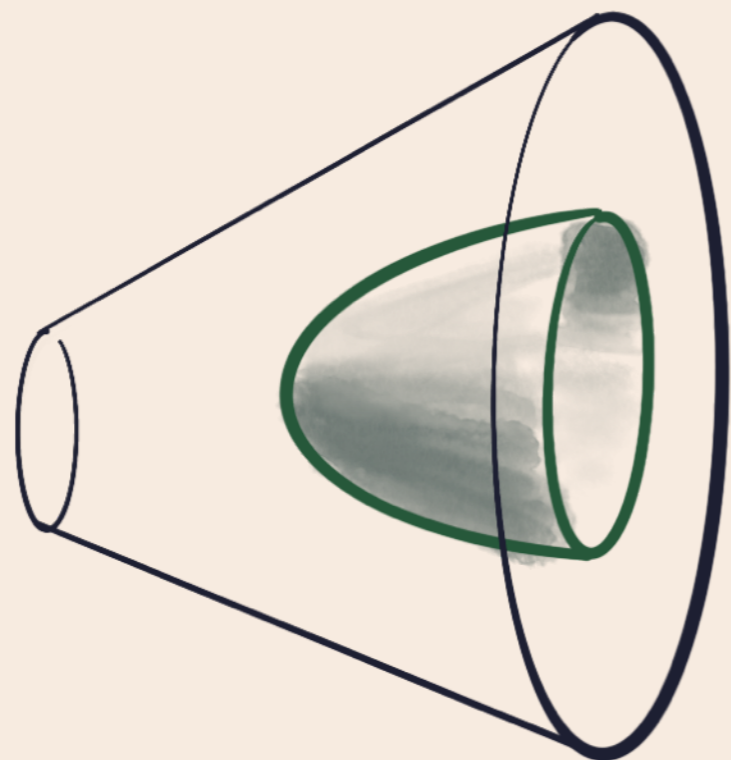
What happens
in the middle?

Symplectization

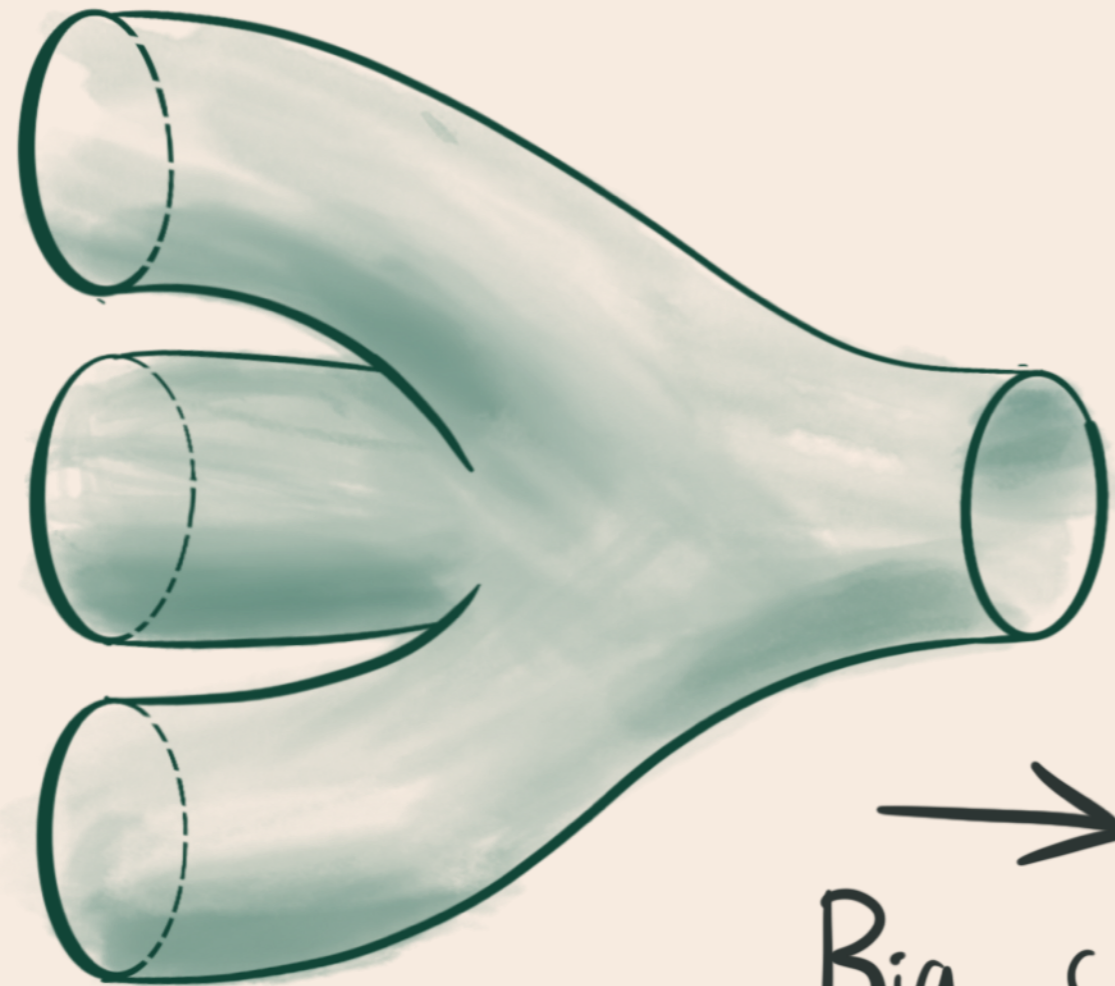
Small side



Big side

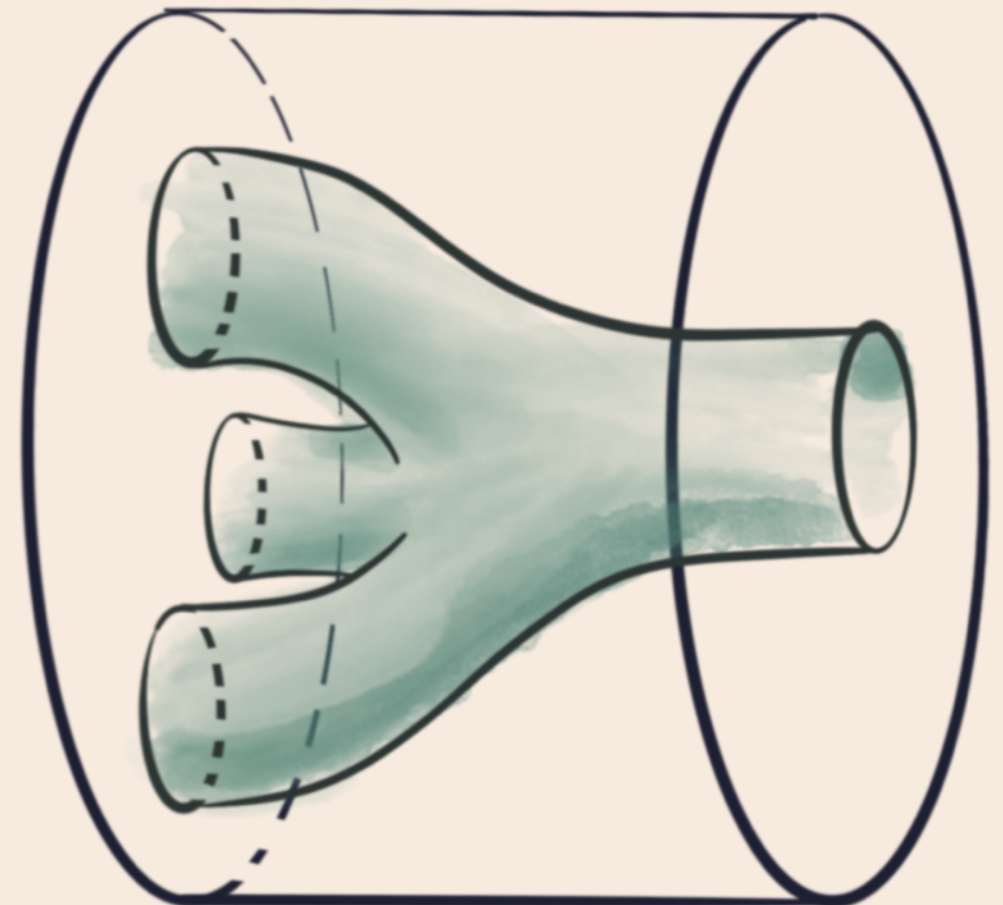
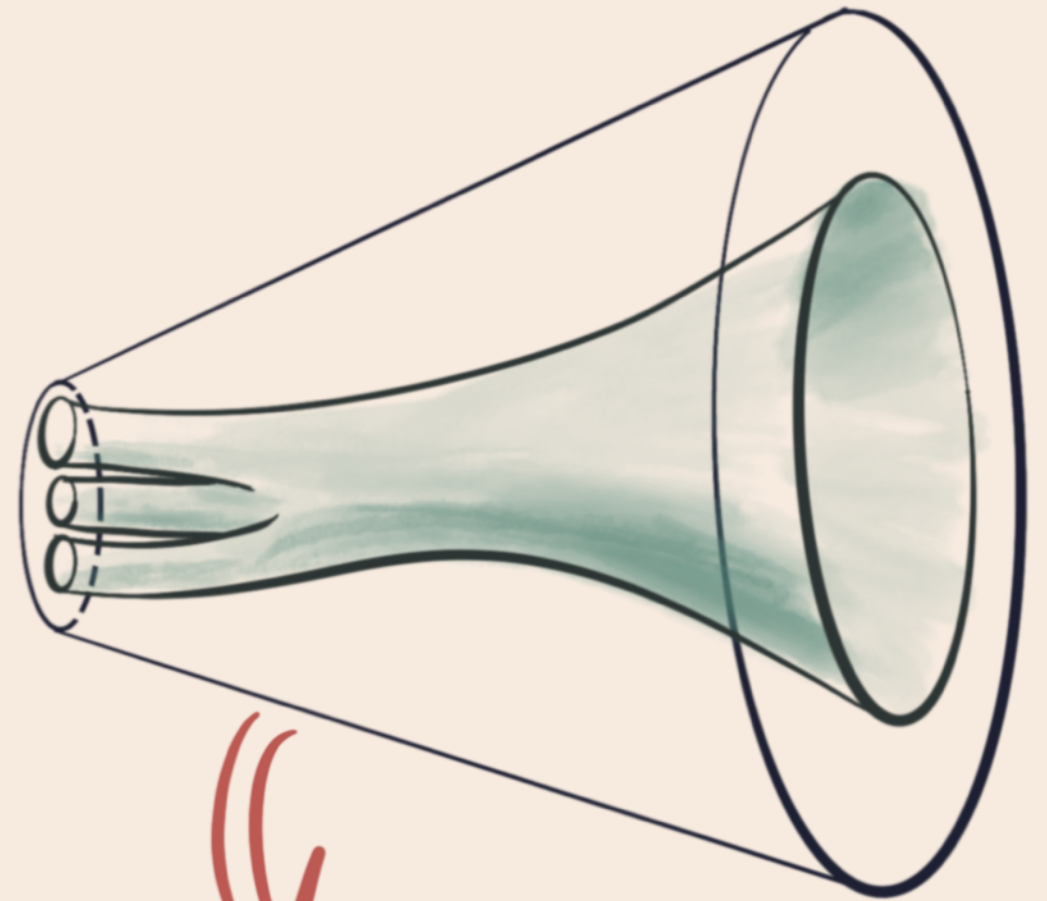


