Is Math BIG OR Small

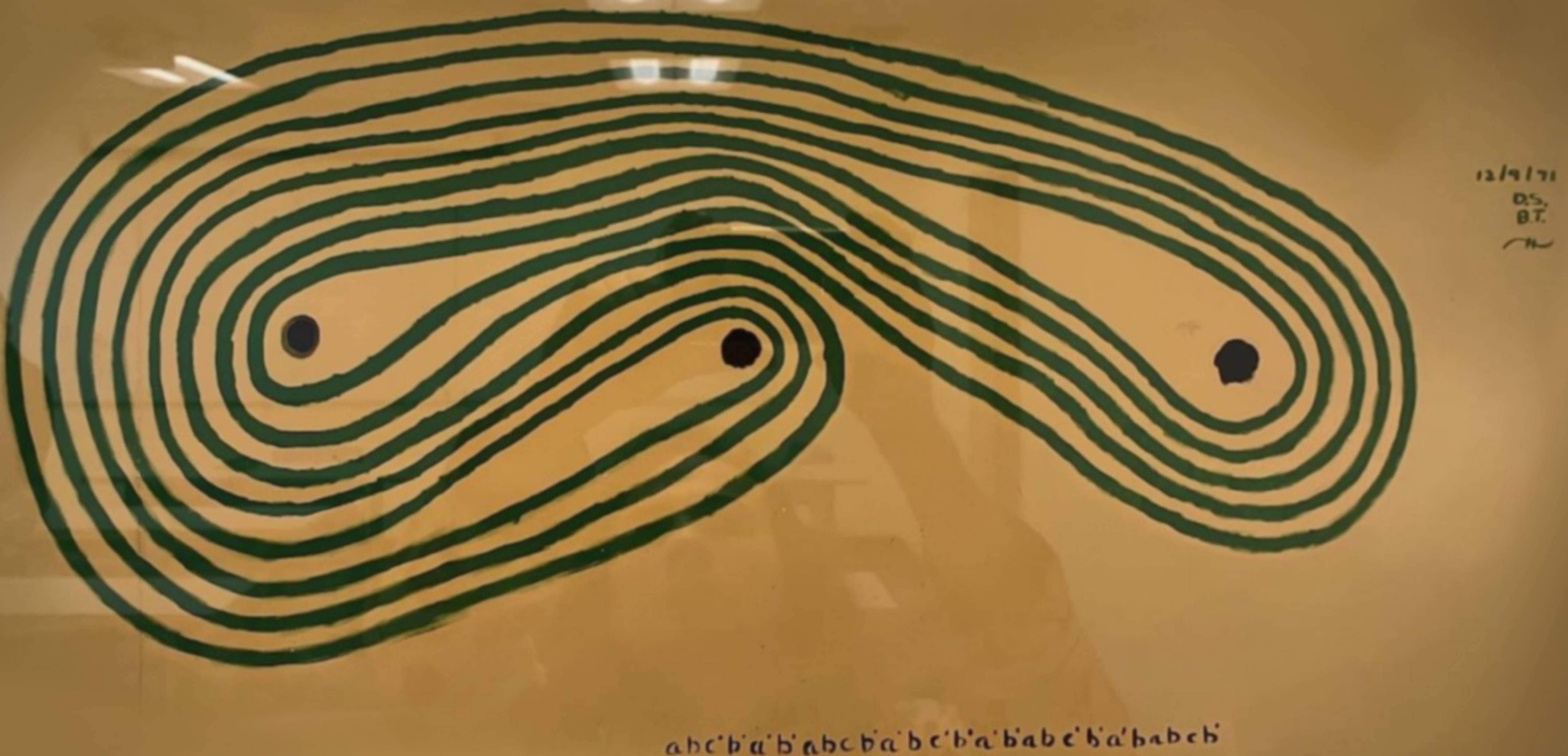
Imagine a torus:

- How big is it?
- Why?

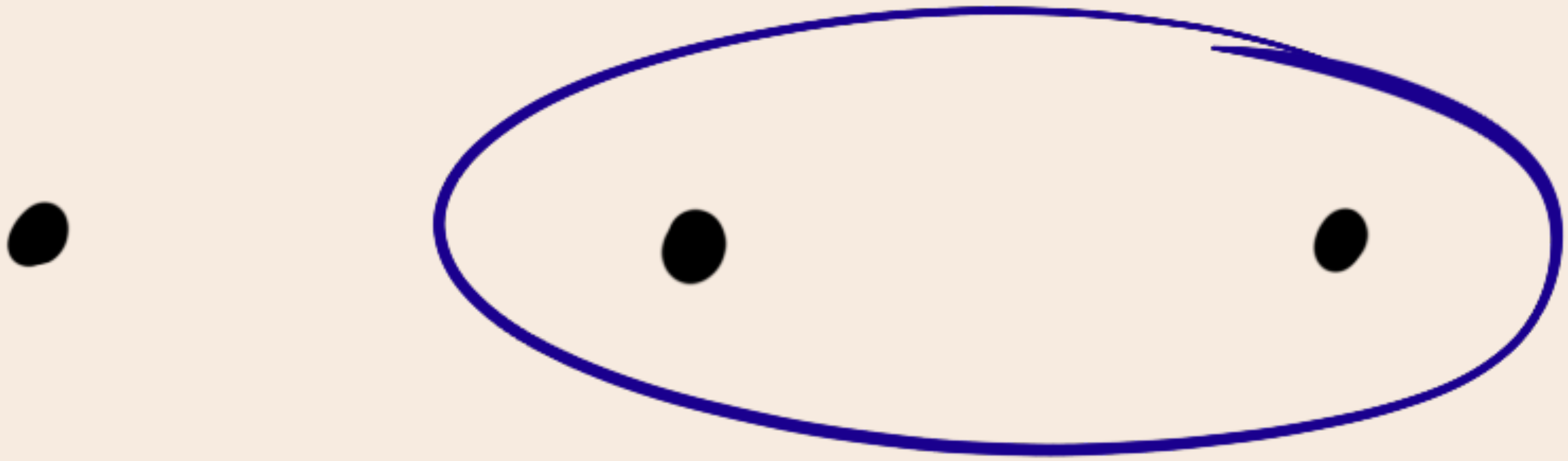
Anecdote 1:

