
Higgs bundles,
Mirror symmetry,
Geometric Langlands

Geometric Langlands Conjecture

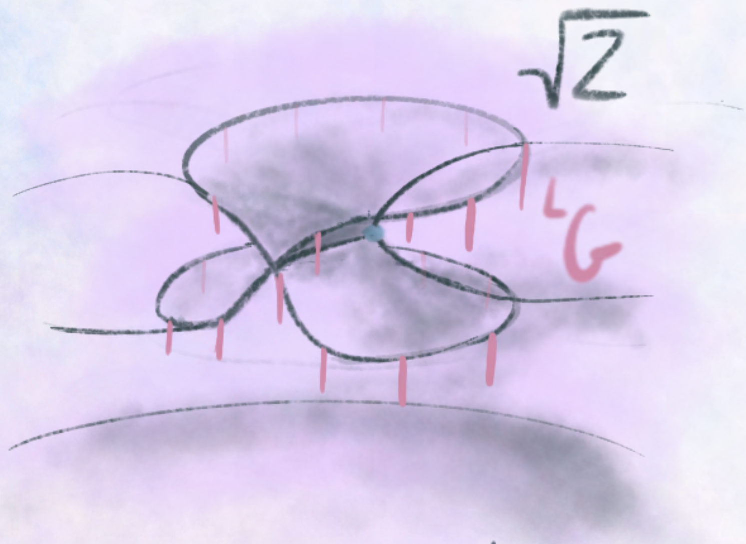
Arithmetic:

$\mathrm{Gal}(\overline{F}/F) \longrightarrow {}^L G$

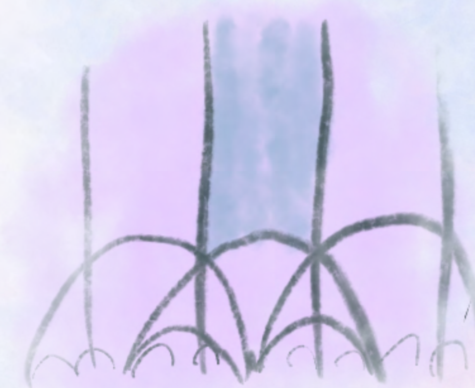
Automorphic:

$L^2(\Gamma \backslash G / K)$

$$\xleftrightarrow[\text{e. fns}]{\text{Hecke}}$$



Rosetta Stone



$SL(\mathbb{Z}) \backslash PGL(2) / SO(2)$

QM of particle on locally symmetric space

$\pi_1(\Sigma) \longrightarrow {}^L G$

Weil Uniformization:

$\Gamma \backslash G / K \longleftrightarrow \mathrm{Bun}_G$

flat ${}^L G$-bundles $\xleftrightarrow[\text{Eigen sheaf}]{\text{Hecke}}$ D-modules on Bun_G

S-Duality:

principal G-bundle $P \longrightarrow M$ $\qquad P \times^{ad} \mathfrak{g}$

fields: $\mathcal{A}$ G-connections $\cong \Omega^1(M, ad(P))$ //

Yang-mills action: $S(A) = \frac{1}{4g^2} \int \underset{\text{curvature}}{F_A} \wedge * F_A + \frac{i\theta}{8\pi^2} \int F_A \wedge F_A$ $\qquad \tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$

$N=4$ Supersymmetry: include extra fields

relevant: $(A, \phi) \in \underset{\text{gauge}}{\mathcal{A}} \oplus \underset{\text{higgs}}{\Omega^1(M, ad(P))} \cong T^*\mathcal{A}$

Twist: convert to TFT

$$(G, \tau) \longleftrightarrow ({}^L G, 1/\tau)$$

Montonen–Olive duality

Q: why ${}^L G$?

$\hookrightarrow$ derive Langlands correspondence from S-Duality?

first step: dim $4 \rightarrow 2$

Compactification

$M = \Sigma \times C$

$\sigma : \Sigma \times C \longrightarrow T^*\mathcal{A}(\Sigma \times C)$

$\sigma_\Sigma : \Sigma \to \{\sigma_c : C \to T^*\mathcal{A}(\Sigma \times C)\}$ reduce $T^*\mathcal{A}(\Sigma \times C) \to T^*\mathcal{A}(C)$

Idea: make energy of σ_c dominated by $T^*\mathcal{A}(C)$

"High" & "Low" energy dimensions: Low energy approximation

$\Rightarrow$ Dimensional reduction

Compactification

$$S(A,\phi) = \int_M dV \; \|F_A\|^2 + \|D^*\phi\|^2 + \text{topological}$$

$$\geq S_\Sigma(A,\phi) + S_C(A,\phi) \geq 0$$

$\mathrm{Vol}(C) \to 0 \;\Rightarrow\; S_C \to \infty$ $S \sim L \cdot \left(\frac{1}{L}\right)^2 = L^{-1}$

$\Rightarrow \int \mathcal{D}(A,\phi)\, e^{-S(A,\phi)}$ localizes to $S_C(A,\phi) = 0$

$$\Rightarrow \sigma : M \to T^*\mathcal{A} \rightsquigarrow \sigma_{\text{eff}} : \Sigma \to S_C^{-1}(0)$$

Classical Vacua

QFT topological $\Rightarrow$ this is exact!

$$F_A - \phi \wedge \phi = 0$$
$$D\phi = D^*\phi = 0$$

HyperKähler!