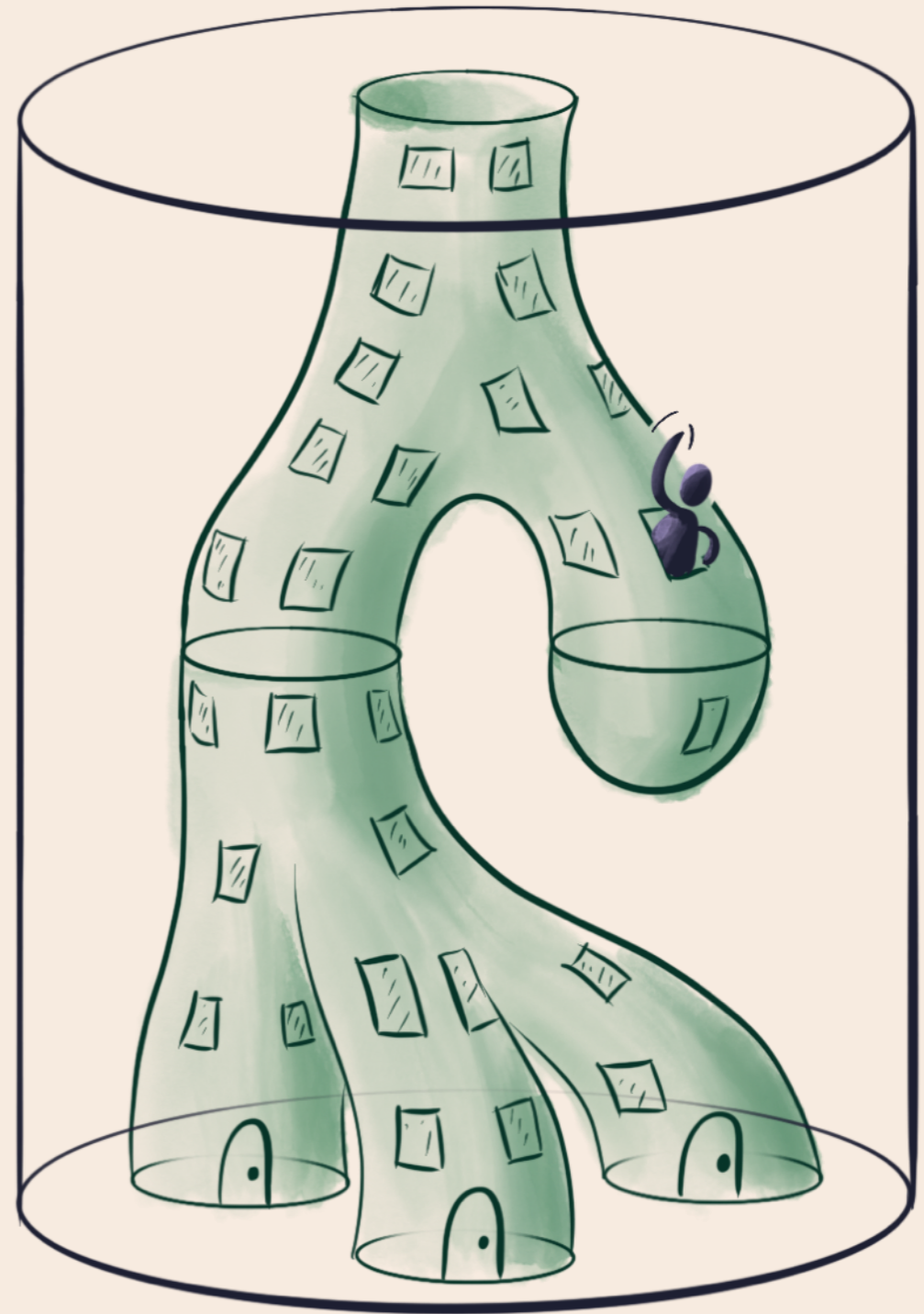
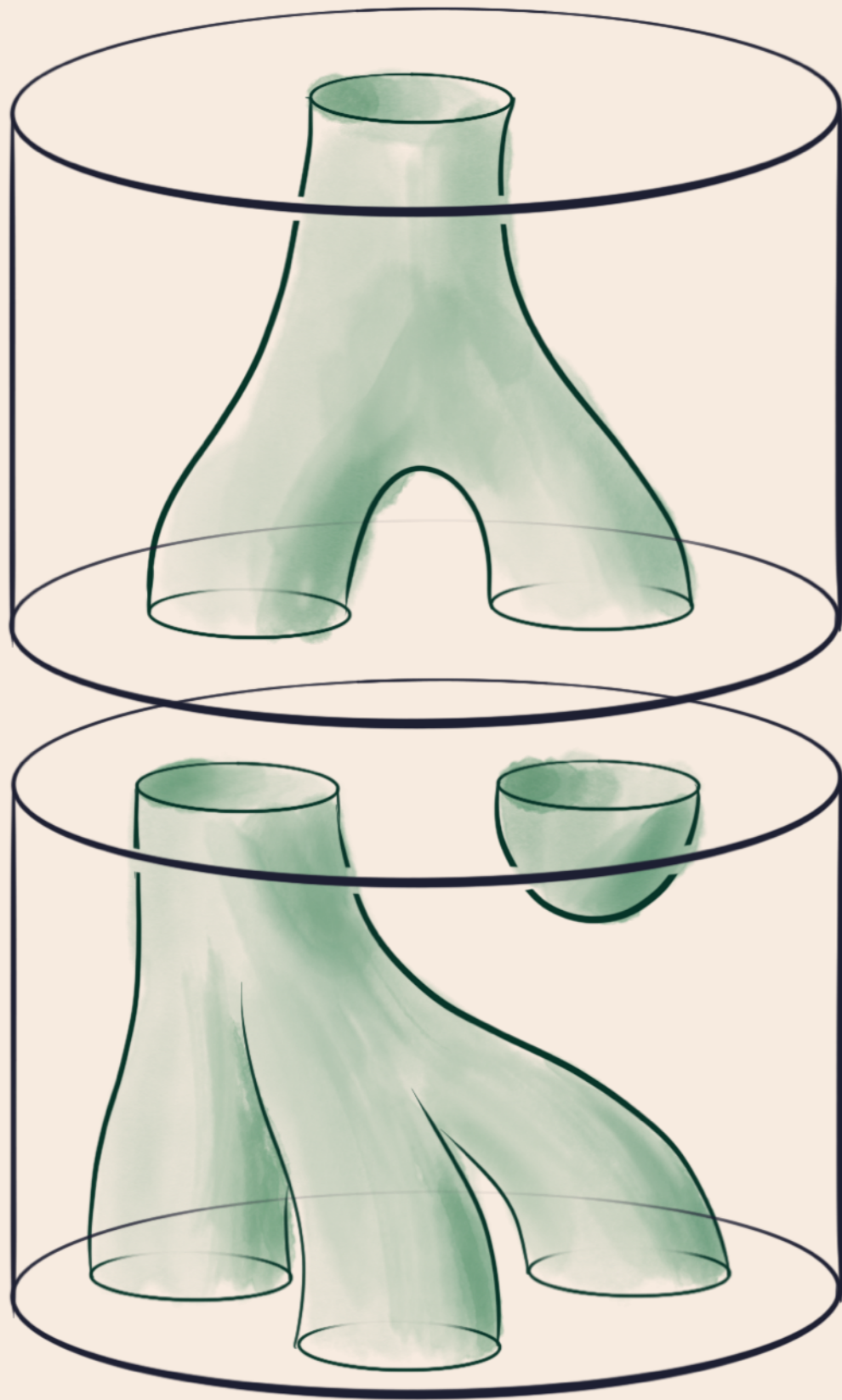
Need to count:



←
Small side

→
Big side





Pseudoholomorphic
Building

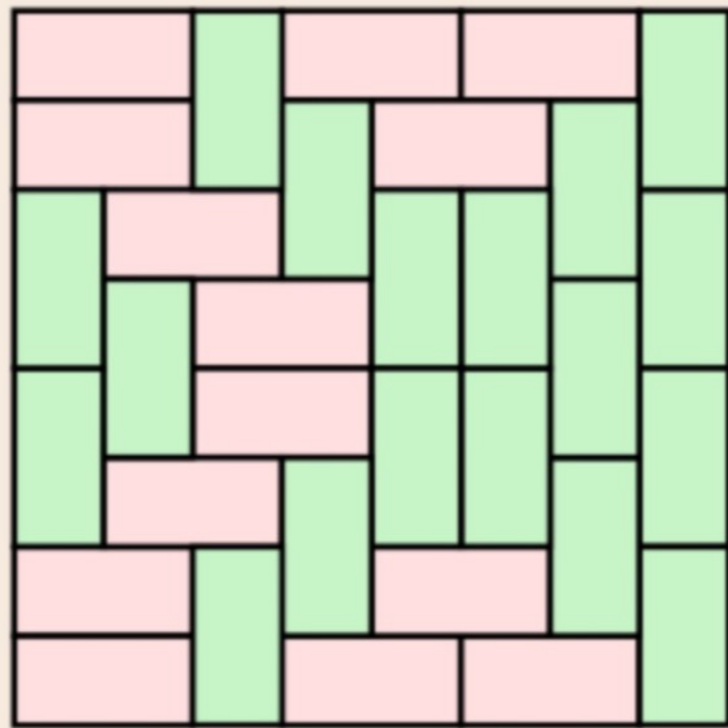
Big Math

- can walk inside
- can draw other math inside

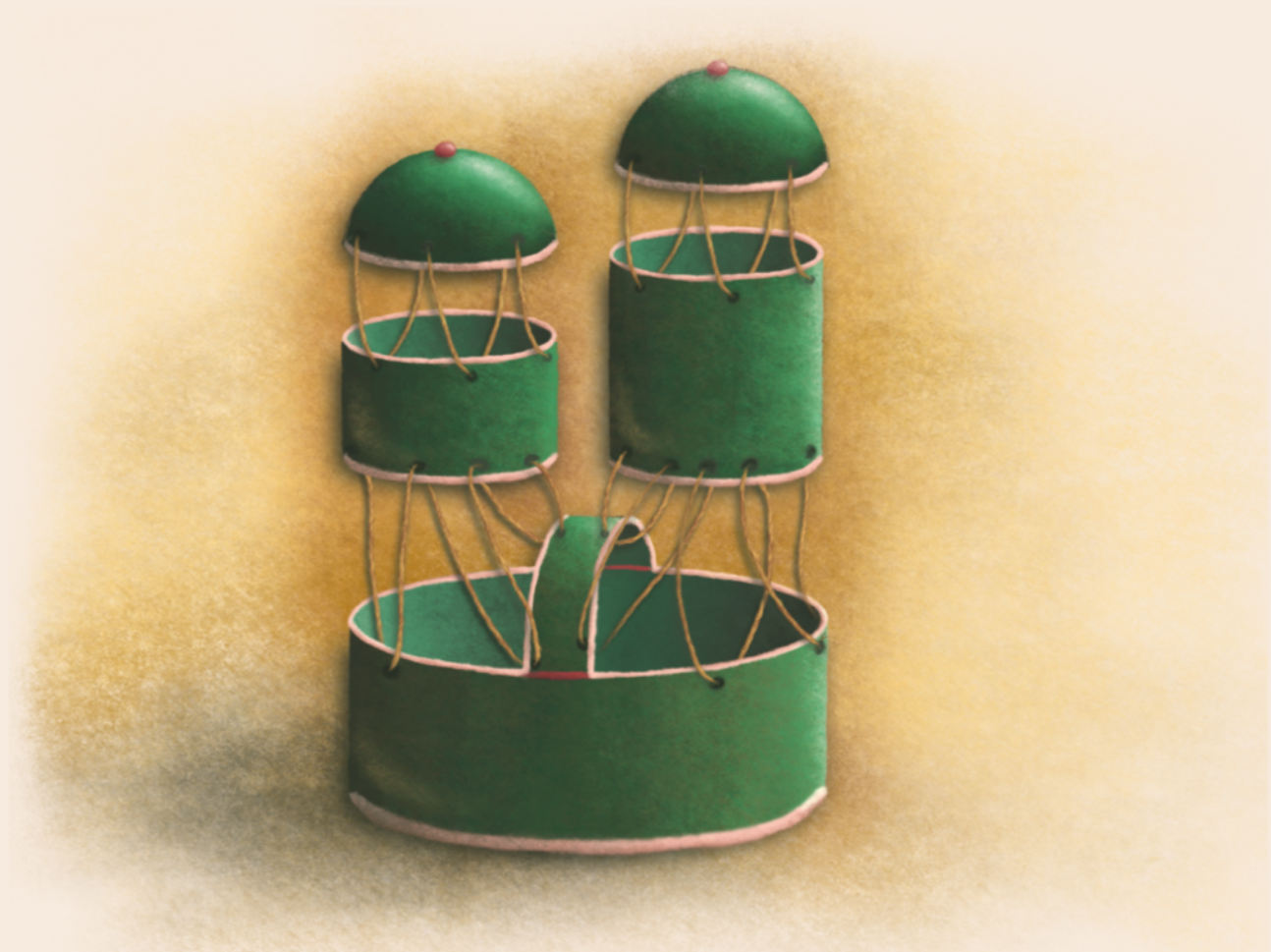
Small Math

- can manipulate
- can draw other math outside

Small Math examples:



Domino Tilings



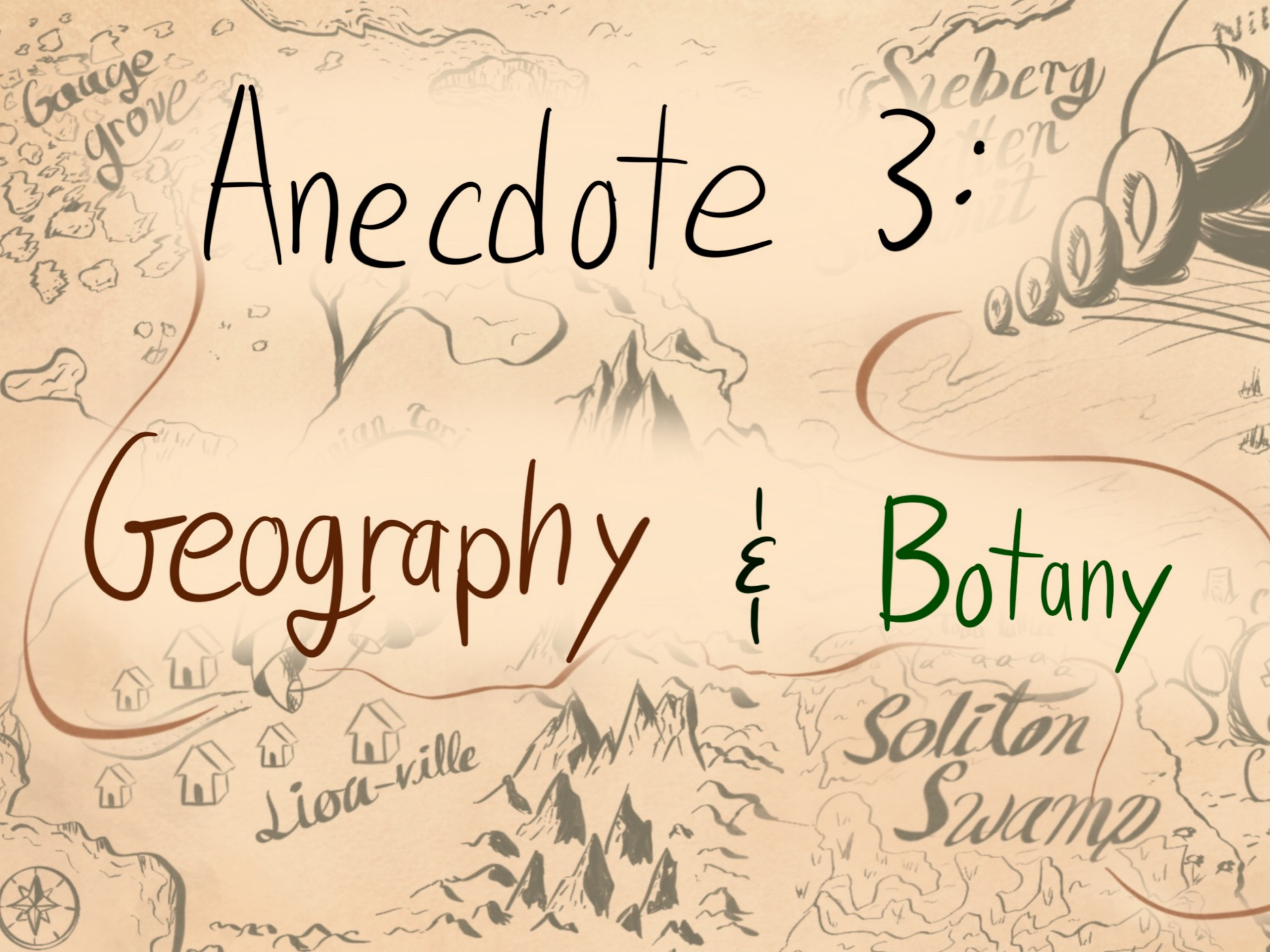
Sewing Manifolds

Math is Both!

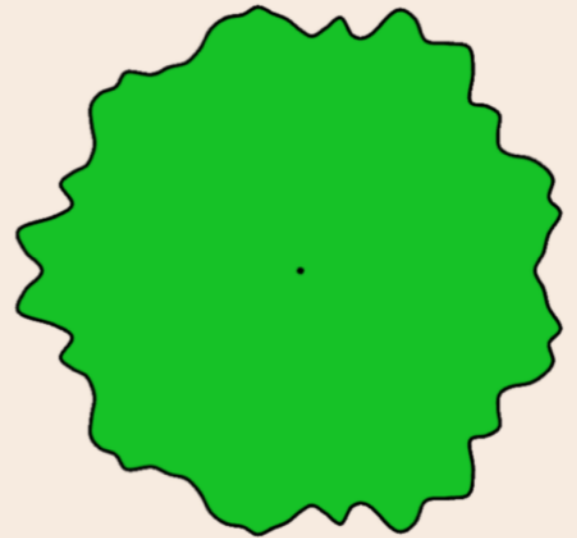
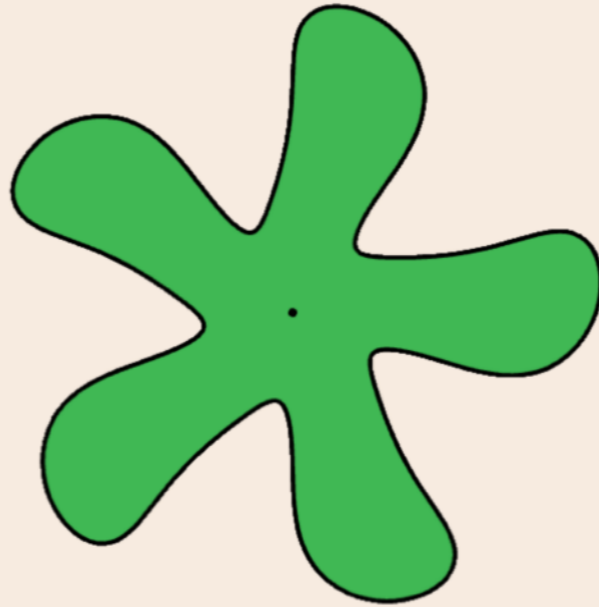
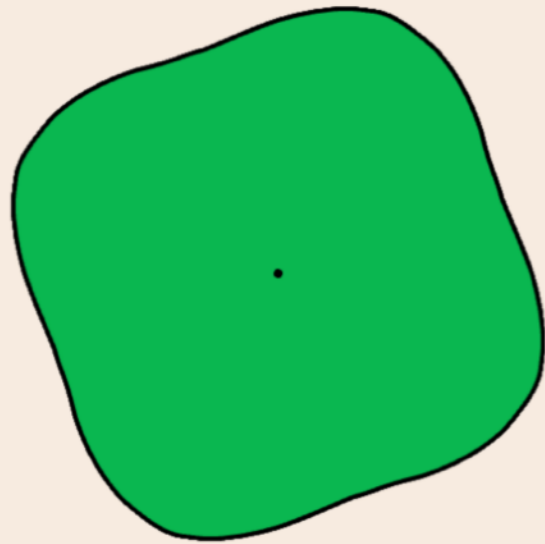


Anecdote 3:

Geography & Botany



Invariants (of curves)



Perimeter

P

Area

A

Goal: Classify all objects

Geography

What values of invariants are realized?

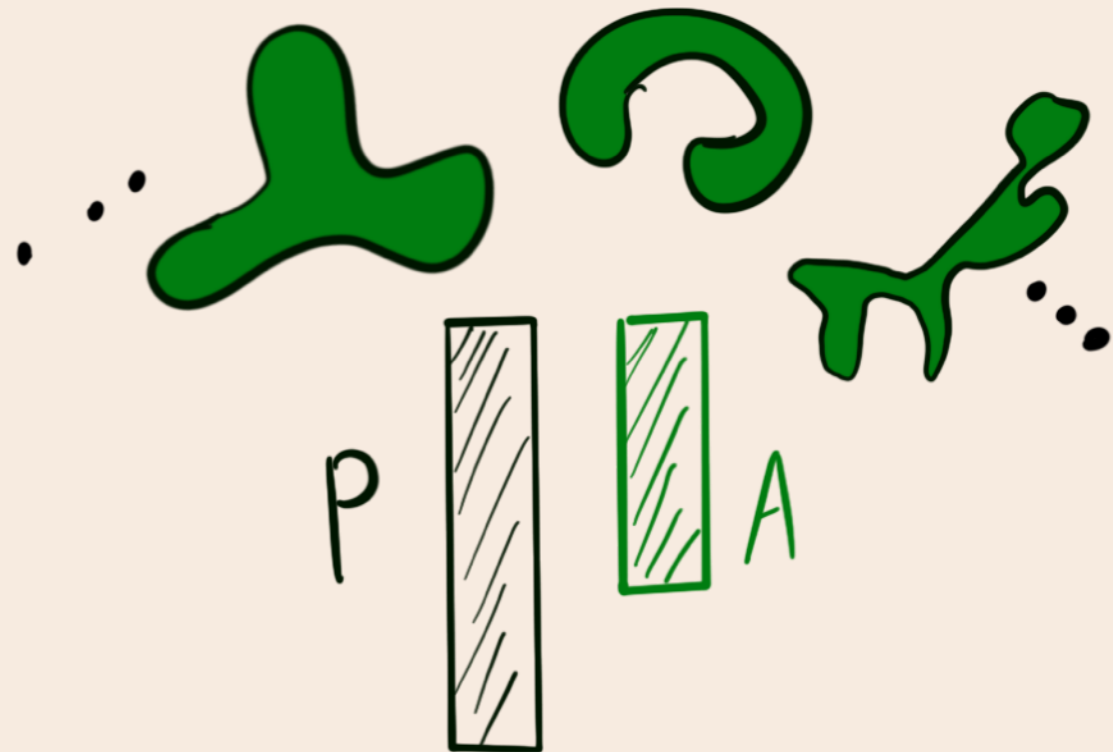
For curves:

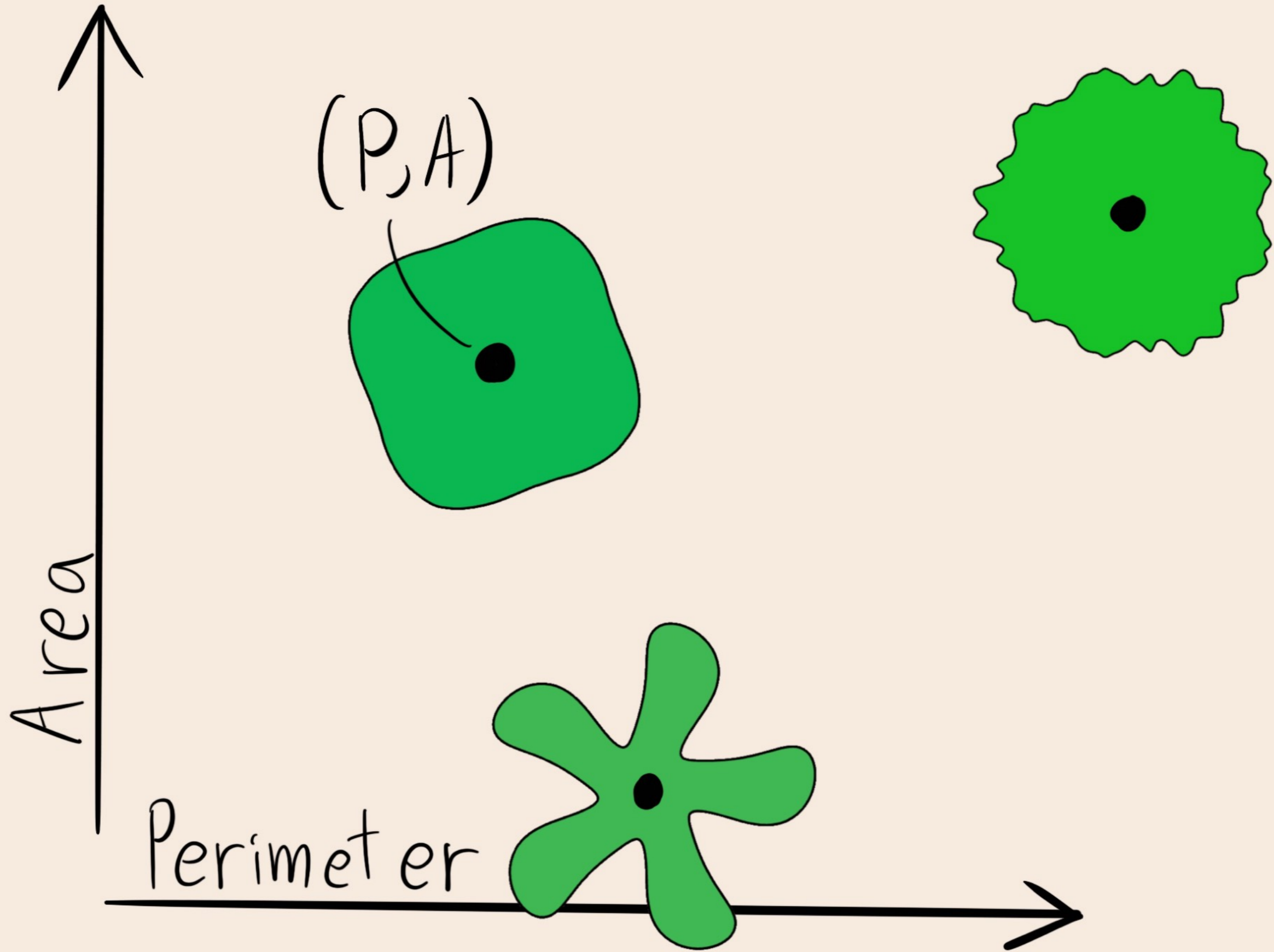
what pairs (P, A) are measurements of a curve?

Perimeter Area

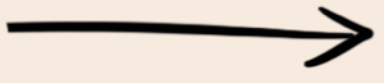
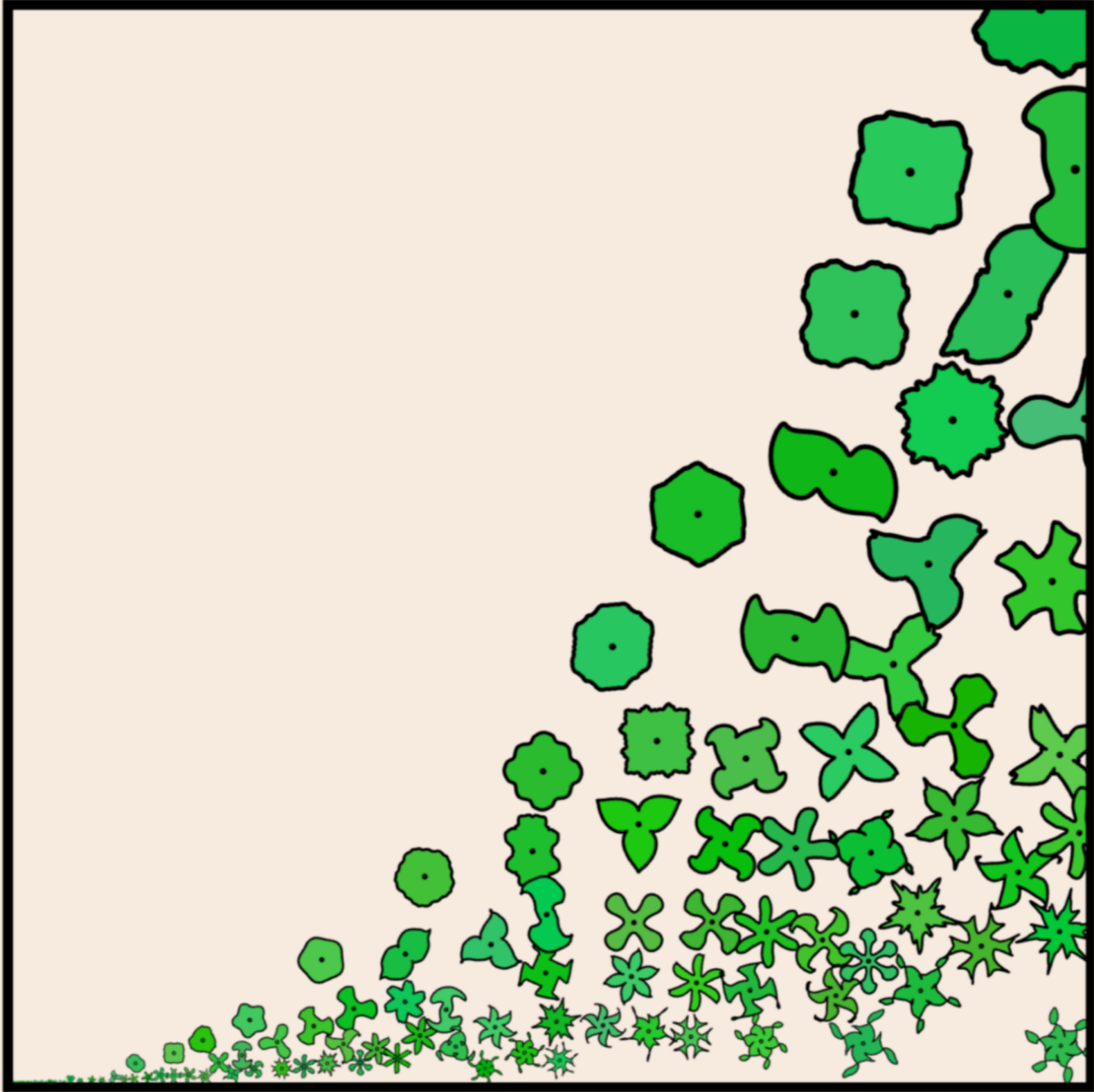
Botany

What objects realize a particular value of the invariants?