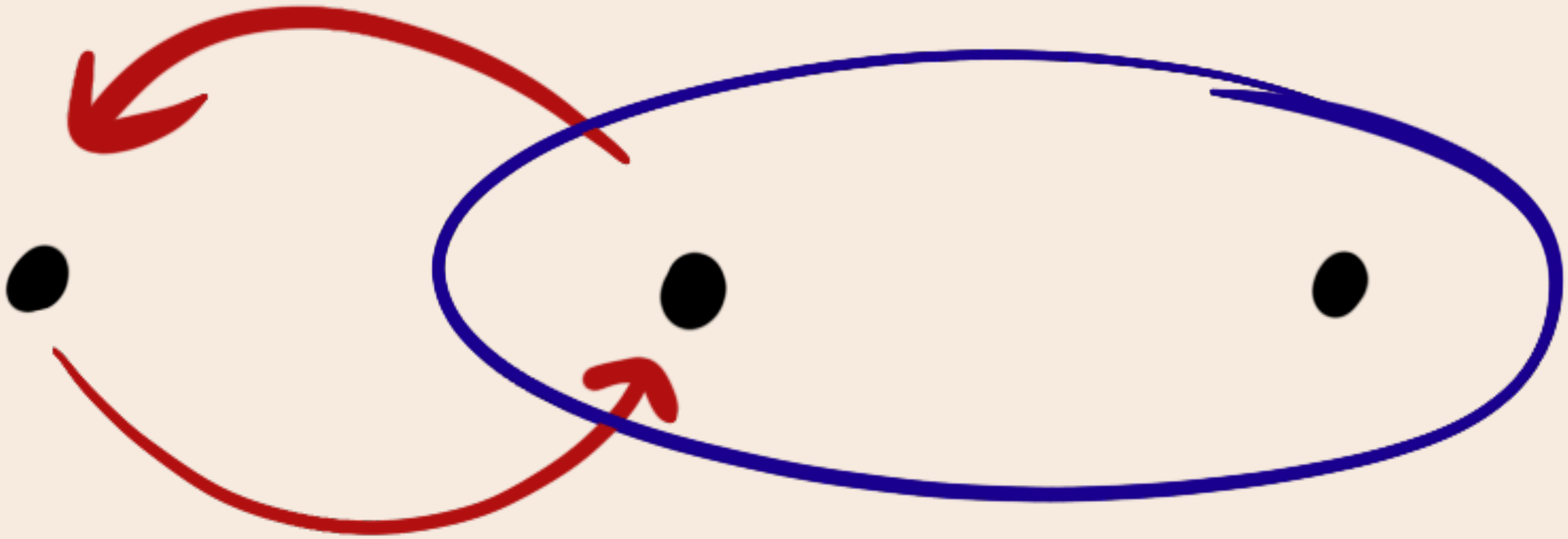
TRAIN TRACKS

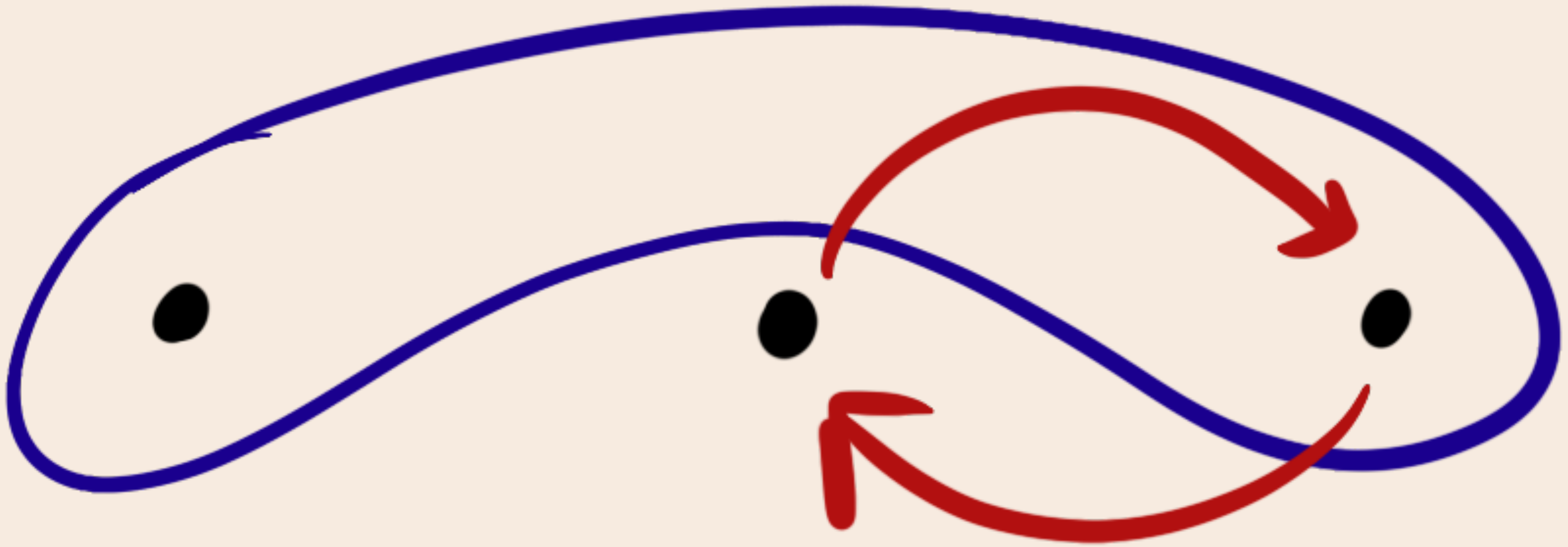
Evans Hall, Built 1971

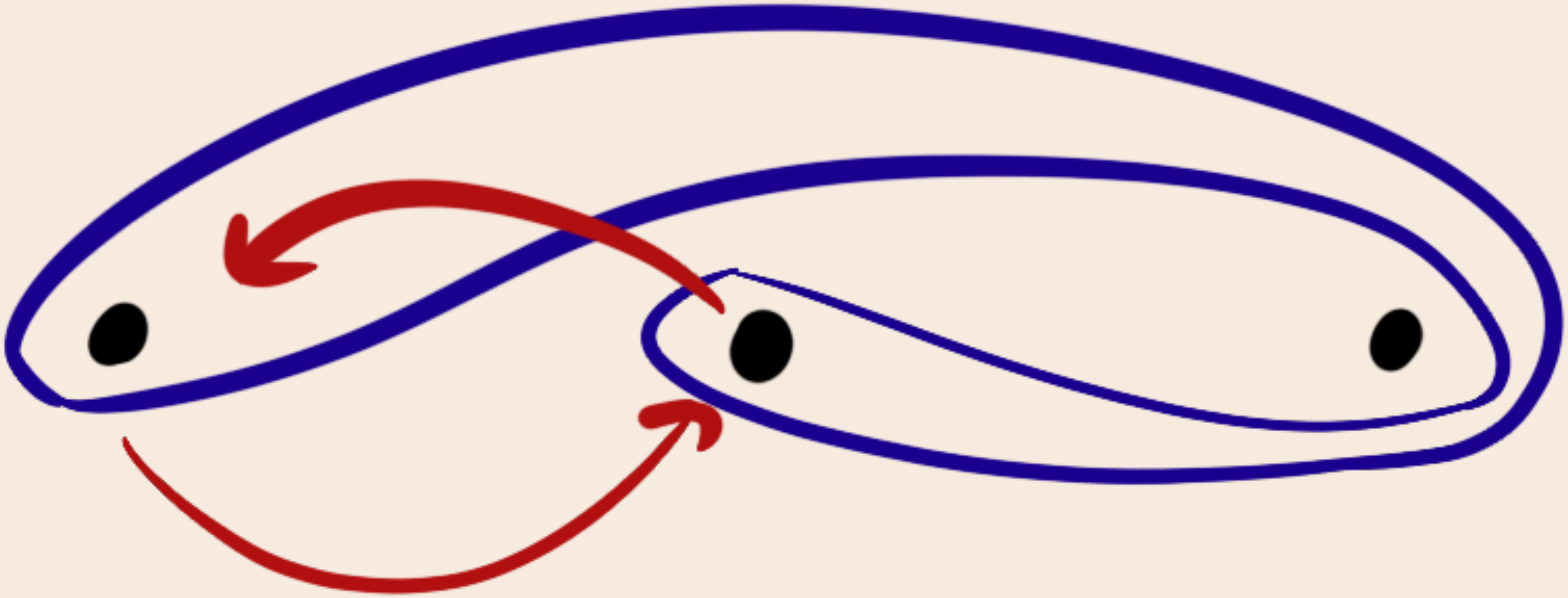


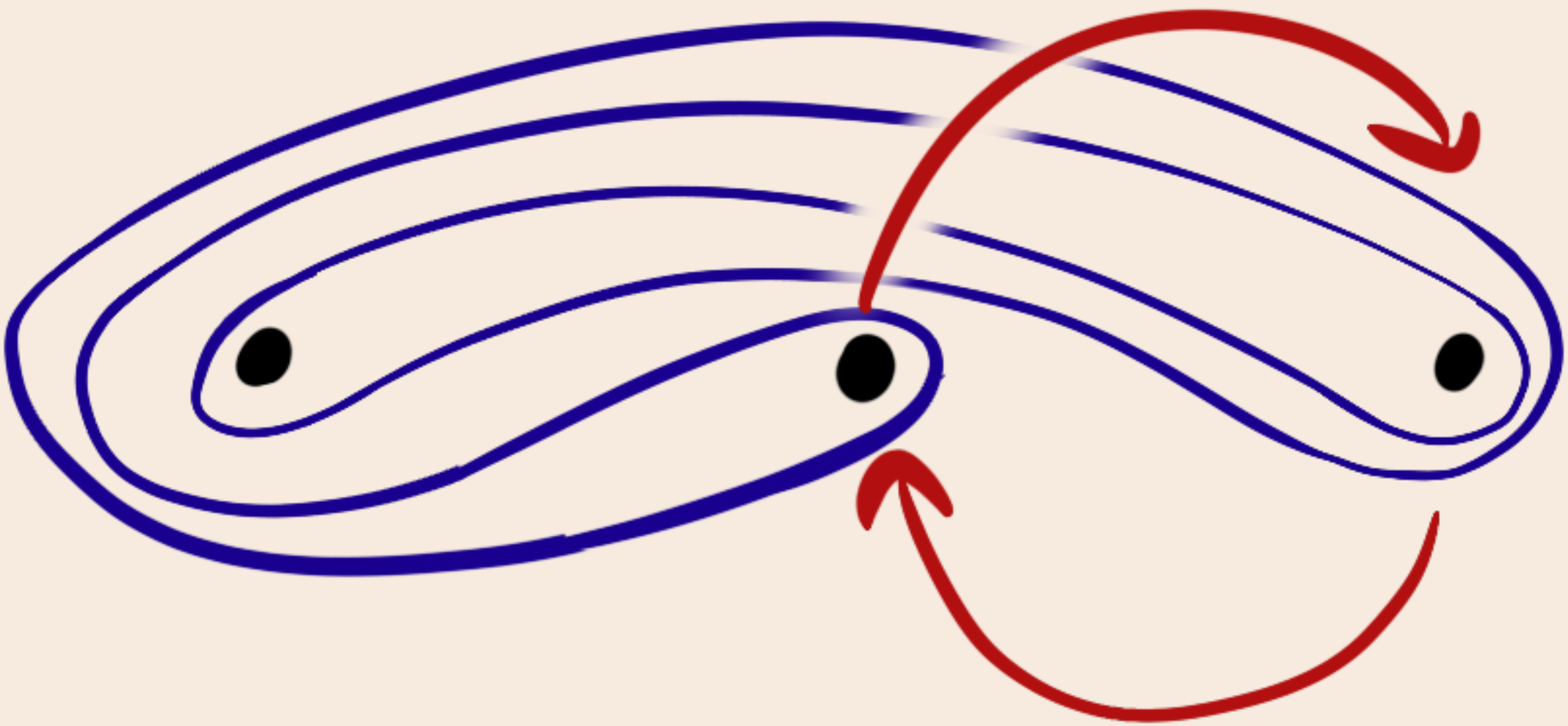
Bill Thurston & Dennis Sullivan, 1971

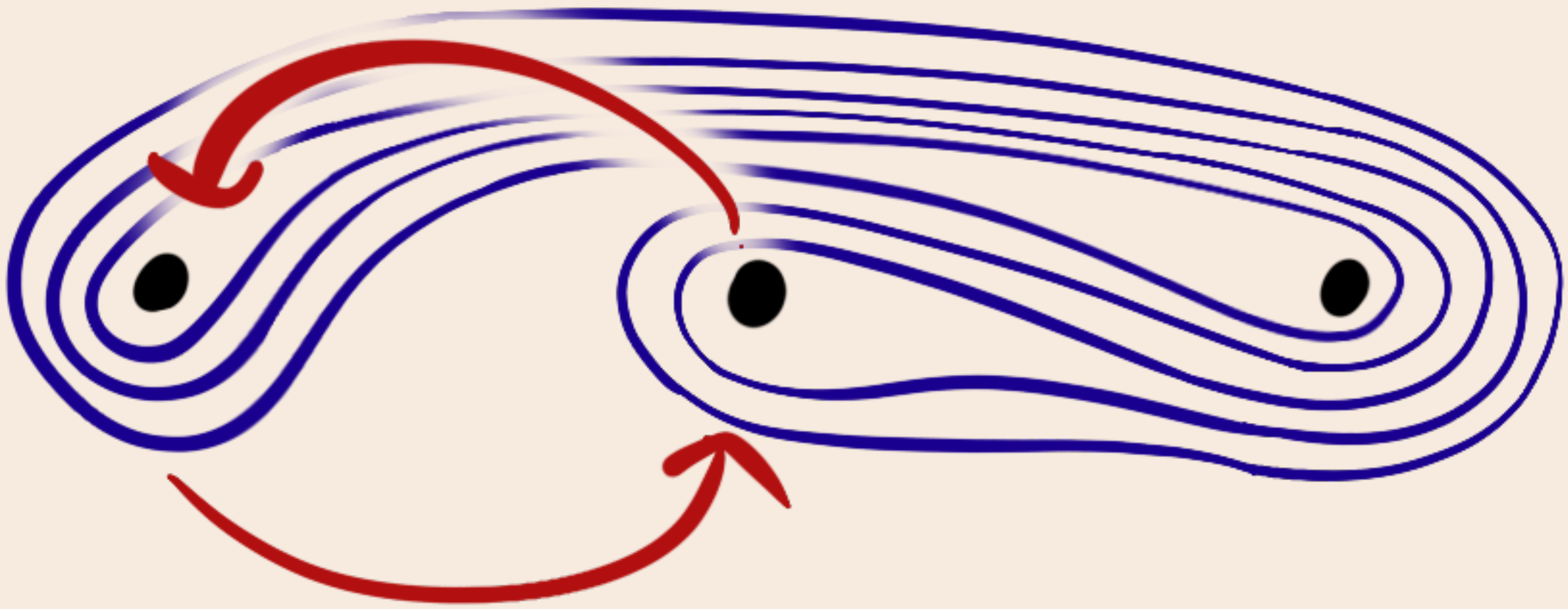


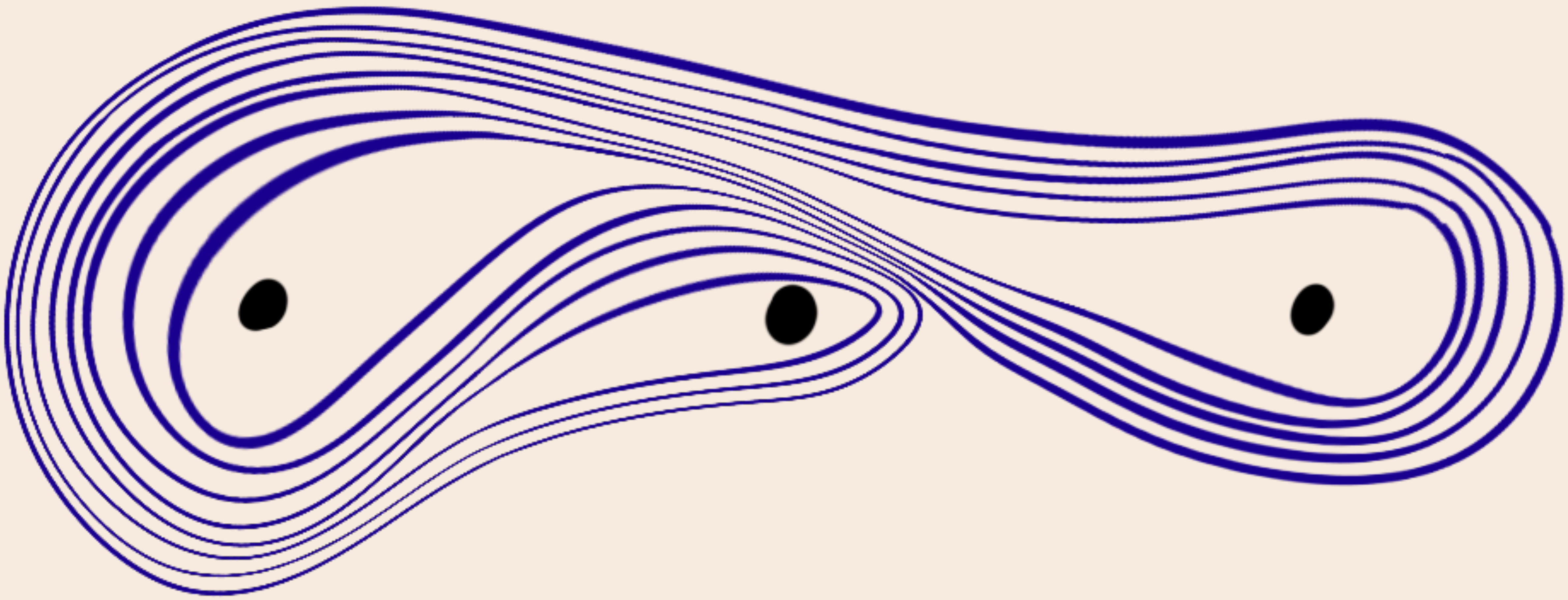