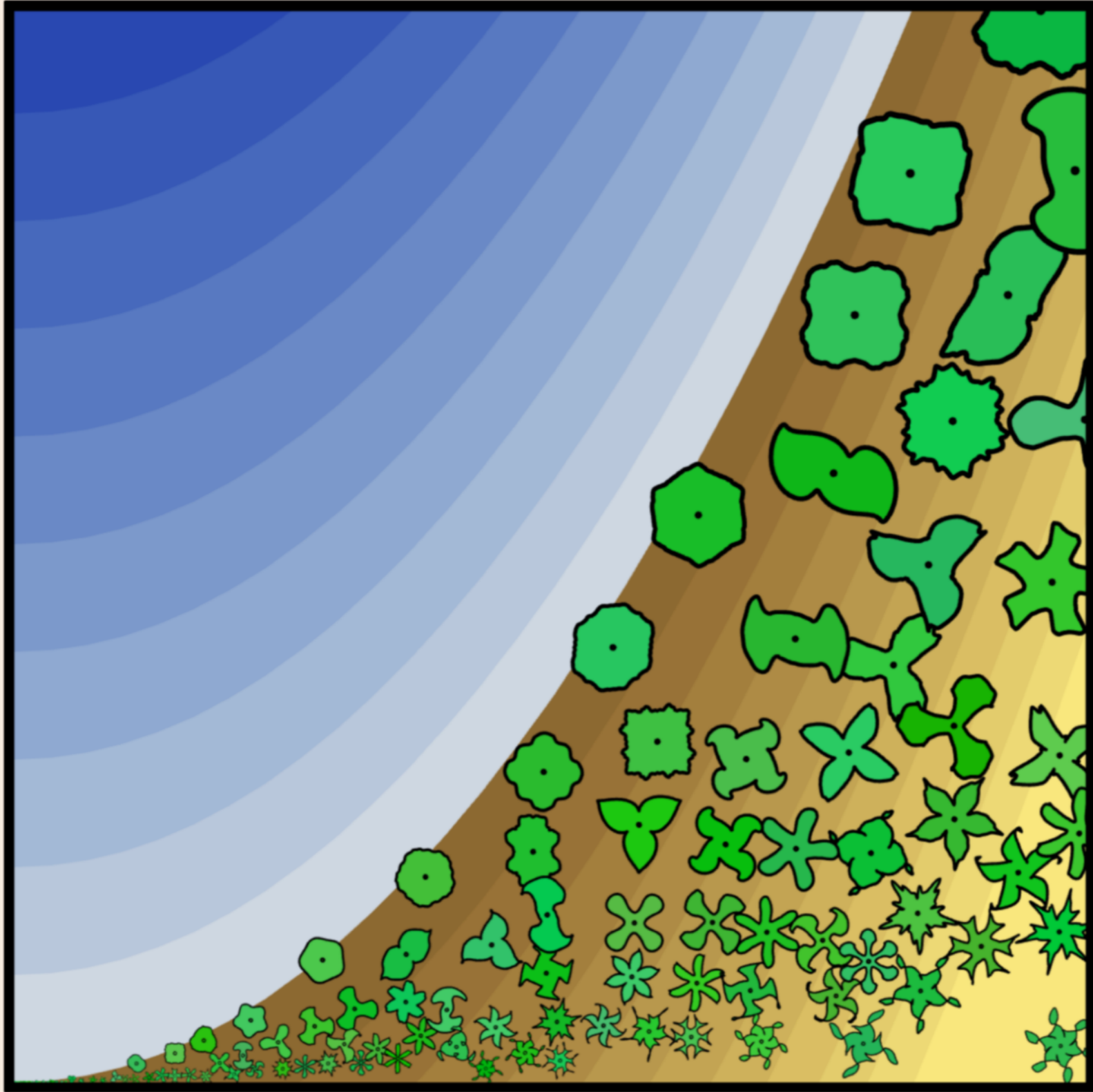


Area



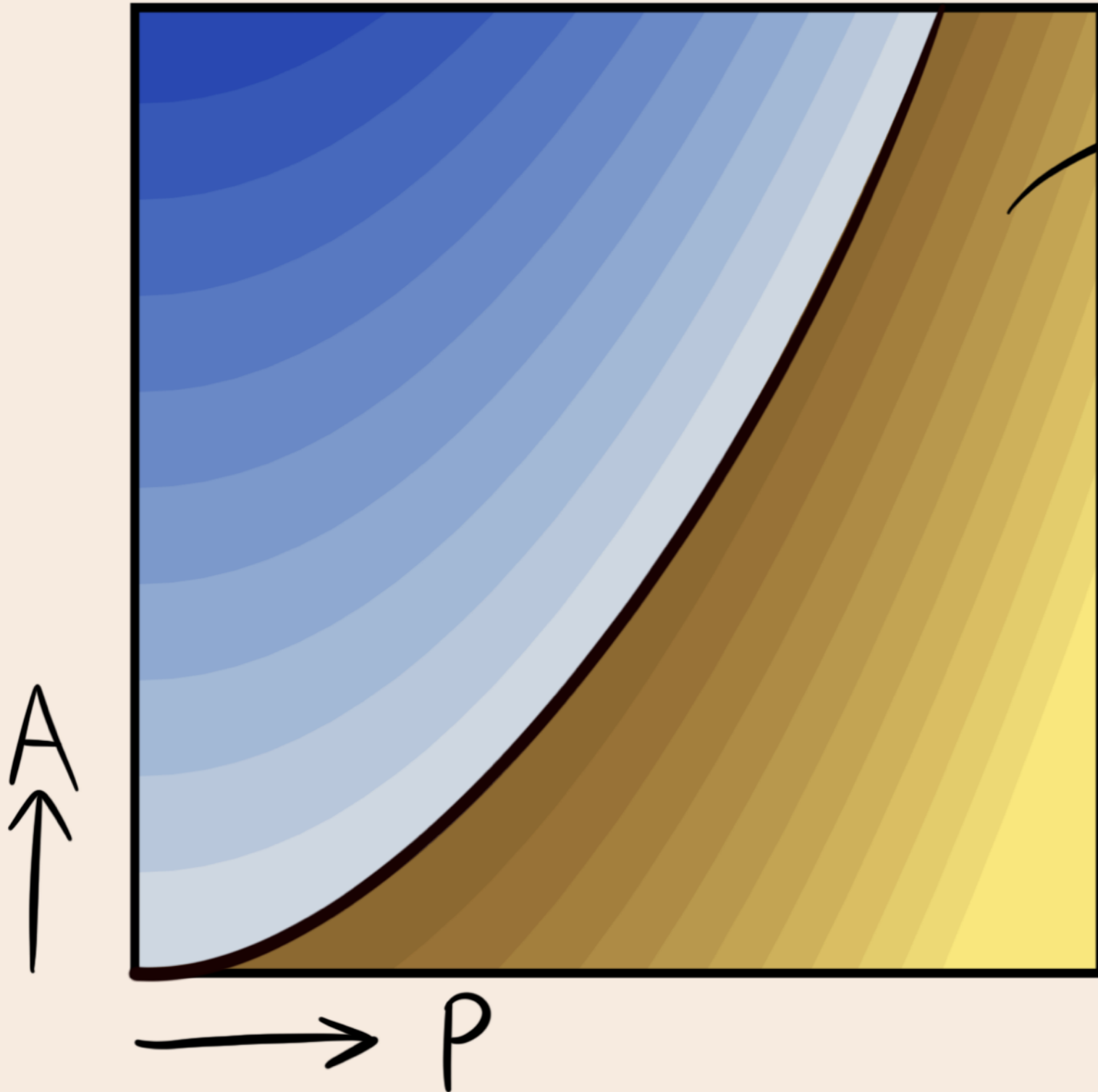
Perimeter

→ Area