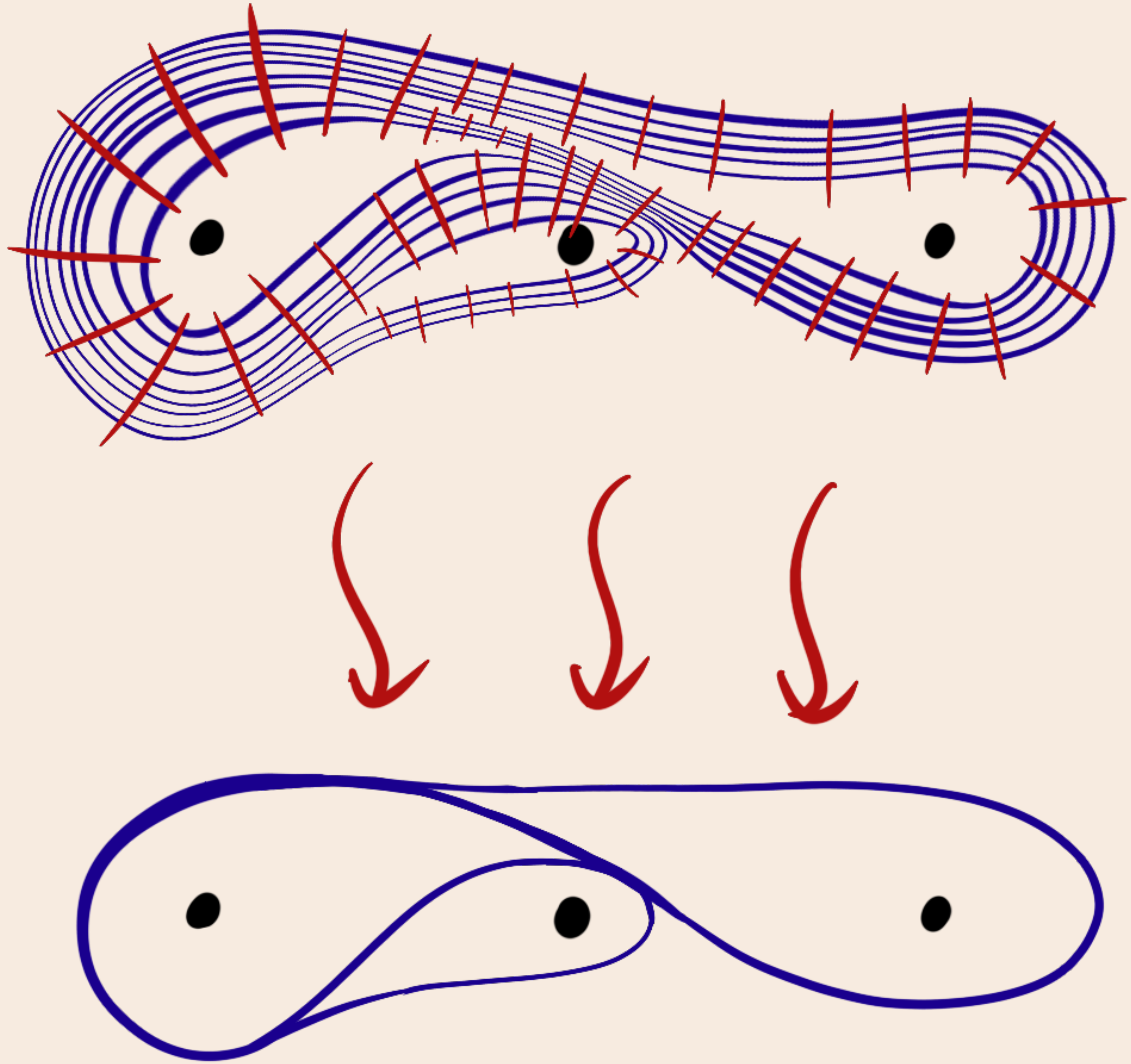




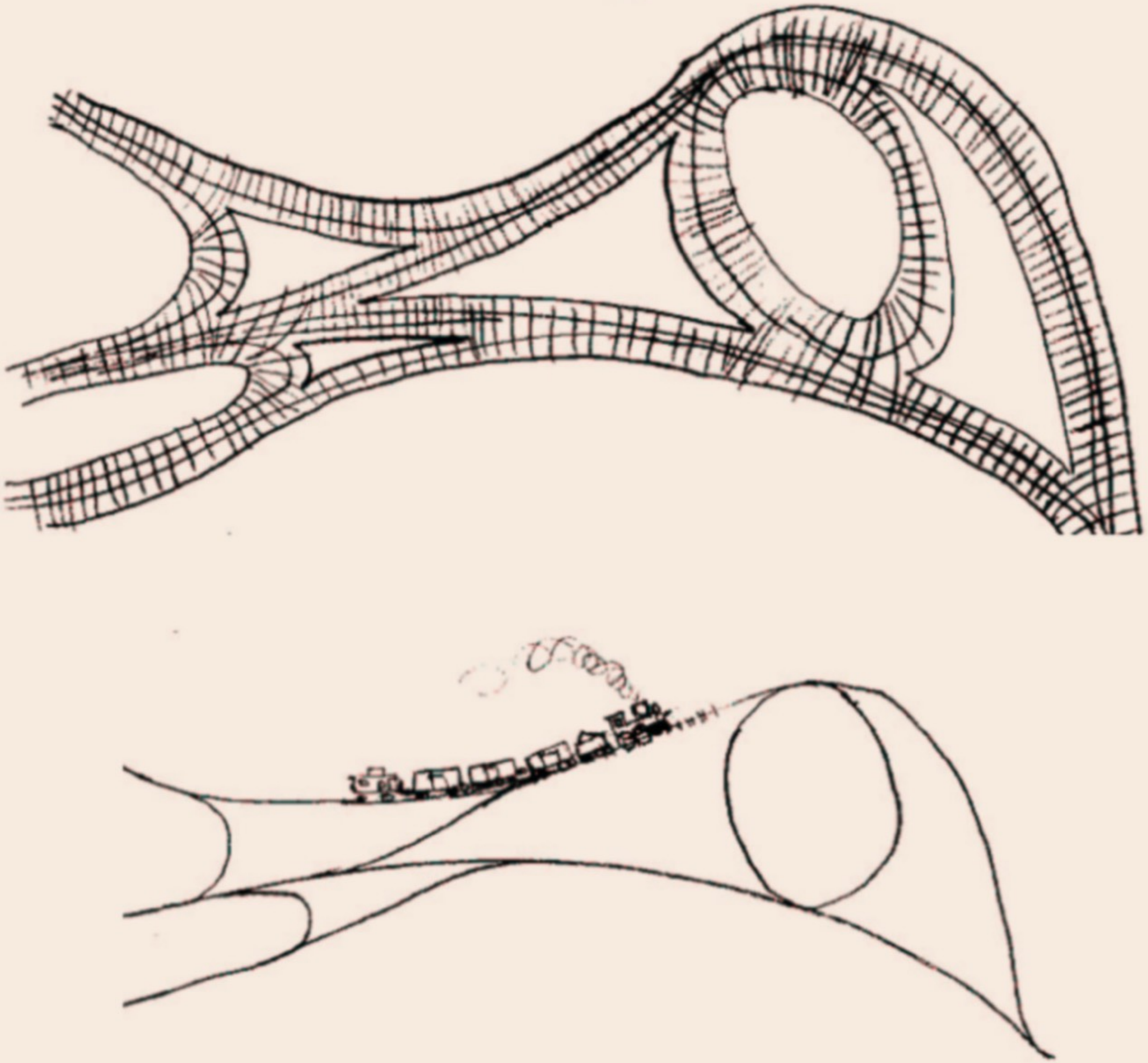






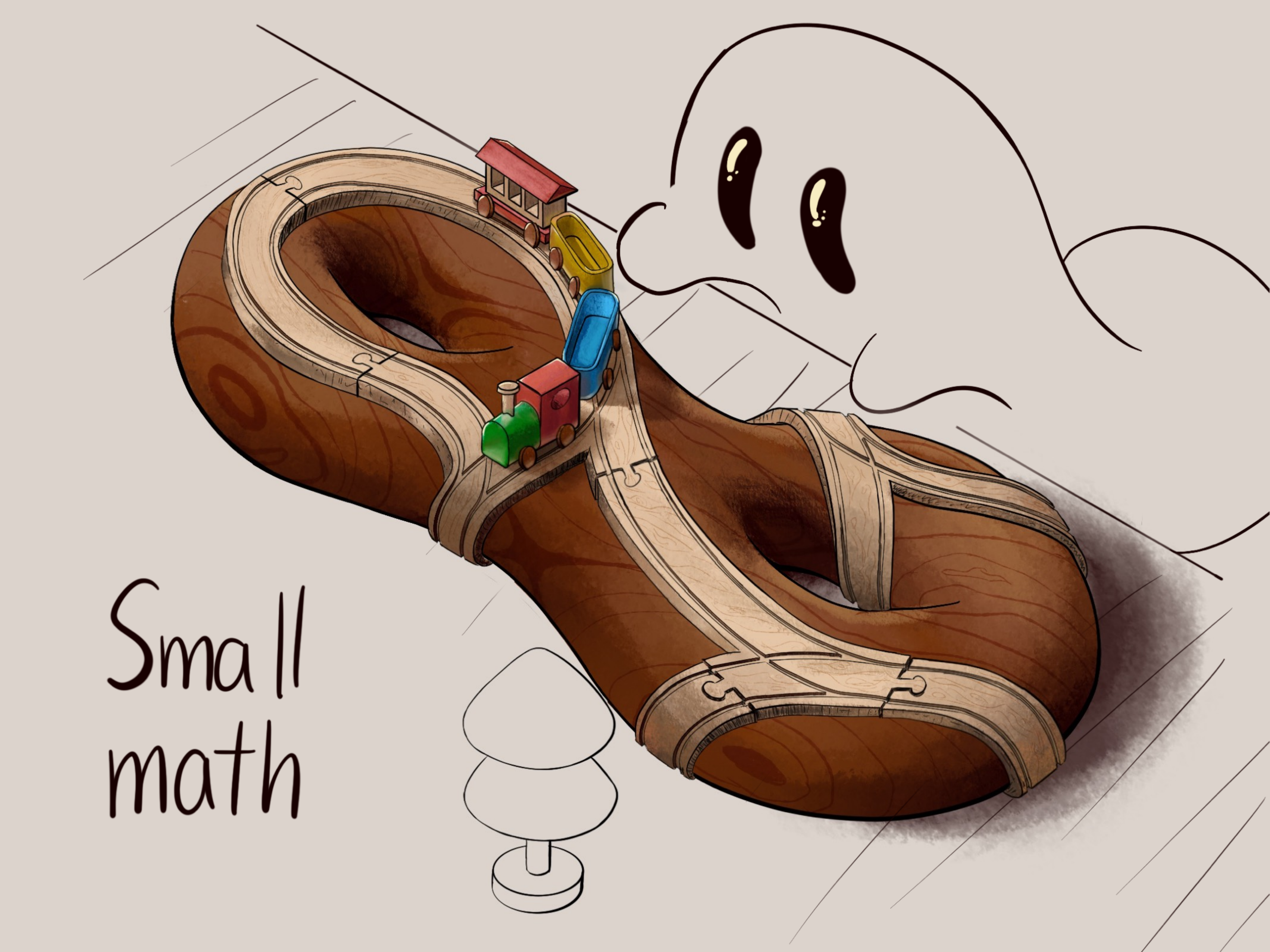


Train Tracks



Thurston, "The geometry & topology of 3-manifolds"

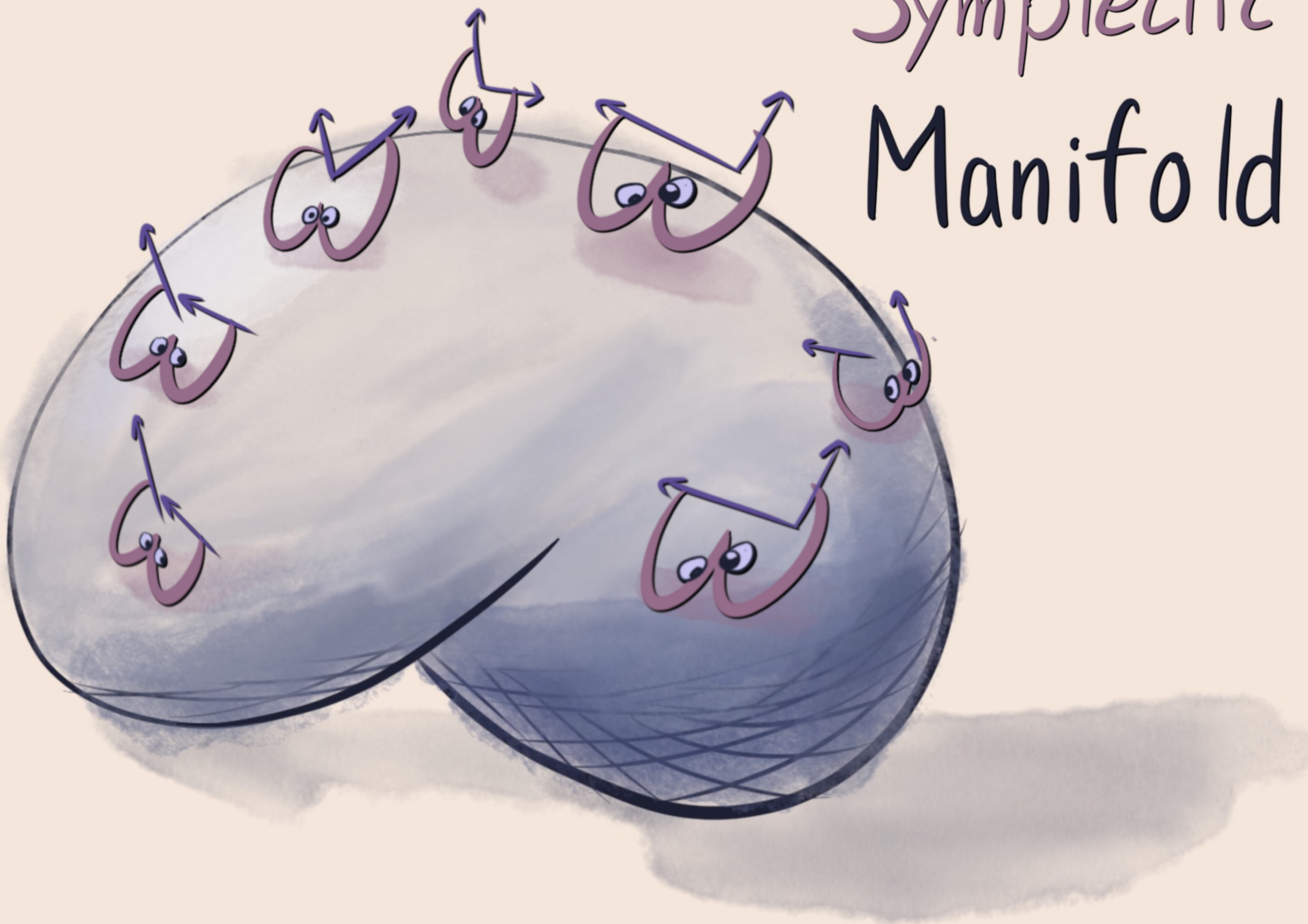

Big Math
Conan Wu

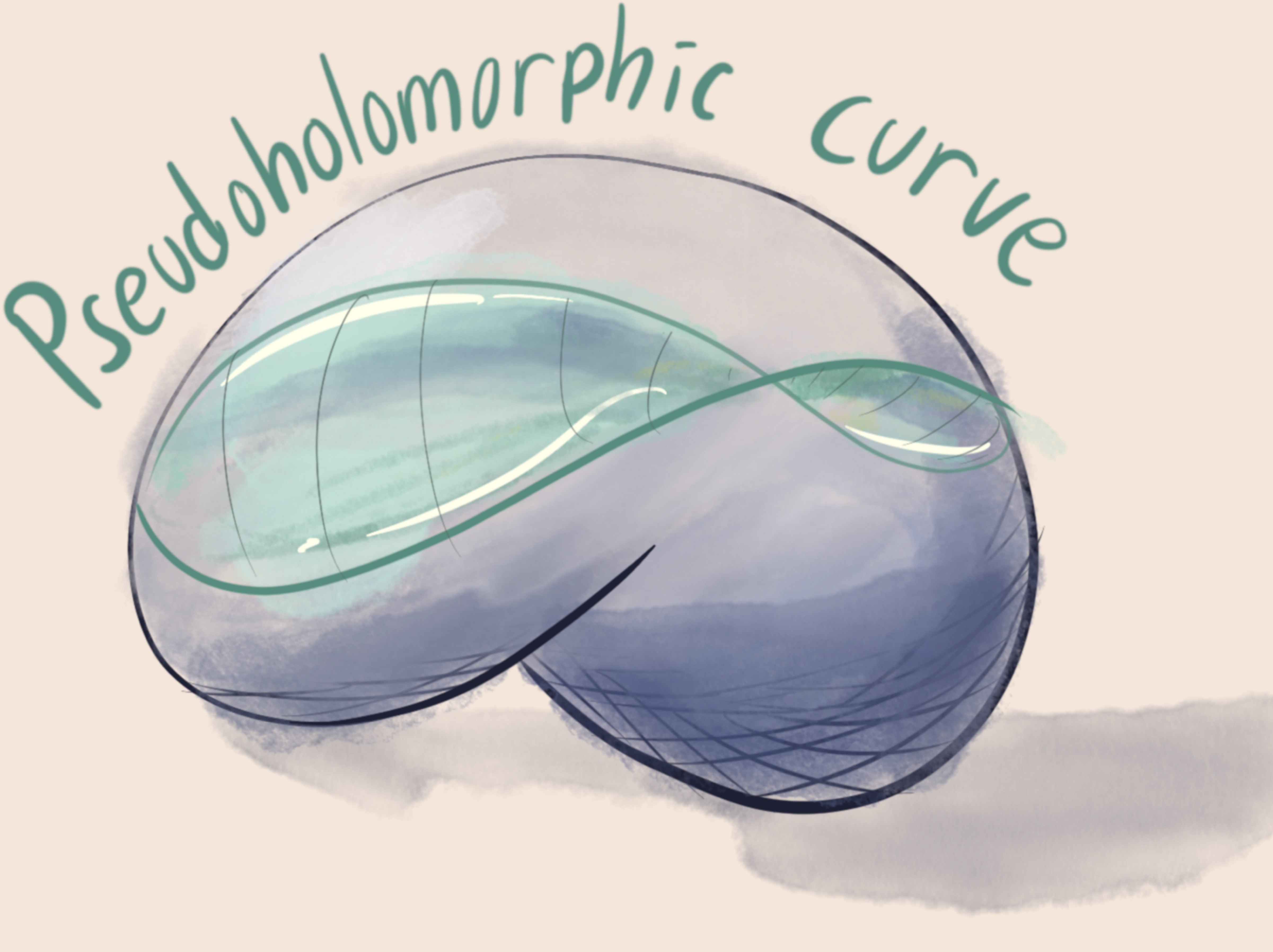

Small
math

Anecdote 2:

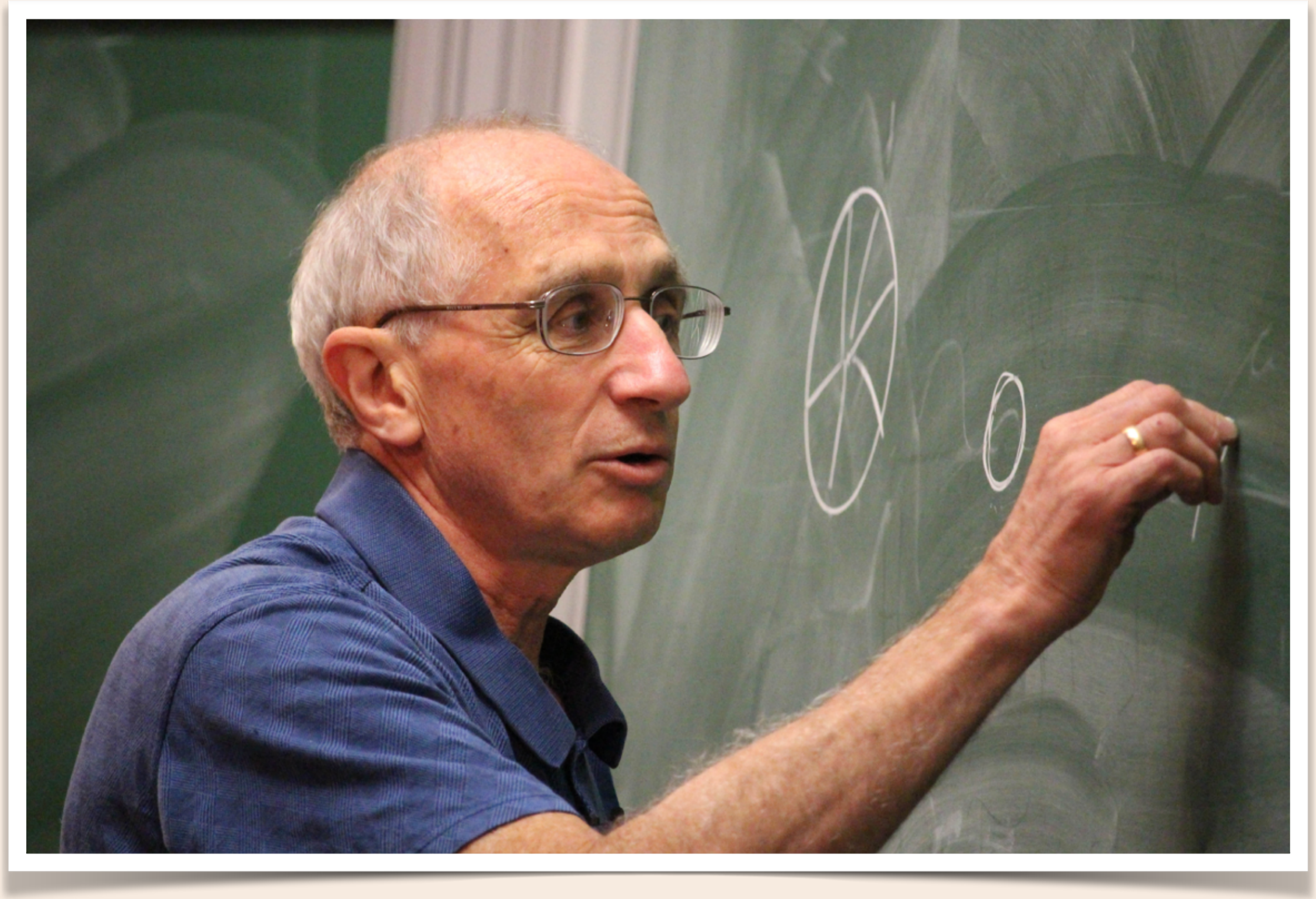
Big Math in symplectic geometry

Symplectic Manifold




Pseudoholomorphic curve

Yasha Eliashberg



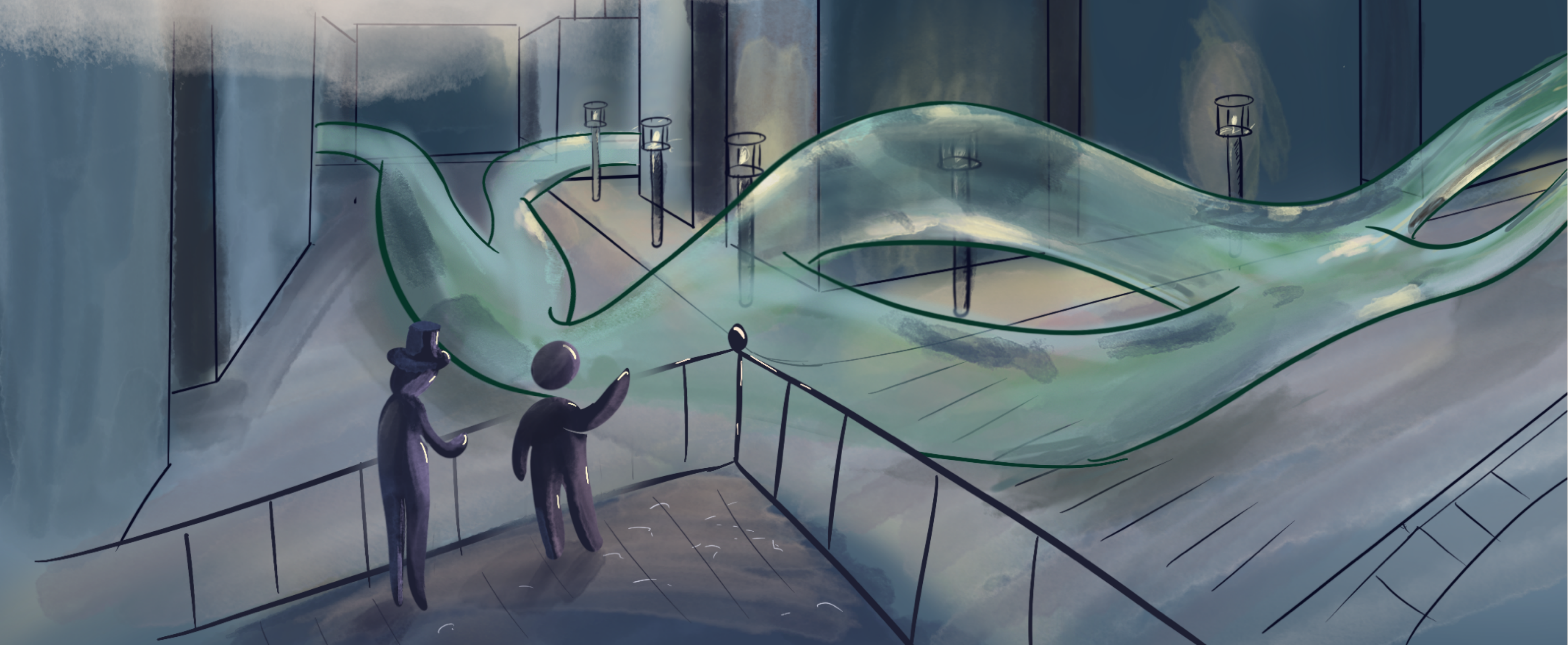
"Yasha's vision of mathematics is so great because he is very small"

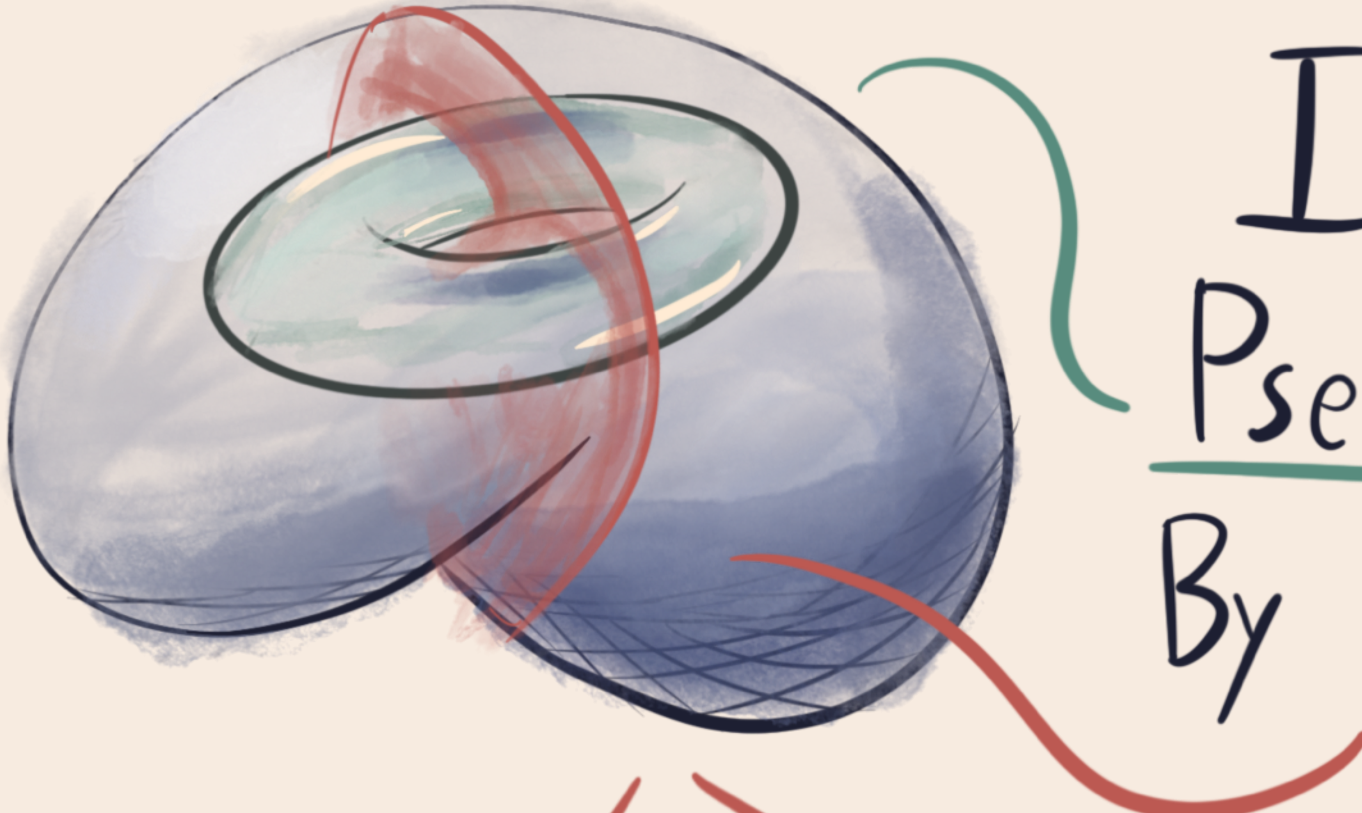
-Yashafest 2007

"... Yasha imagines mathematical objects as being much larger than himself, so that he can visualize all their details and develop a good intuition.

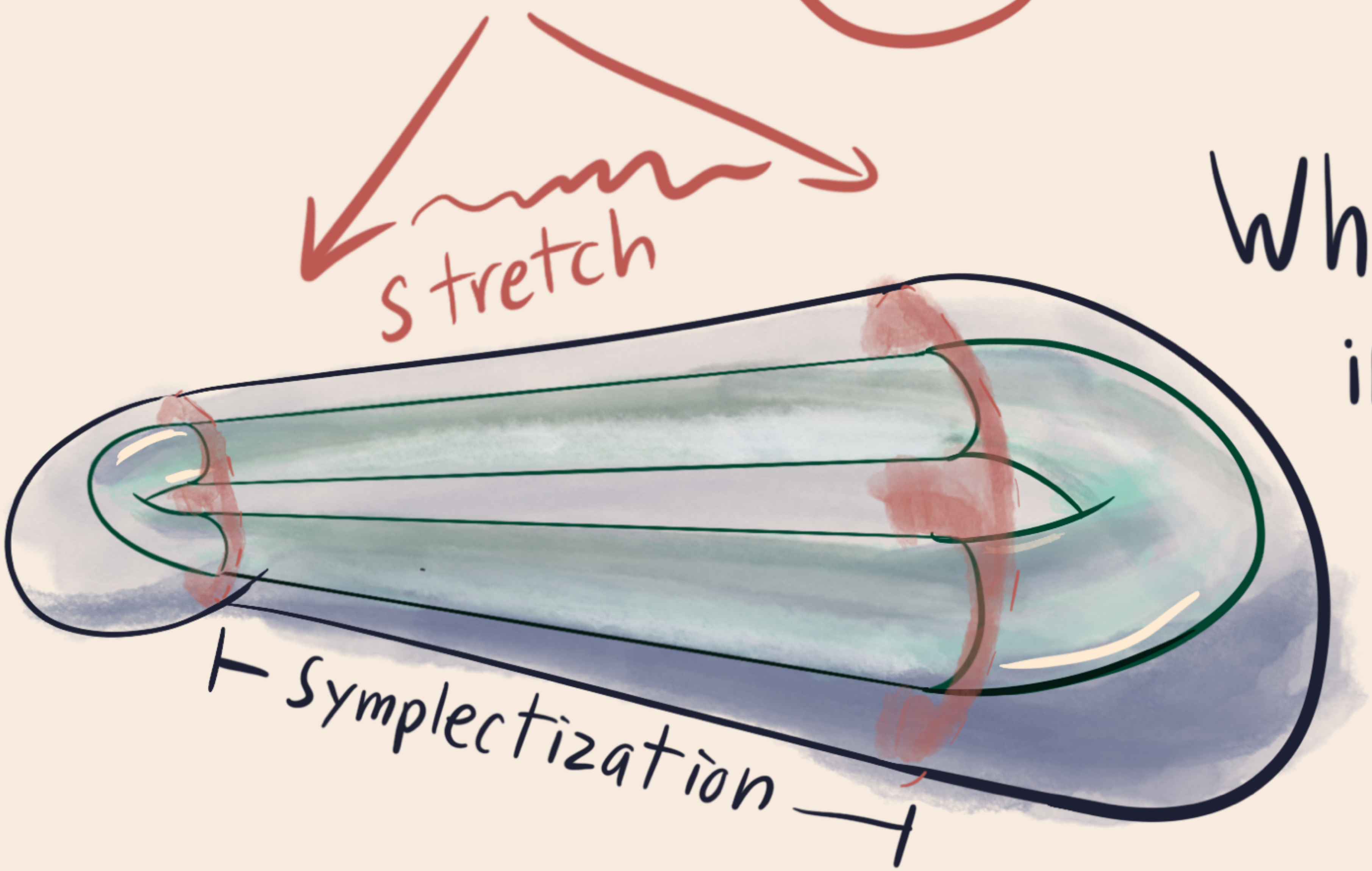
This is obvious for anyone who has had the chance to discuss mathematics with Yasha on a sidewalk terrace, where it would be typical for him to describe a huge pseudoholomorphic curve, say, coming from the other side of the street"