→ Perimeter

Geography

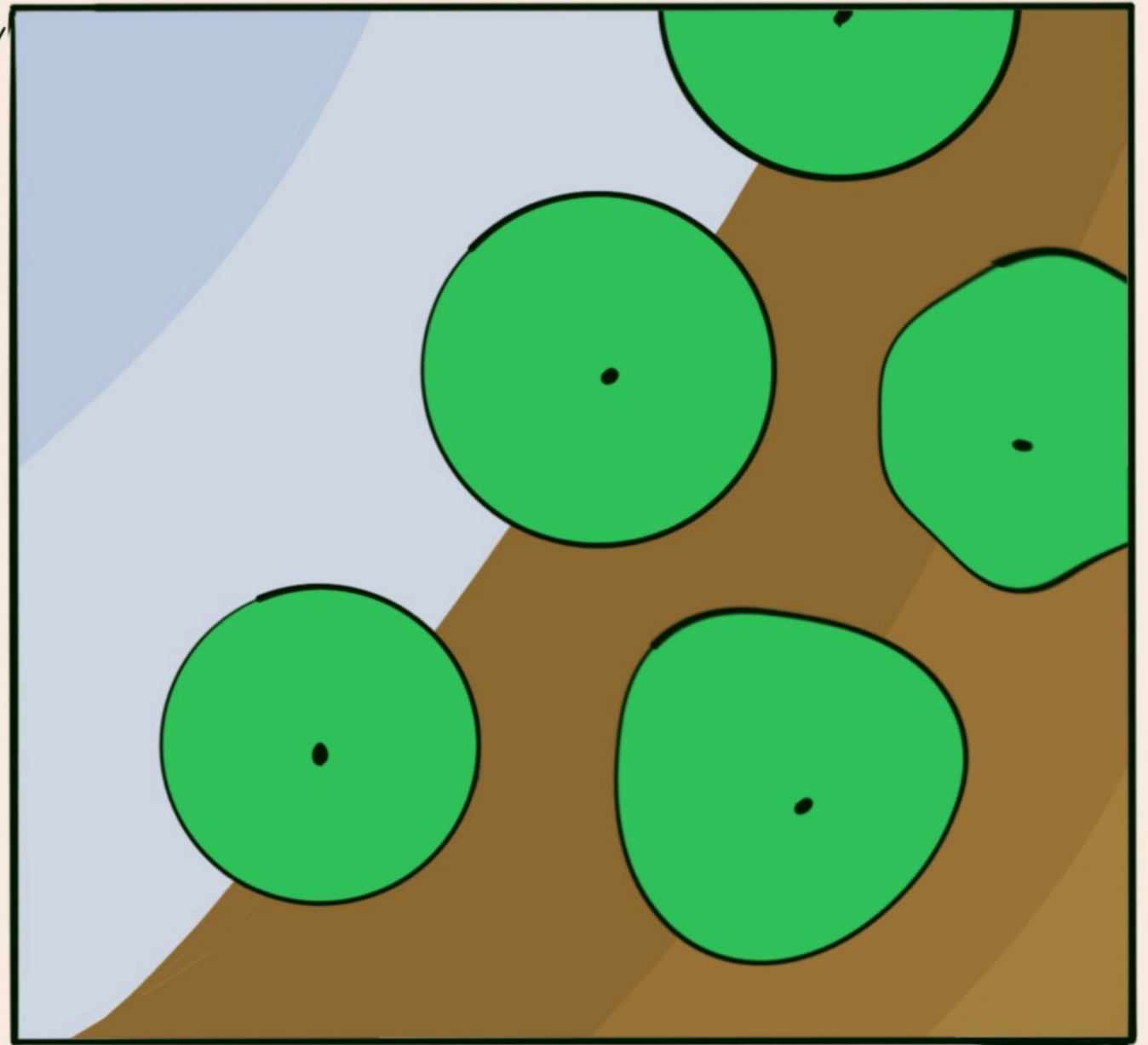
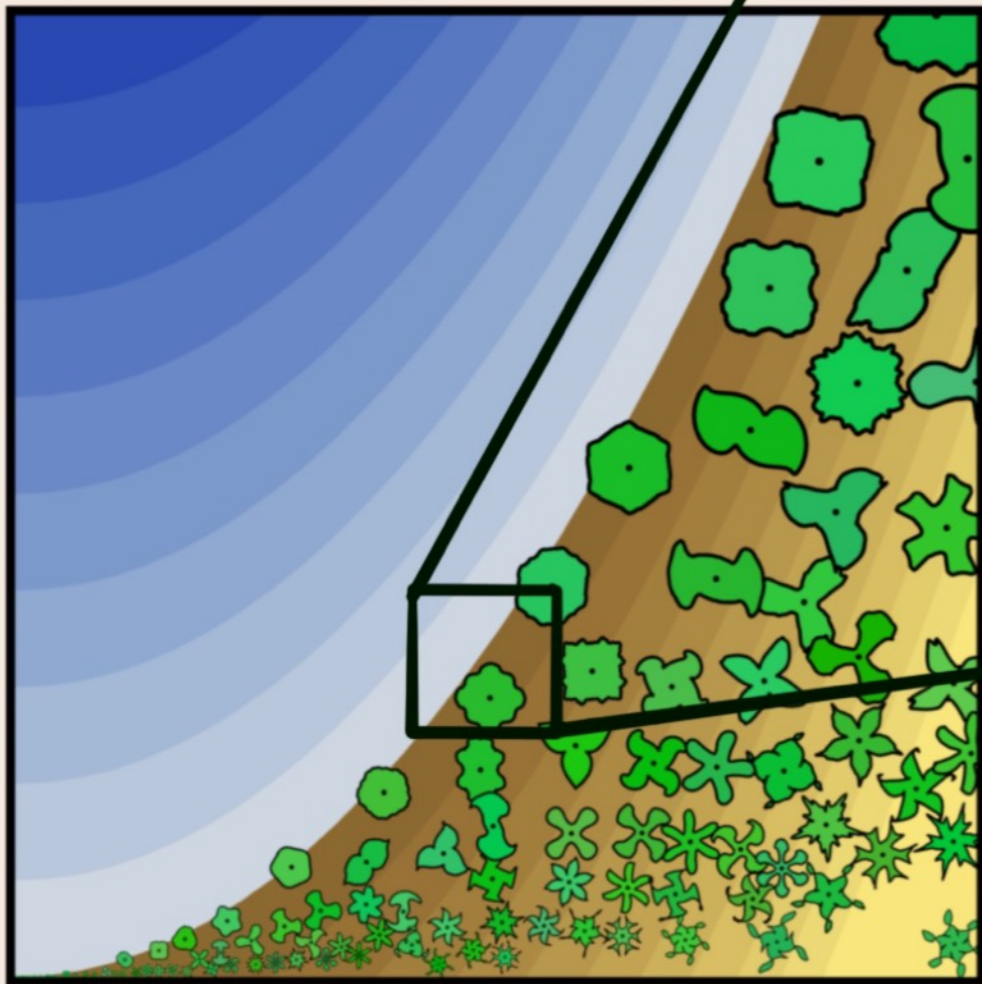