- Frédéric Bourgeois

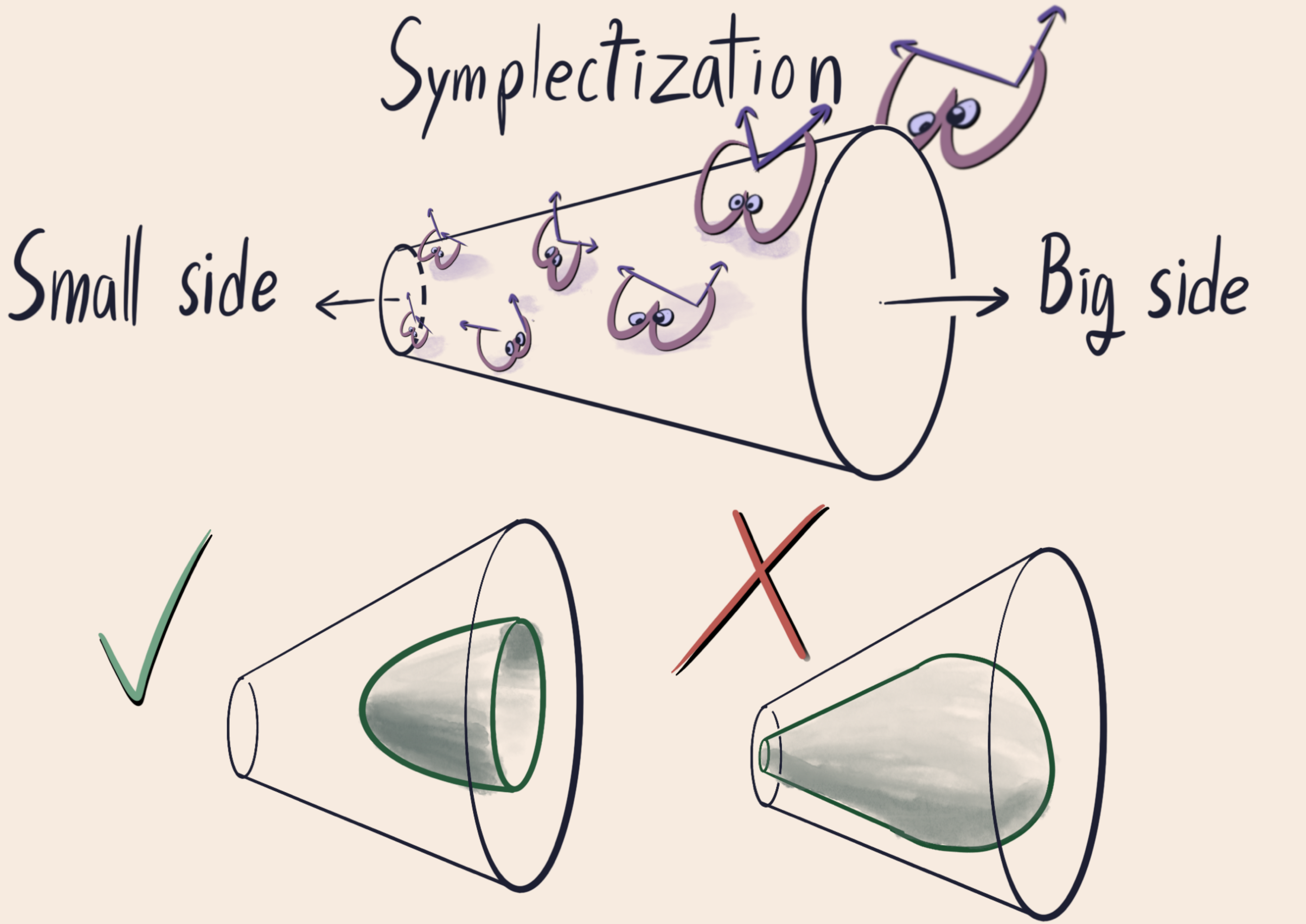




Idea: Count Pseudoholomorphic curves By decomposing manifold

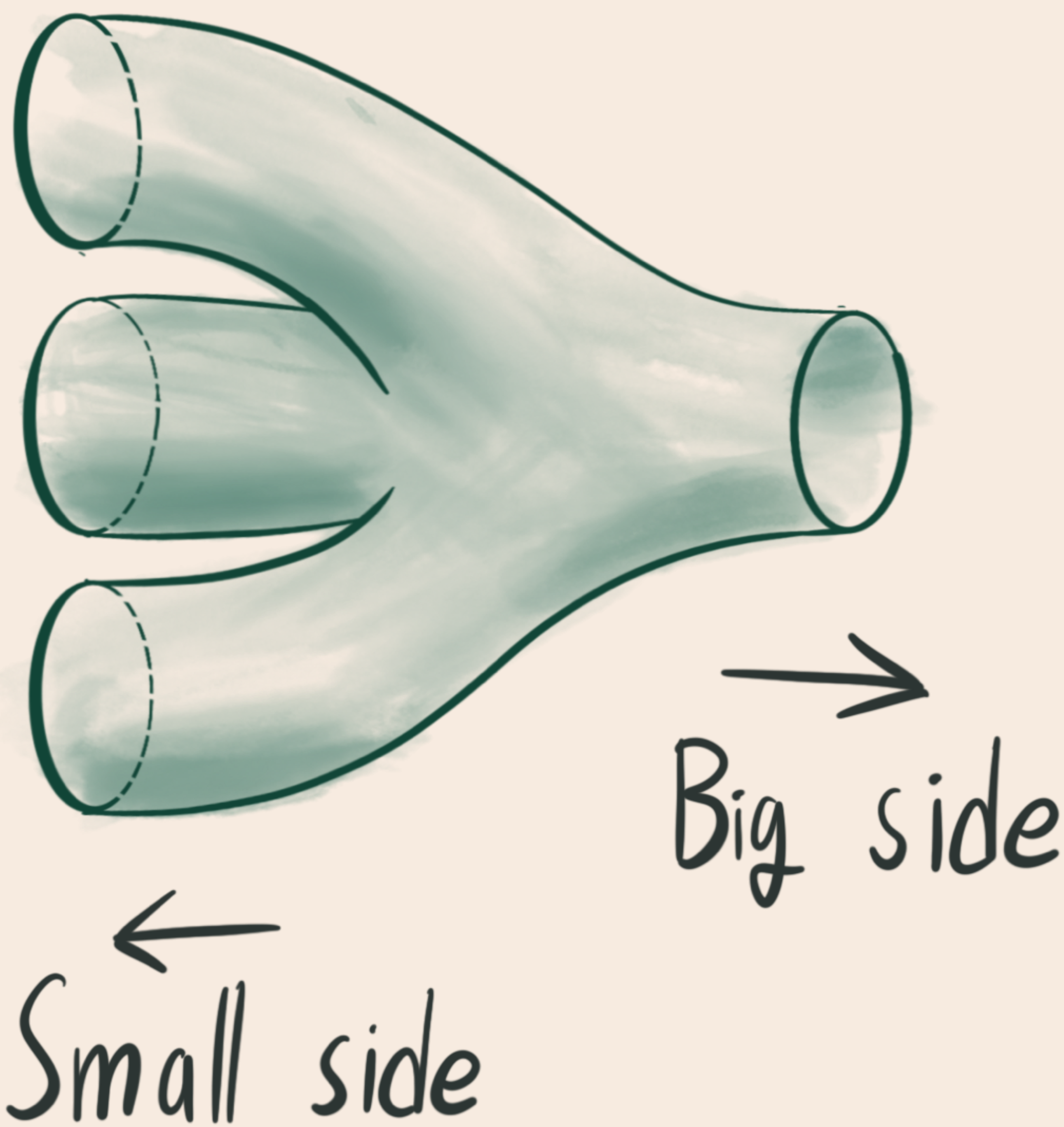


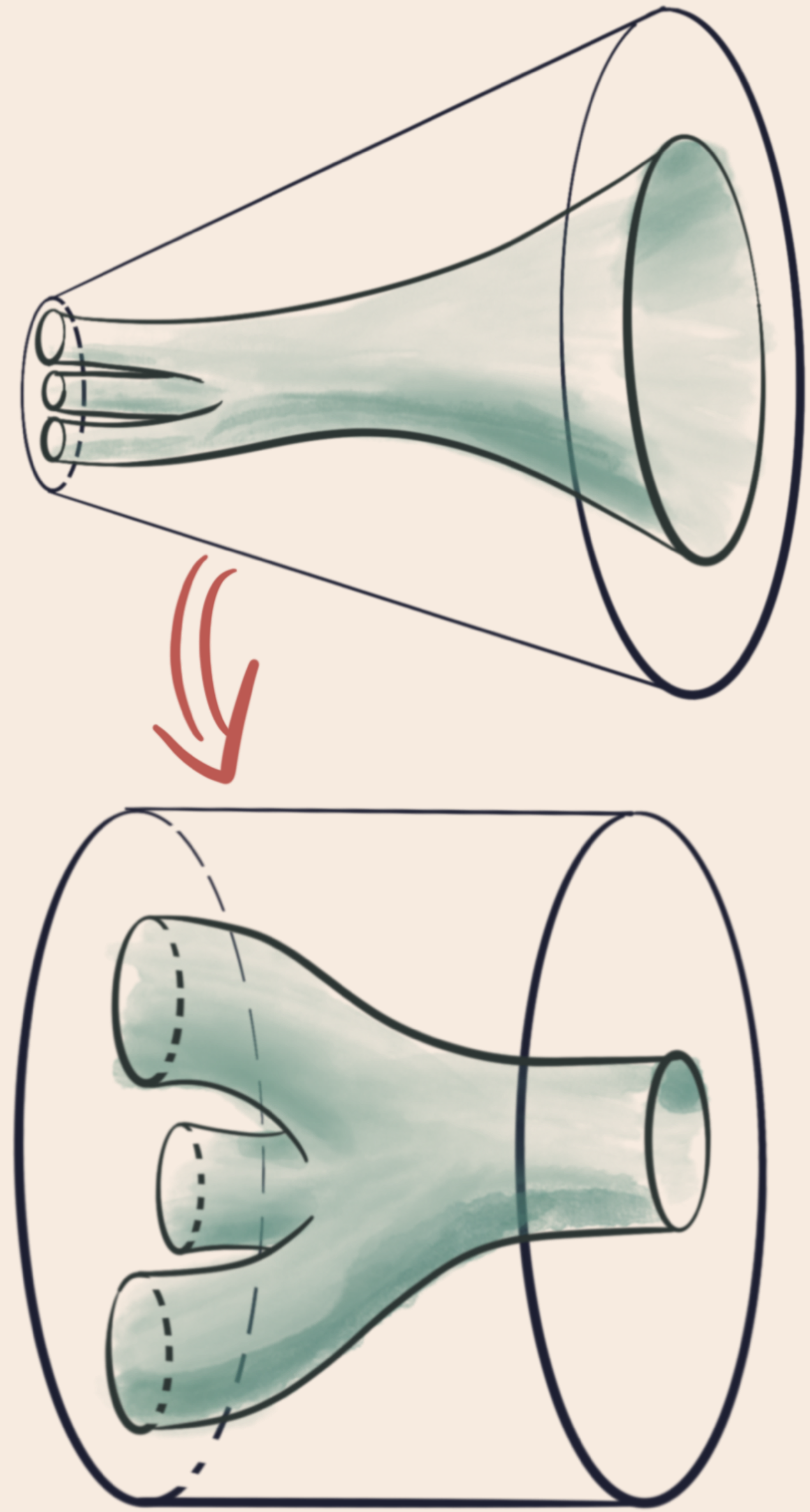
What happens in the middle?

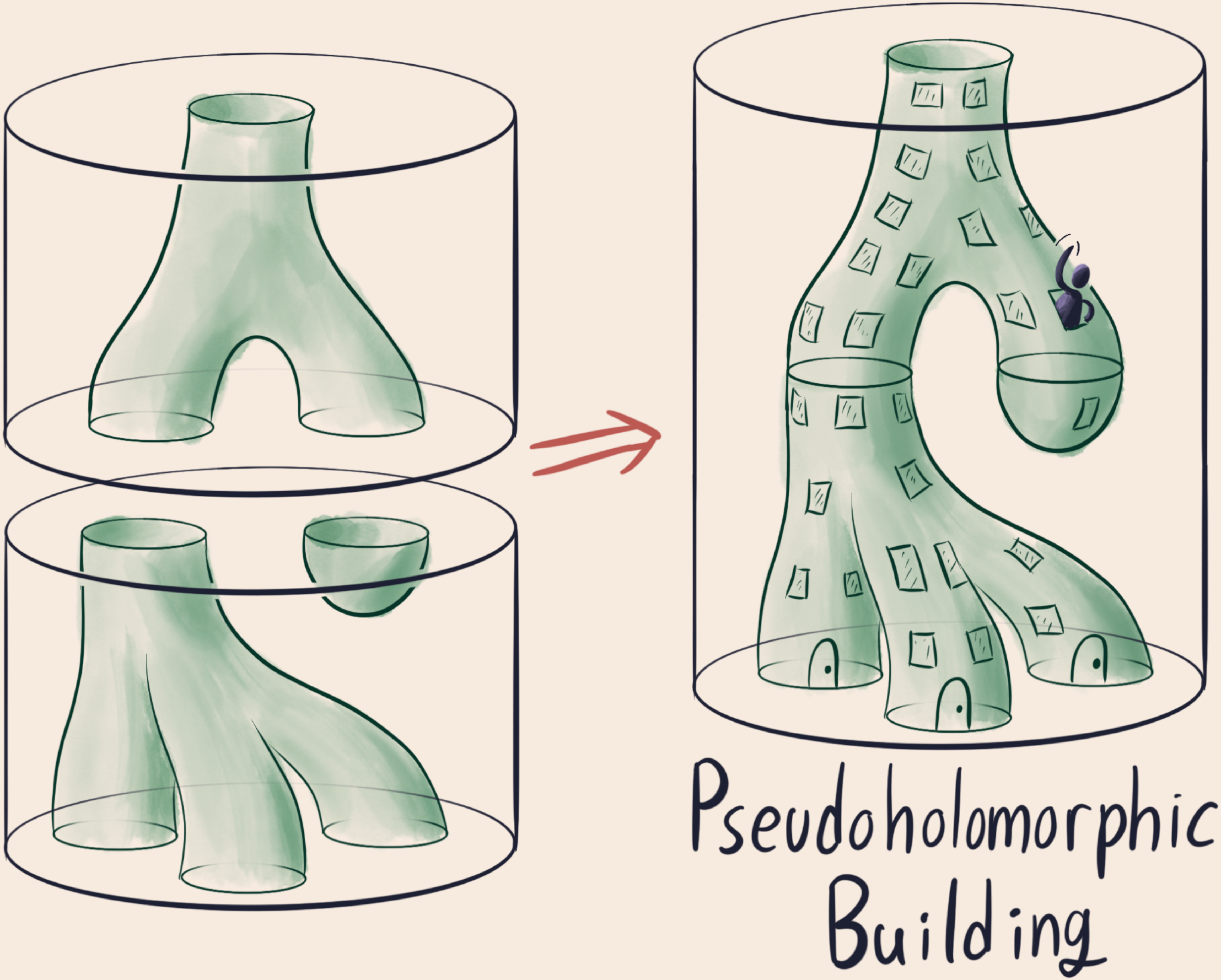


Symplectization
Small side
Big side

Need to count:






Pseudoholomorphic
Building

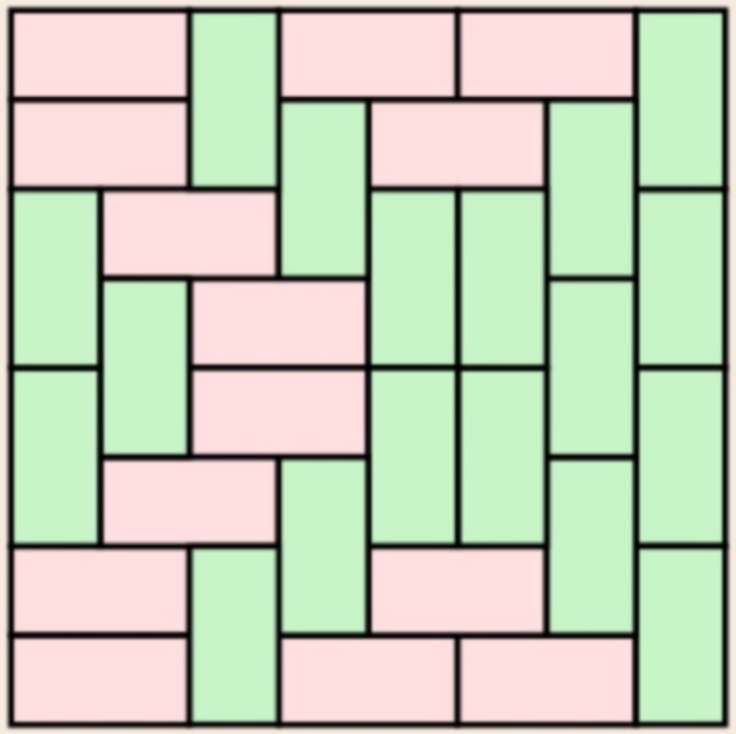
Big Math

- can walk inside
- can draw other math inside

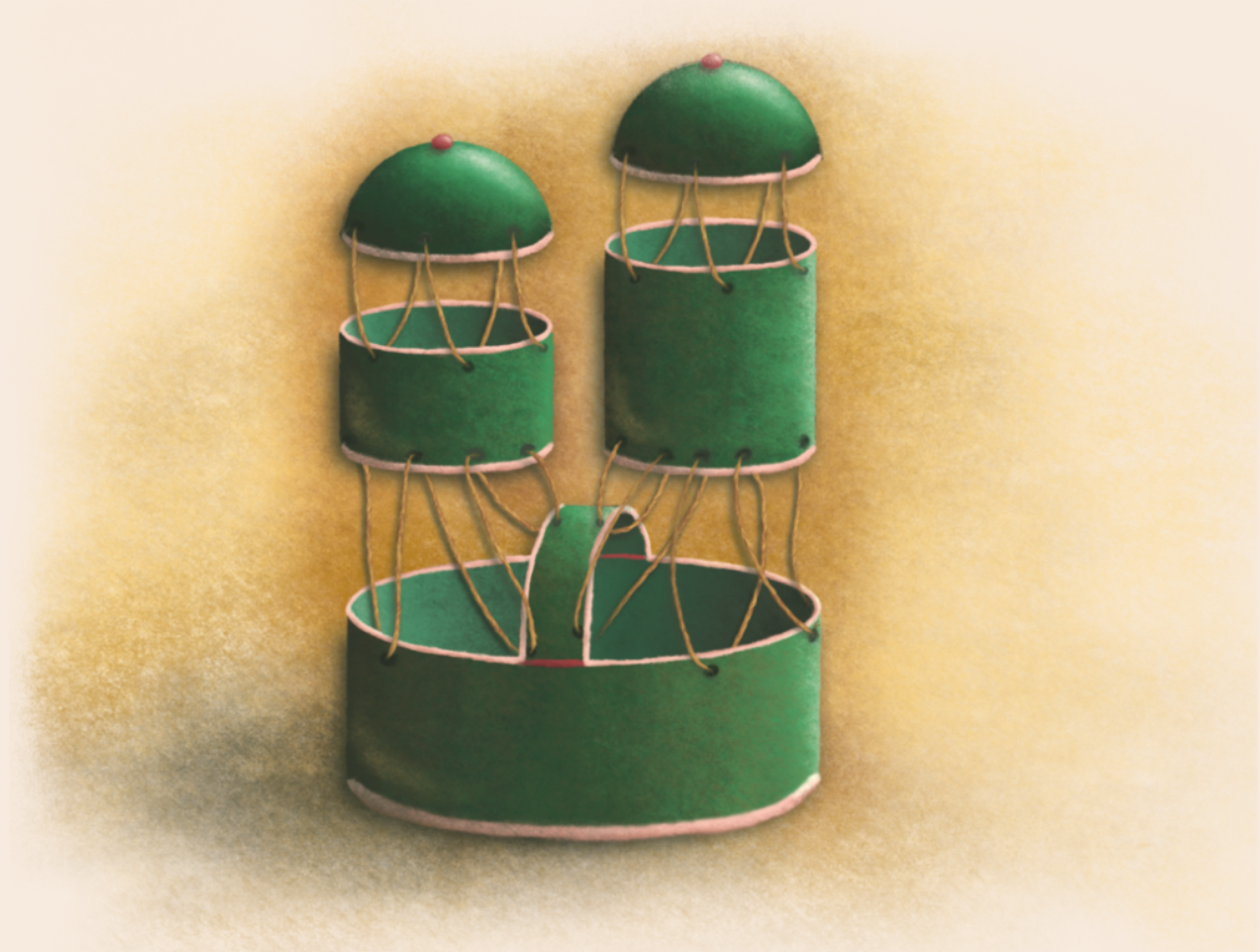
Small Math

- can manipulate
- can draw other math outside

Small Math examples:



Domino Tilings

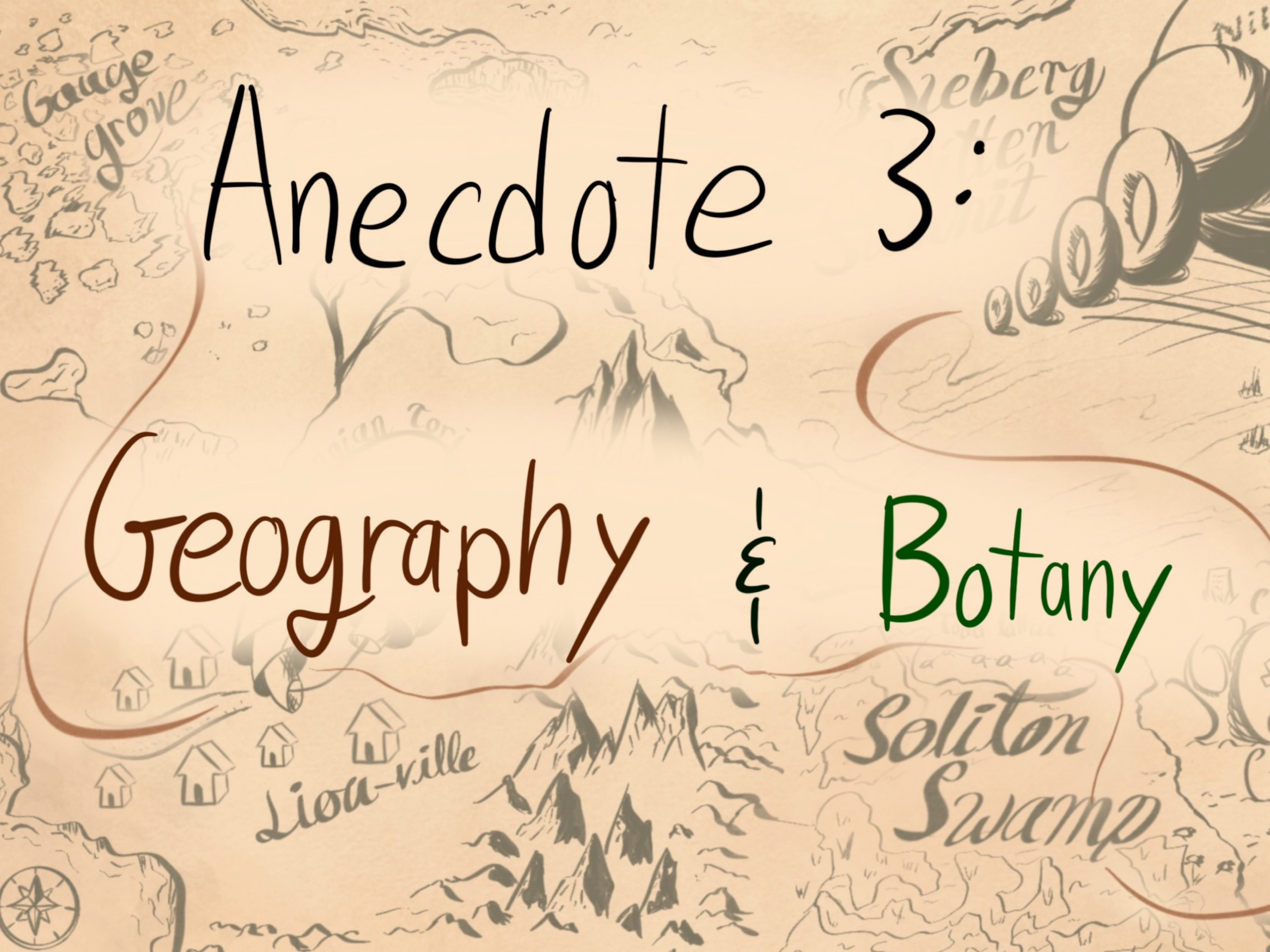


Sewing Manifolds

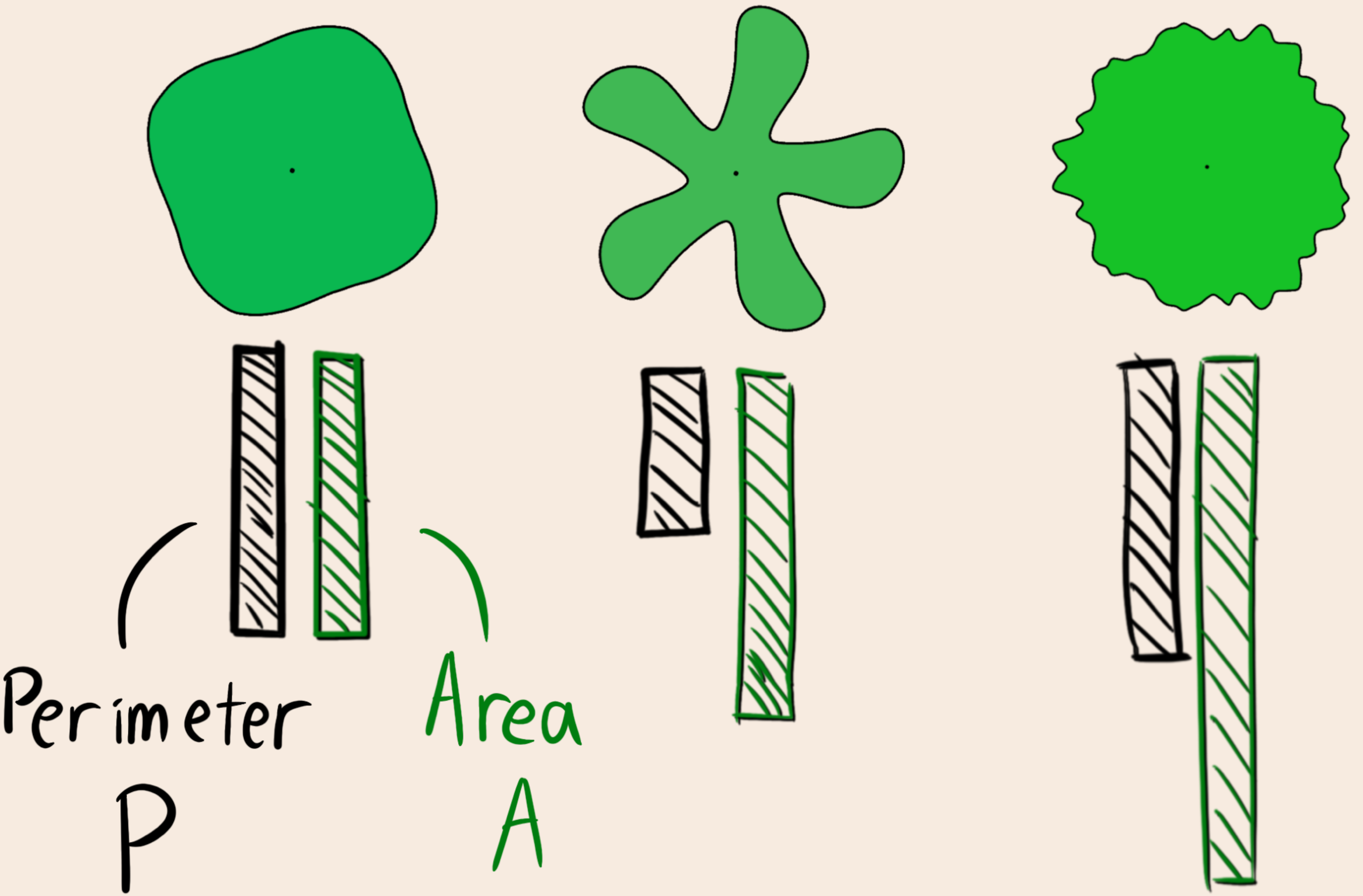
Math is Both!
B-twist
4D N=4 SUSY yang-mills
A-twist
Kapustin-Witten B_G
Kapustin-Witten A_G
3D N=4
X////G_3 HyperKahler
Coulomb Branch
3D Monopoles
Product
B σ-model
constant
Fueter
A σ-model
Rozansky-Witten 2-Category
holo lagrangians
holomorphic lagrangians
Fueter 2-category
Ω-Deformation
Squish
Raviolo
2D N=(2,2)
X//G_2 Symplectic
B σ-model
constant
holomorphic curve
A σ-model
2D Vorticies
Category O over C_h[M_H]
Derived category of Coherent sheaves
holomorphic submanifold
Lagrangian
Fukaya category
Dimensional Reduction
Symplectic Duality
Deformation Quantization
X/G_1 Complex
constant
Boundary Conditions
X/G_1 Riemannian
constant
1D N=2
B σ-model
A σ-model
N=(2,0)
N=(1,1)
Dolbeault Cohomology
de Rham Cohomology
Category O over C_h[M_C]

Anecdote 3:

Geography & Botany



Invariants (of curves)



Goal: Classify all objects

Geography

What values of invariants are realized?

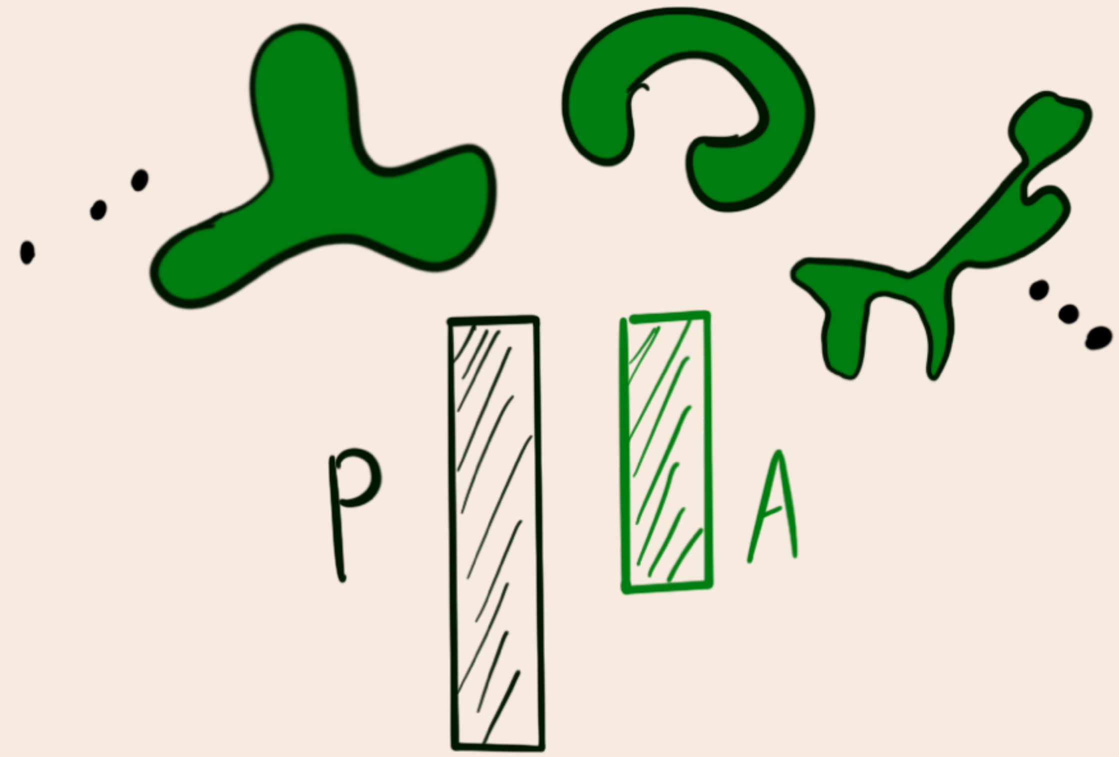
For curves:

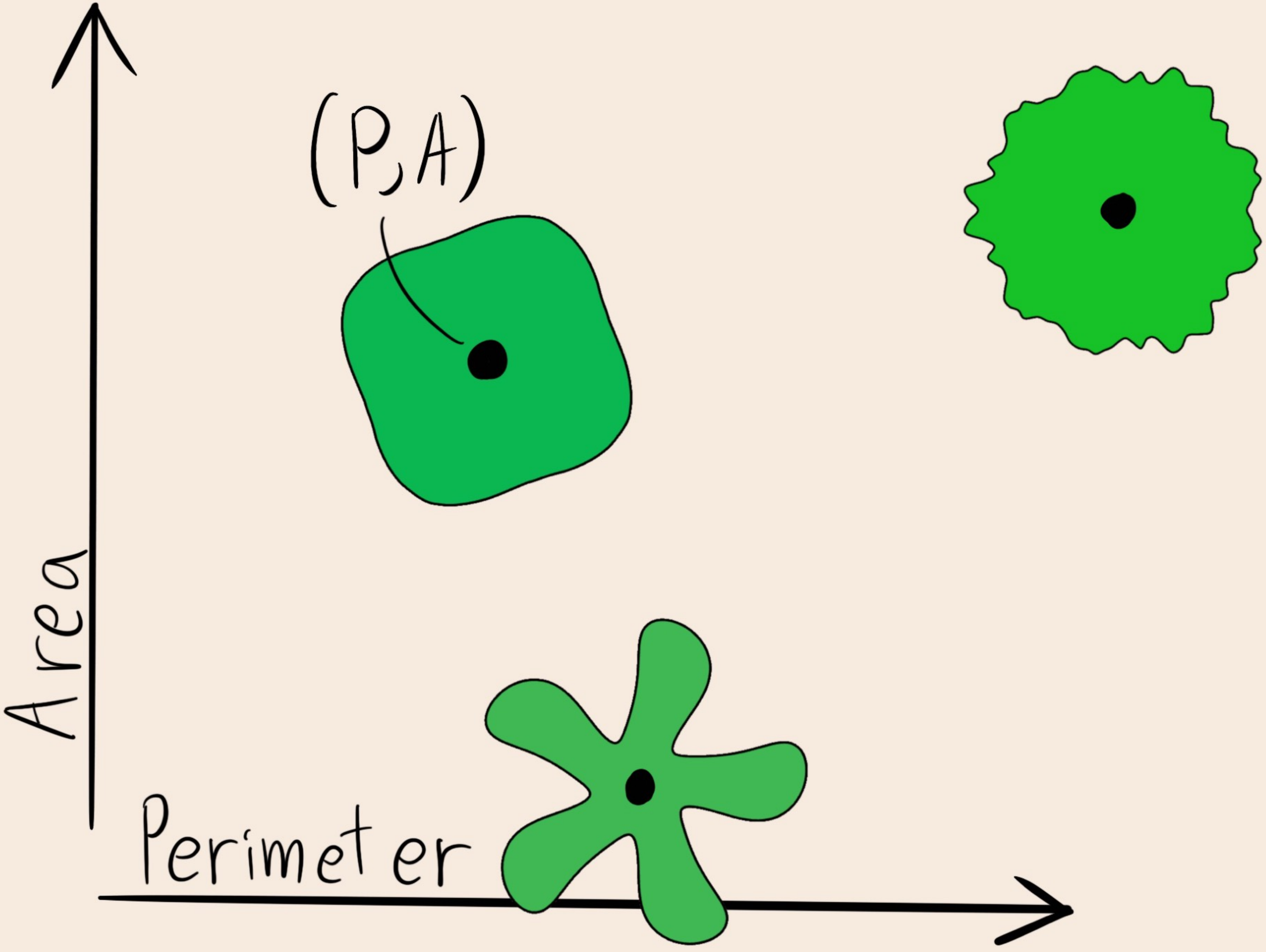
What pairs (P, A) are measurements of a curve?

(P = Perimeter, A = Area)

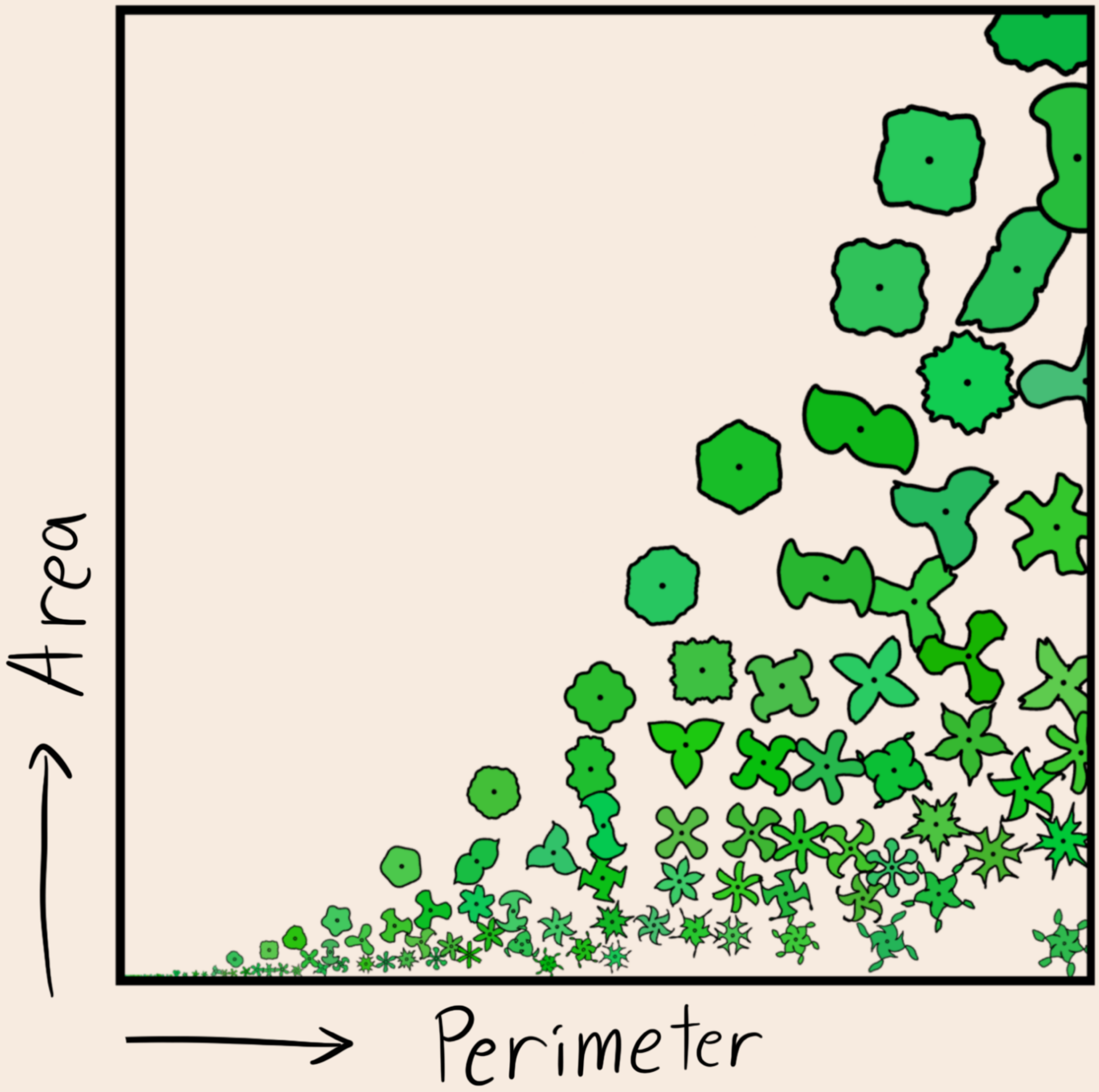
Botany

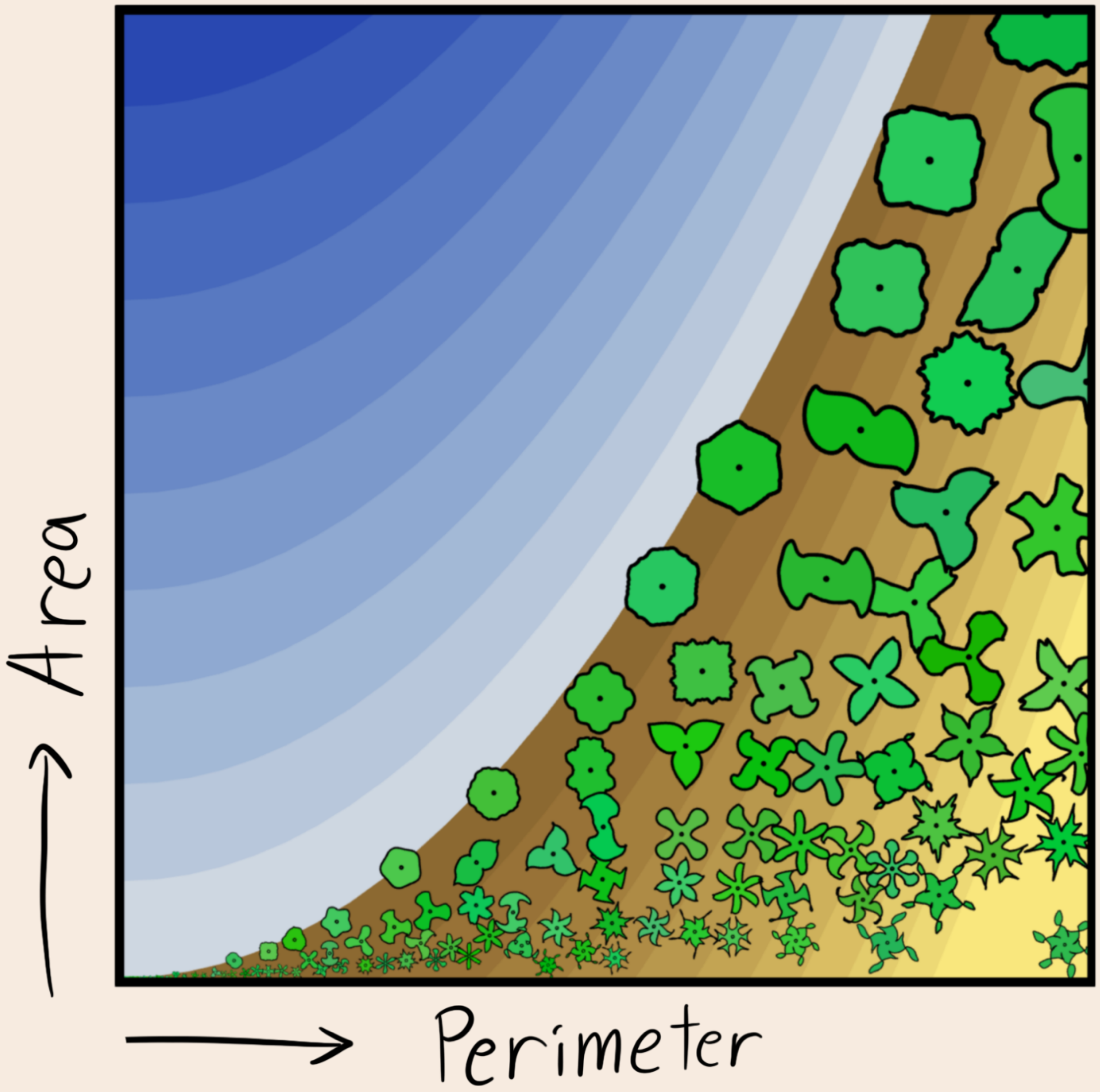
What objects realize a particular value of the invariants?