$$4\pi A \leq P^2$$

Isoperimetric
inequality

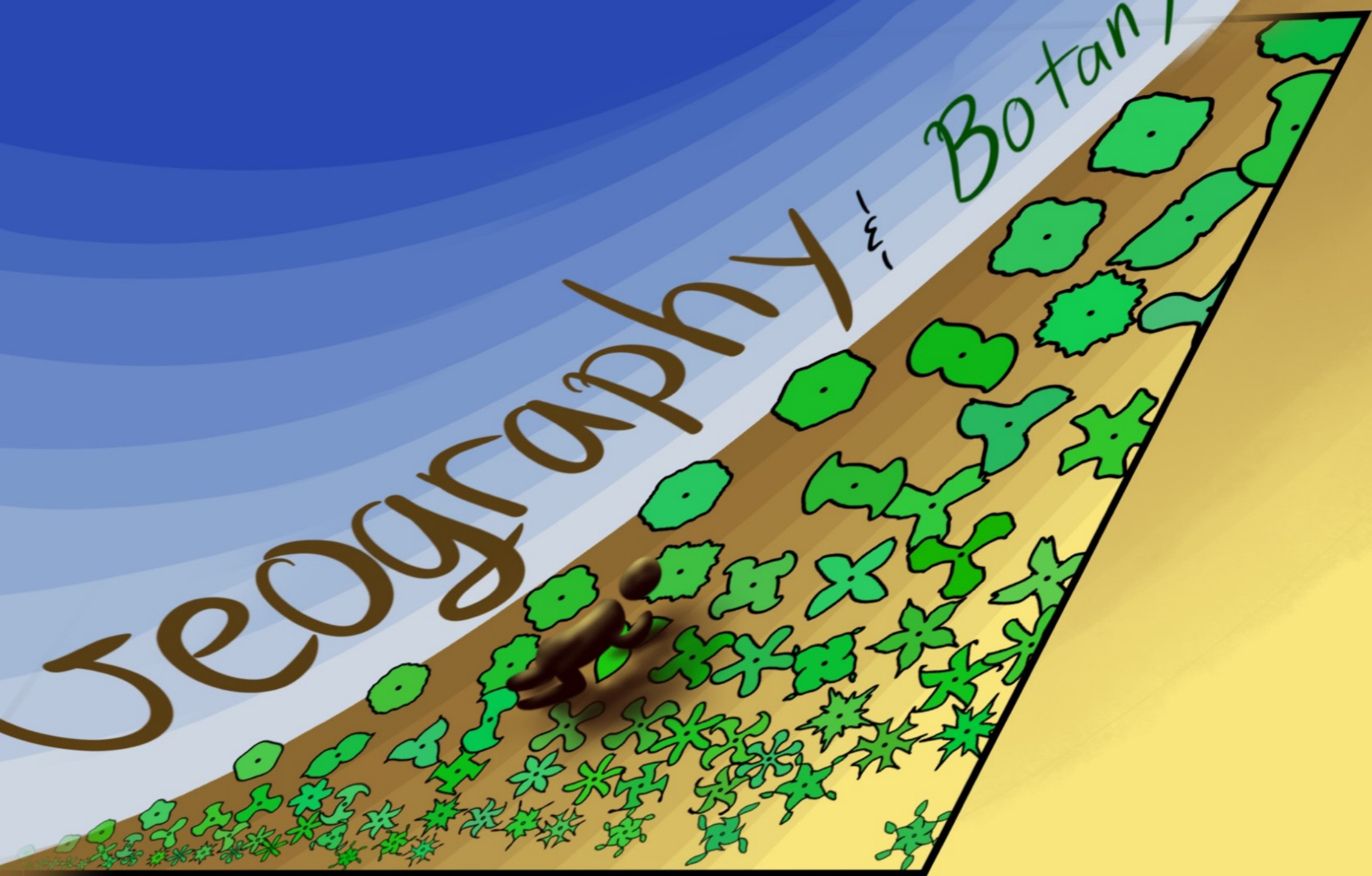
Botany

$4\pi A = P^2 \Rightarrow$
Curve is a circle

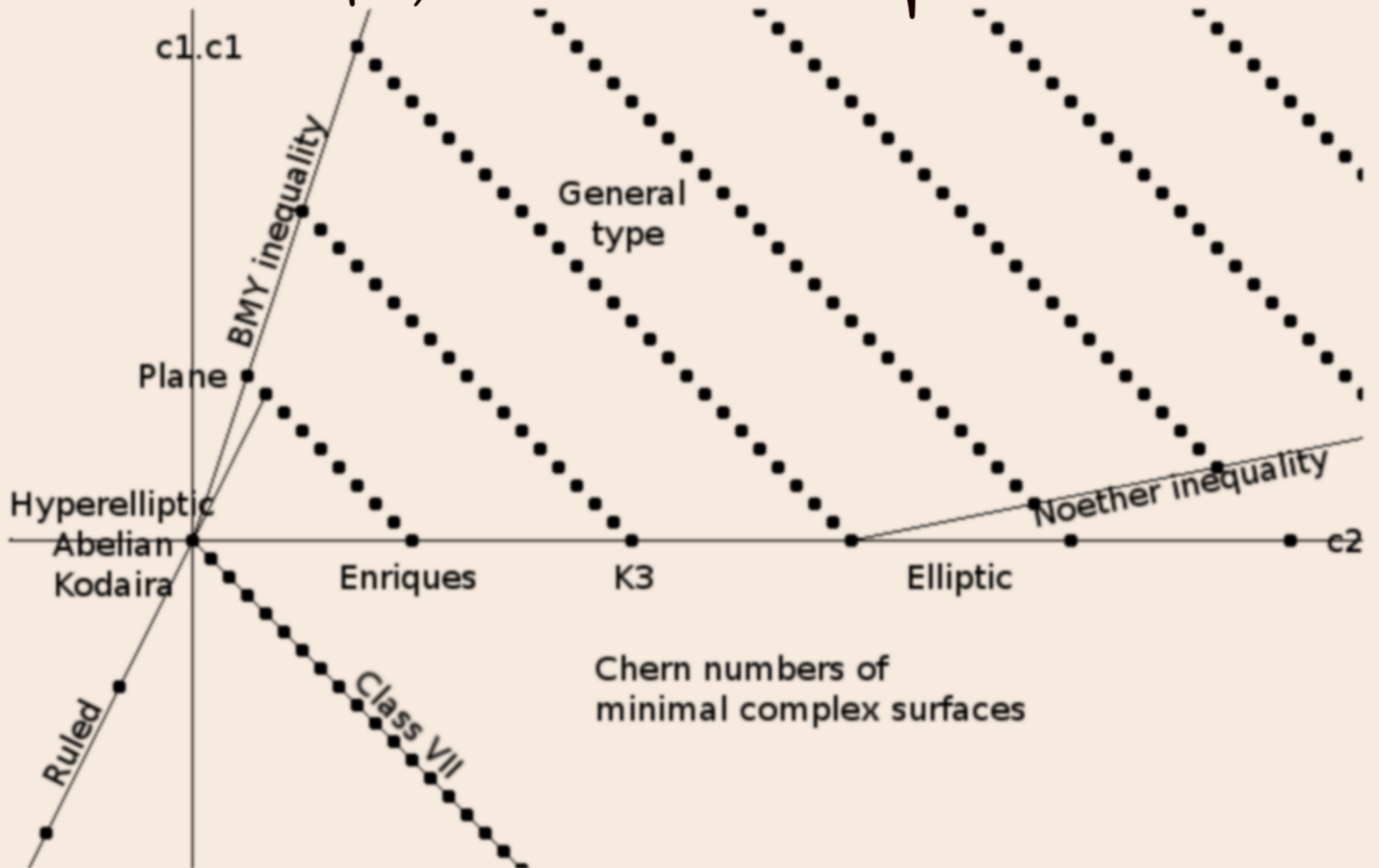


$4\pi A \approx P^2 \Rightarrow$
Curve is almost a circle

Geography ~ Botany



Geography of complex surfaces



Chern numbers of minimal complex surfaces

MAP OF THE MANDELBROT SET

Defined as the set of points in the complex plane that do not escape to infinity under iterations of

$$z_{n+1} = z_n^2 + C$$

where z and C are complex numbers, $z=0$, and C is determined by the initial position in the complex plane.

Rendered with 9,699,690 iterations, using distance estimation to draw the boundary.

mapped by Bill Javis
additional illustrations: Louis Martin

The Mandelbrot set boundary can be thought of as a circle that has been pinched in from the sides. The complex boundary is more infinitely close to touching itself, but never crosses. The external angles that map the boundary to a circle are measured here as whole number ratios between 0 and 1. This mapping is proven for all such rational fractions.

The boundary of the Mandelbrot set contains many patterns, some of which become quite complex when the boundary is magnified. The path taken when zooming determines the characteristics of the pattern. For example, the local bulb numbering can be observed by counting the number of branches at the tip of (and including) the local neck. The densely covered region to the right was found by zooming to the tip of local bulb 6 of local bulb 5 of local bulb 3 of bulb 5 on the main cardioid. The resulting period of the bulb is then 560, the product of 6-5-4-5-4.

External angles of 0 through 25 are 24, in a counter-clockwise direction.

(below) The largest bulb between any two bulbs has a period equal to the sum of the first two. In this way, the Fibonacci sequence can be found. Between 2 and 3, the largest bulb is period 5; between 5 and 3, the largest bulb is period 8; and after that, the largest bulbs are 13, 21, 34, 55, etc.

The Needle begins at $C = -1.01155$, where the bulbs to the west of the Main Cardioid continue to double their periods until reaching infinity. The view below (only 10° arc) shows how the branches continue to pile up. In the limit, they will become infinitely dense at this point, known as the Myrberg-Fractalbulb point.

The border has a caving off of the bulbs, the area in number as they go into the valleys, forming spirals and arcs (upper right).

JULIA MEDALLIONS

Medallions can be found along the extra tendrils that grow off every minibrot, and are related with patterns inherited from the minibrot. For example, the medallions below have lightning shaped tendrils inherited from the Lightning minibrot around which they are found (locations indicated by colors). The medallions resemble the Julia sets that correspond to their location. Julia sets are fractals that use the same equation as the Mandelbrot set, but differ in that C is constant for the entire image and z varies with the initial position in the complex plane.

THE NEEDLE

The bifurcation diagram below shows the period doubling cascade, which occurs along the real axis in the Needle.

Elephant Medallion, Brain Medallion, Branch Medallion, Triple Spiral Medallion, Lightning Minibrot, Seahorse Medallion, Double Spiral Medallion, Needle Medallion, Sceptor Medallion

Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line. -Benoit Mandelbrot, father of fractal geometry

MINIBROT ISLANDS

The border of the Mandelbrot set contains an infinite number of minibrots - small islands that resemble the whole, surrounded by intricate patterns unique to each island. This self-similarity at every scale level is an important characteristic of fractal geometry. The minibrot to the left was found at the center of an elephant medallion from a tendril of the Lightning minibrot. It is surrounded by more and more holes of the medallion, which double to infinity at the border.

PERIOD 2 MAIN BULB

The Mandelbrot set is a geometric shape much like a circle as a whole. It differs in that the boundary is infinitely complex and reveals endless detail upon magnification.

PERIOD 5 BULB

Orbits in the west side of the Main Cardioid have star-shaped patterns. This example orbit, for $C = -0.15 + 0.1i$, creates a five-pointed star because it begins beneath the Period 5 bulb.

Orbits in the east side of the Main Cardioid have spiral-shaped patterns. The example orbit above, for the point $C = 0.15 + 0.1i$, creates a four-sided spiral because it originates beneath the Period 3 bulb.

The example orbit below, for the point $C = 0.285 + 0.03i$, creates a small logarithmic spiral inward, exiting and reentering the set.

PERIOD 1 MAIN CARDIOID

Complex numbers have a real part and an imaginary part, represented as $a + bi$, where a is the real part, b is the imaginary part, and i is the square root of -1. A complex number can then be located in the plane by forming the coordinate point (a, b) . To square a complex number, it is written $(a + bi)(a + bi)$, which becomes $a^2 + abi + bai - b^2$, simplifying in the new complex number of $(a^2 - b^2) + (2ab)i$. Testing a point to see if it belongs in the Mandelbrot set is done by squaring the current value and adding a constant value, which is determined by the initial location in the complex plane. After many iterations, if the point is at a distance of less than 2 from the origin, it is considered part of the set.

OCEAN OF FINITELY

A point in the center of any given bulb has a periodic orbit, meaning that it will repeat its values under iteration. The orbit will always pass through the origin, and the number of times it moves before repeating determines the period number. This can be seen in the example orbit for the center of the Period 3 bulb (right).

Equipotential lines show how many iterations it takes for a point to reach a very large radius (here approximately 10^{100}).

As soon as a point becomes further than a distance of 2 away from the origin, it is guaranteed to never come back and will eventually envelope an infinity.

Spirals (above) can be found on the Mandelbrot set, perpendicular to the boundary.

Zooming in on a point of a complex, together are spirals are not and others do.

Looking closer, spirals are doubled in the complex plane, the same manner.

The spirals can go to infinity as the spiral, it is guaranteed to never come back and will eventually envelope an infinity.