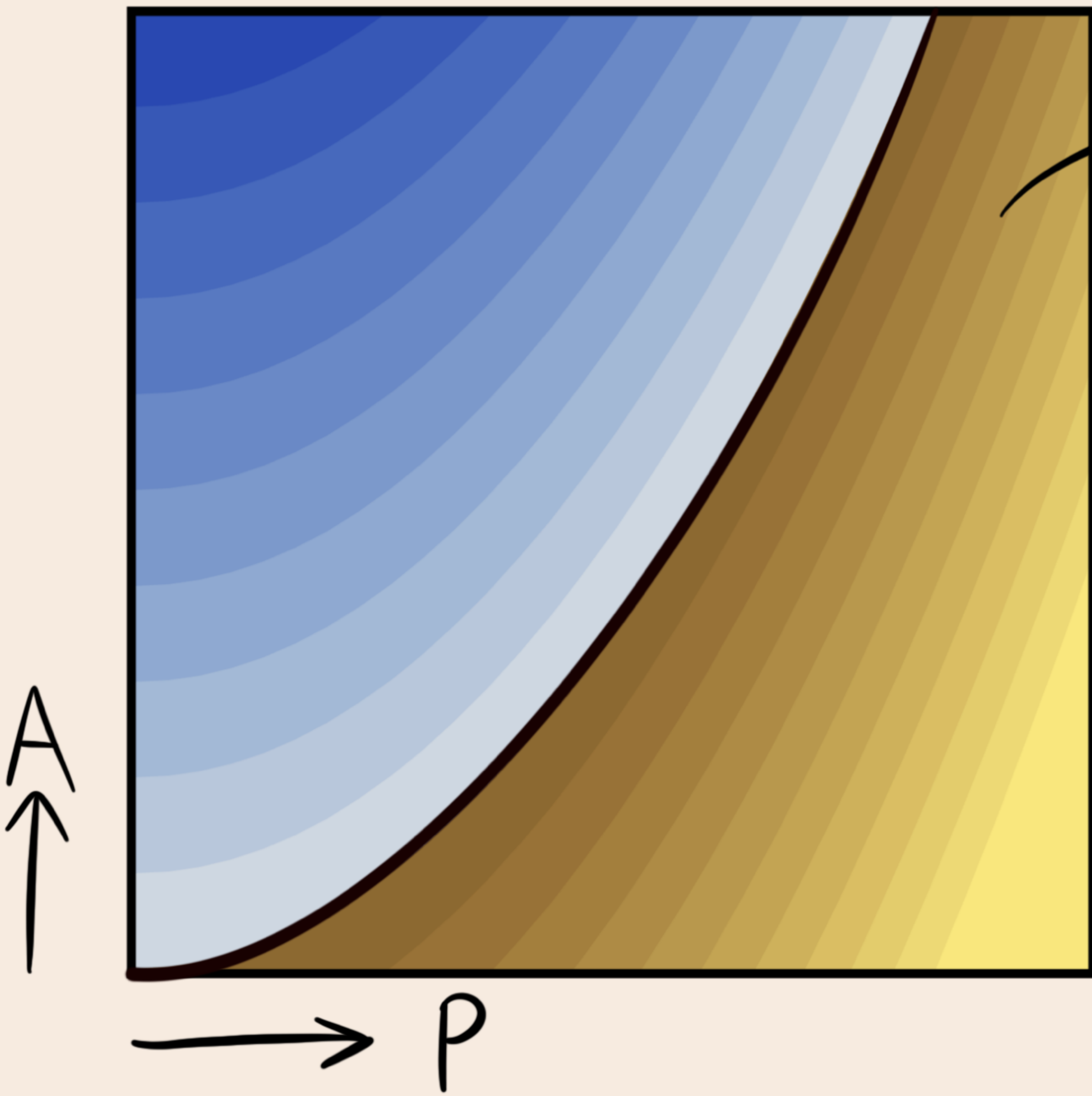


(P, A)
Area
Perimeter


Area
Perimeter


Area
Perimeter

Geography



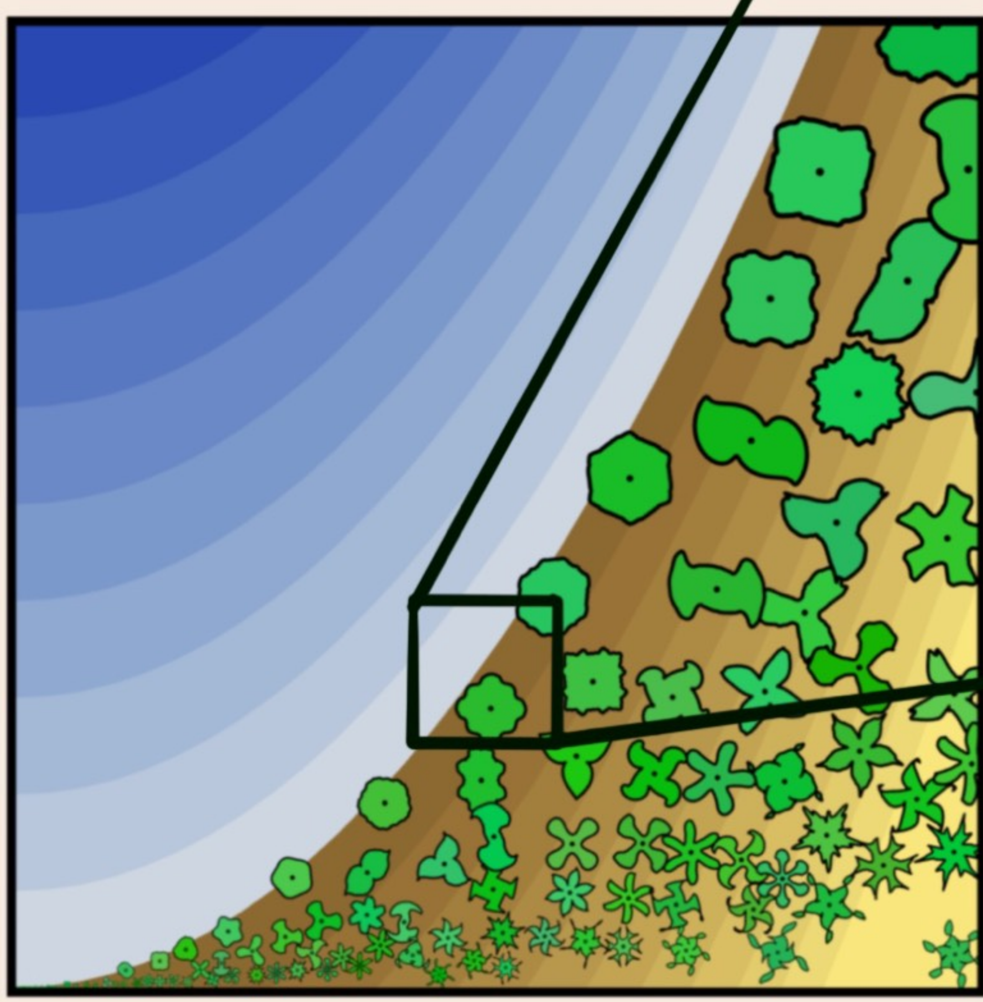
$$4\pi A \leq p^2$$

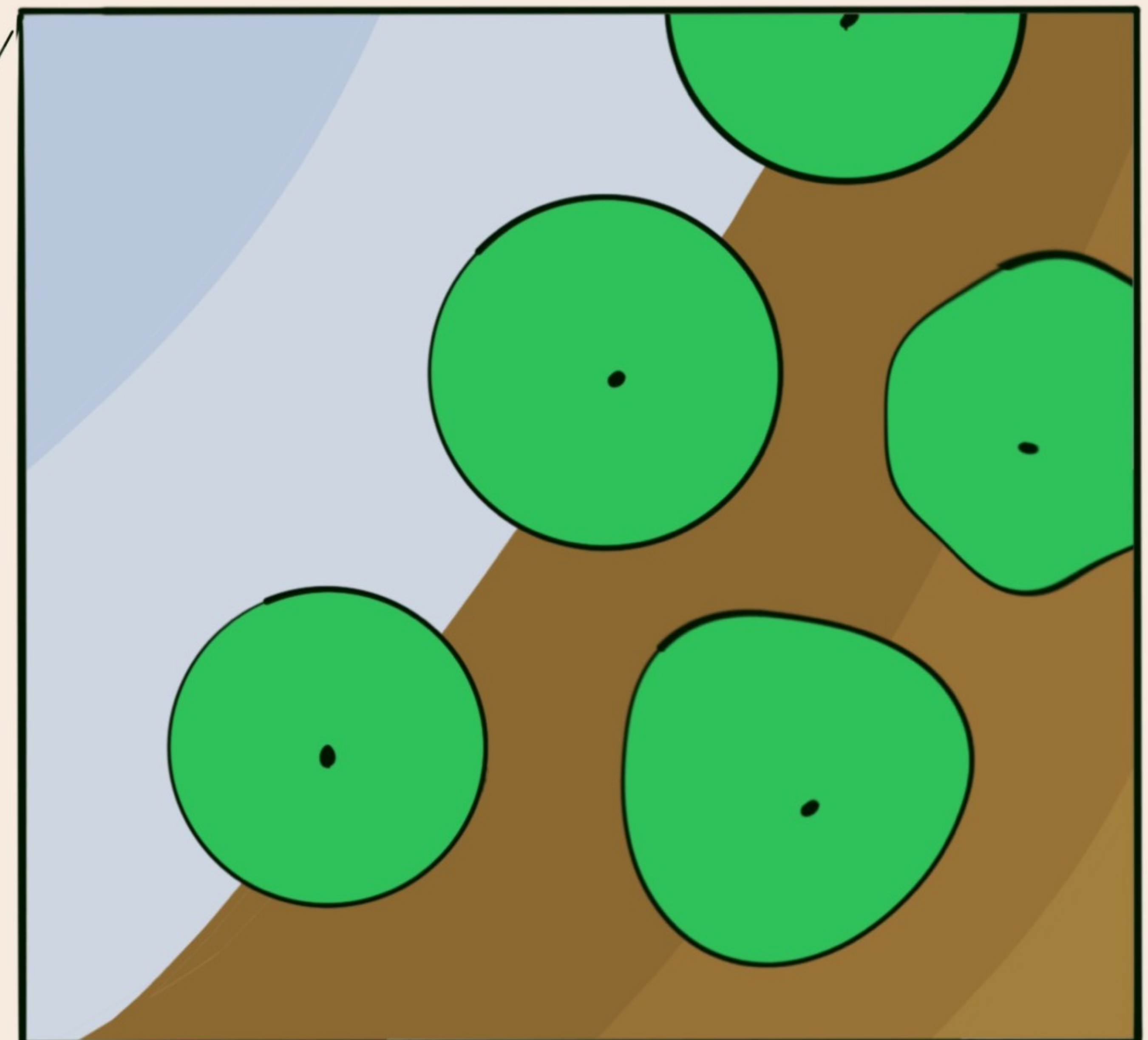
Isoperimetric inequality

Botany

$4\pi A = P^2 \Rightarrow$

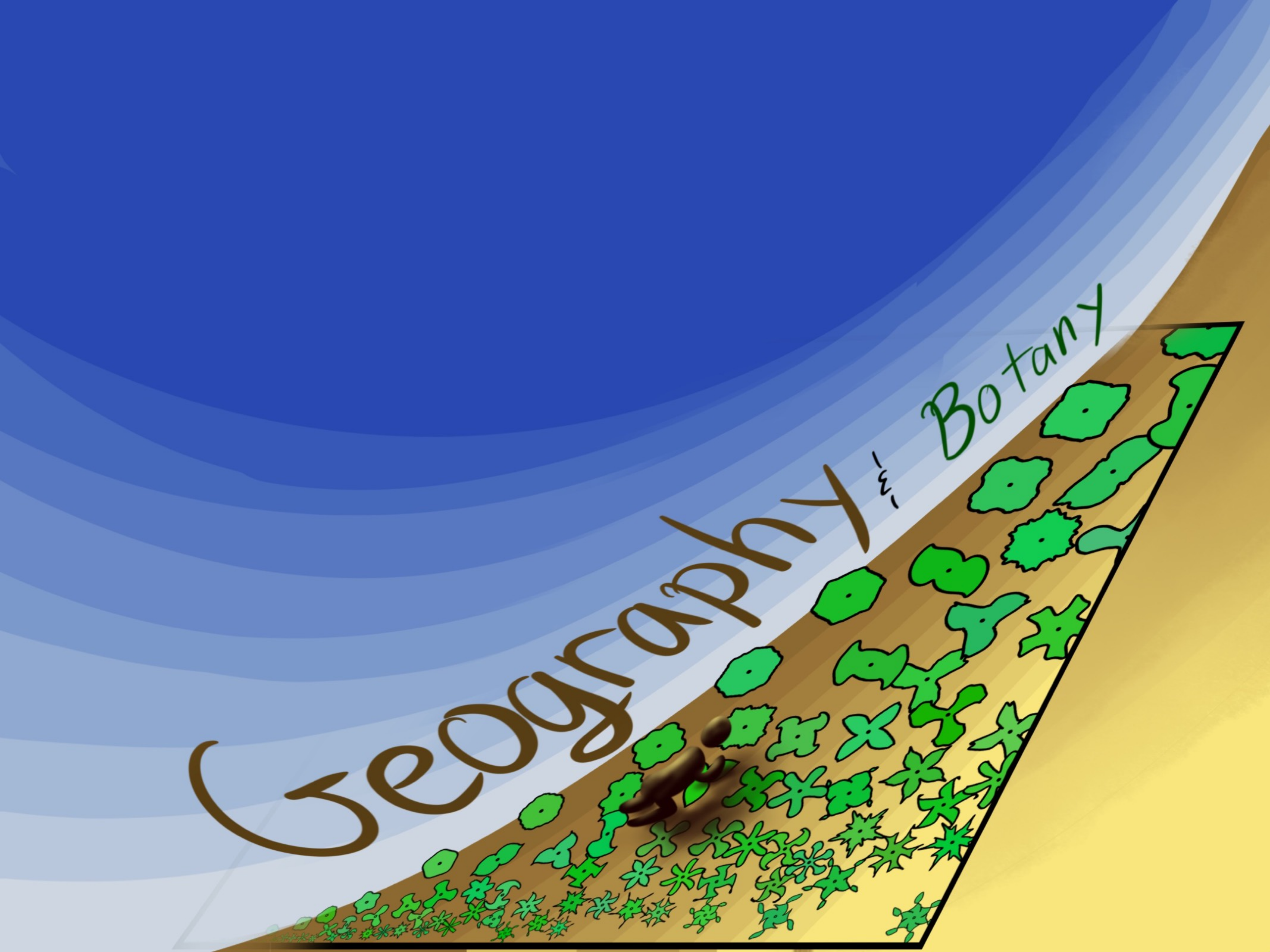
curve is a circle



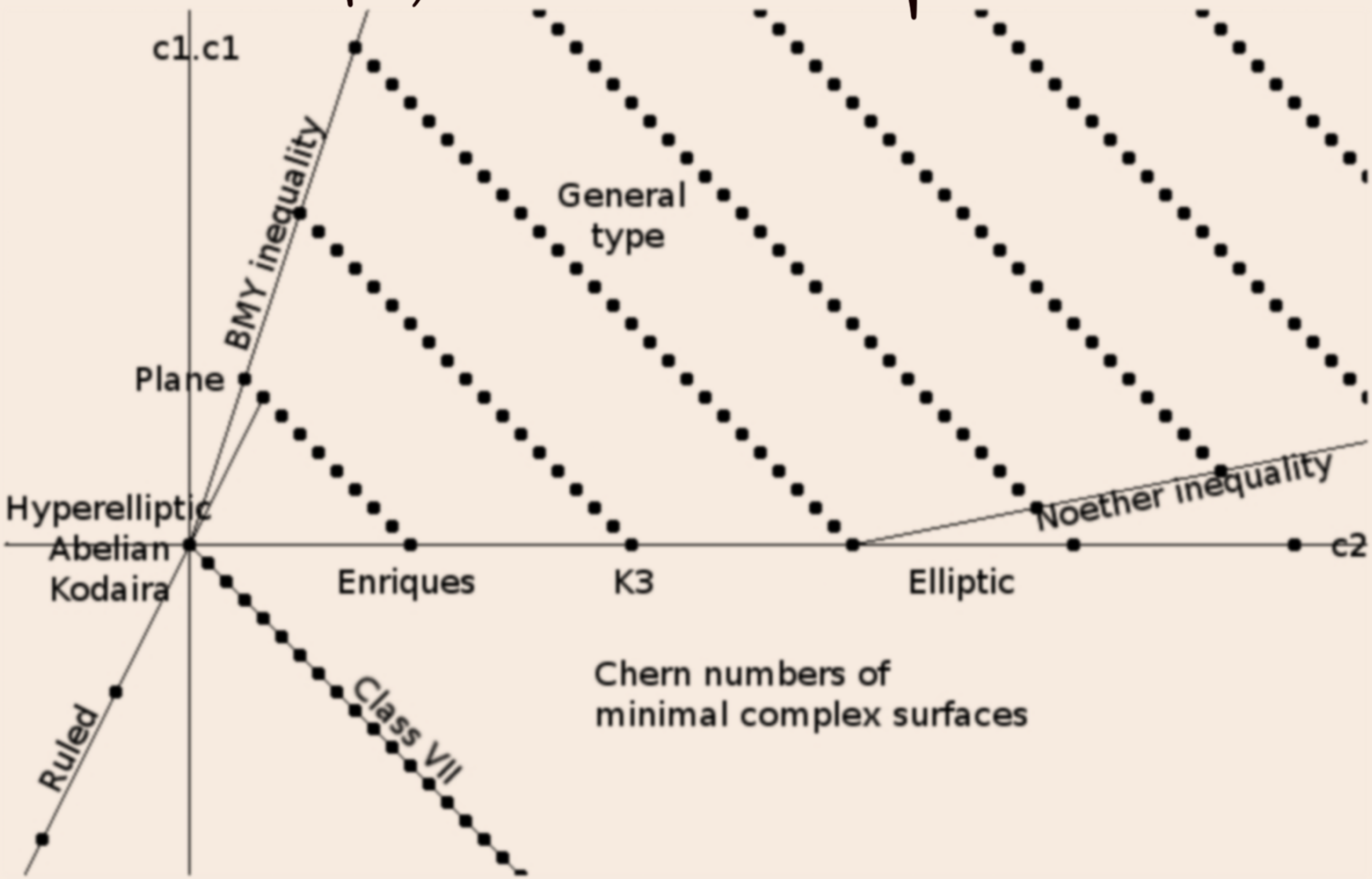


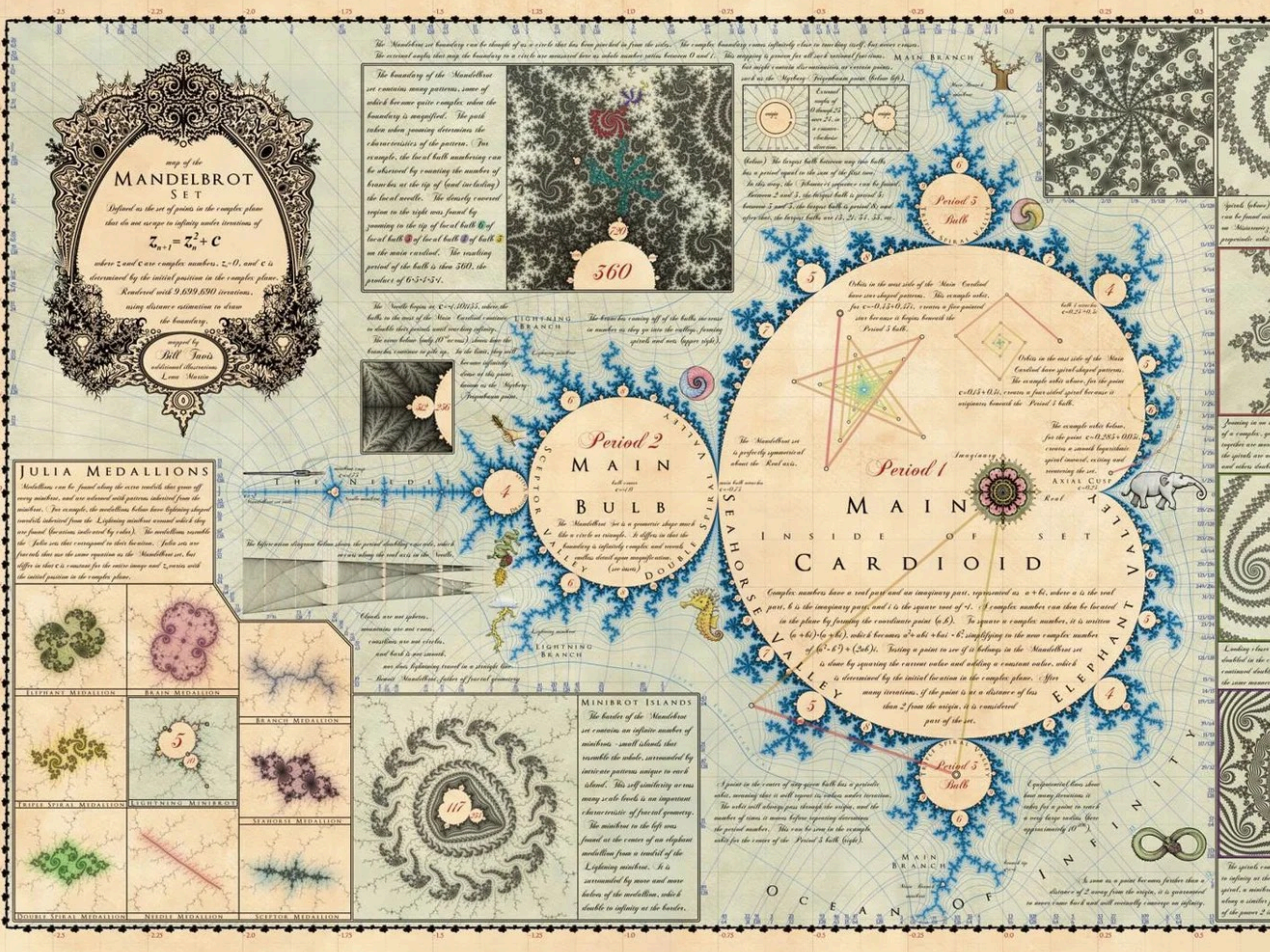
$4\pi A \approx P^2 \Rightarrow$

curve is almost a circle


Geography & Botany

Geography of complex surfaces





map of the
MANDELBROT
SET
Defined as the set of points in the complex plane that do not escape to infinity under iterations of
$z_{n+1} = z_n^2 + c$
where z and c are complex numbers, $z_0 = 0$, and c is determined by the initial position in the complex plane. Rendered with 9,699,690 iterations, using distance estimation to draw the boundary.
mapped by
Bill Tavis
The boundary of the Mandelbrot set contains many patterns, some of which become quite complex when the boundary is magnified. The path taken when zooming determines the characteristics of the pattern. For example, the local bulb numbering can be observed by counting the number of branches at the tip of (and including) the local needle. The densely covered region to the right was found by zooming to the tip of local bulb 6 of local bulb 3 of local bulb 4 of bulb 5 on the main cardioid. The resulting period of the bulb is then 360, the product of 6·5·4·3·4.
360
JULIA MEDALLIONS
Medallions can be found along the extra tendrils that grow off every minibrot, and are adorned with patterns inherited from the minibrot. For example, the medallions below have lightning shaped tendrils inherited from the Lightning minibrot around which they are found (locations indicated by color). The medallions resemble the Julia sets that correspond to their location. Julia sets are fractals that use the same equation as the Mandelbrot set, but differ in that c is constant for the entire image and z_0 varies with the initial position in the complex plane.
ELEPHANT MEDALLION
BRAIN MEDALLION
BRANCH MEDALLION
TRIPLE SPIRAL MEDALLION
LIGHTNING MINIBROT
SEAHORSE MEDALLION
DOUBLE SPIRAL MEDALLION
NEEDLE MEDALLION
SCEPTOR MEDALLION
MINIBROT ISLANDS
THE NEEDLE
LIGHTNING BRANCH
MAIN BRANCH
Period 2
MAIN
BULB
SCEPTOR VALLEY
DOUBLE SPIRAL VALLEY
SEAHORSE VALLEY
Period 1
MAIN
INSIDE OF SET
CARDIOID
ELEPHANT VALLEY
Period 3
Bulb
TRIPLE SPIRAL VALLEY
Complex numbers have a real part and an imaginary part, represented as a + bi, where a is the real part, b is the imaginary part, and i is the square root of -1. A complex number can then be located in the plane by forming the coordinate point (a,b). To square a complex number, it is written (a + bi)·(a + bi), which becomes $a^2 + abi + bai - b^2$, simplifying to the new complex number of $(a^2 - b^2) + (2ab)i$. Testing a point to see if it belongs in the Mandelbrot set is done by squaring the current value and adding a constant value, which is determined by the initial location in the complex plane. After many iterations, if the point is at a distance of less than 2 from the origin, it is considered part of the set.
OCEAN OF INFINITY