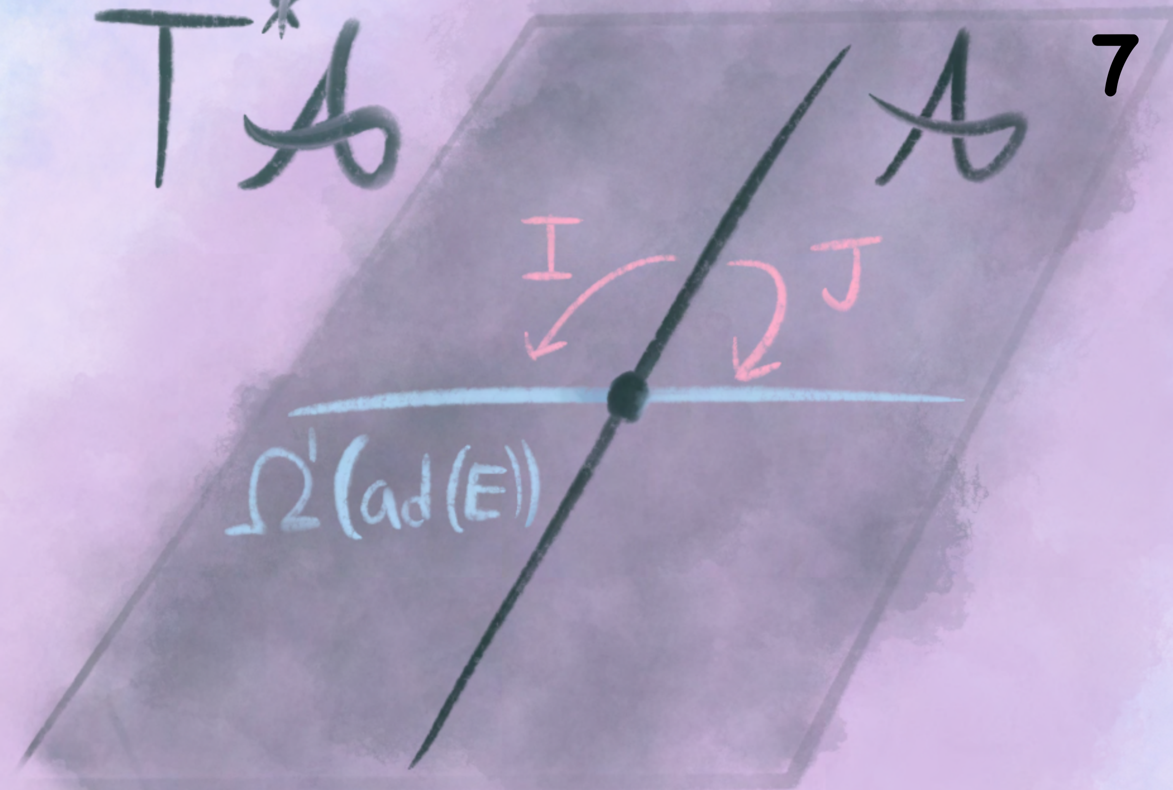
$$T^*\mathcal{A} \ni (A,\phi) \longleftrightarrow d_0 + A + i\phi \in \mathcal{A}_{\mathbb{C}}$$

holomorphic Symplectic structures

Ω_I	Ω_J
$\int_C \mathrm{Tr}\,\delta A \wedge \delta\phi$	$\int \mathrm{Tr}\,\delta A \wedge \delta A$
(from T^*)	(from $\mathcal{A}$)

Atiyah-Bott

$T^*\mathcal{A}$



Ω_I holo. in I: $A + i\phi \to i(A + i\phi)$

Ω_J holo. in J: $\begin{pmatrix} A \\ \phi \end{pmatrix} \to \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A \\ \phi \end{pmatrix}$

$\hookrightarrow$ hyperKähler structure

Linear complex structures:

$I \quad J \quad K = IJ$

Real Symplectic structures:

$\omega_I \quad \omega_J \quad \omega_K$

$\Omega_I = \omega_J + i\omega_K$, etc...

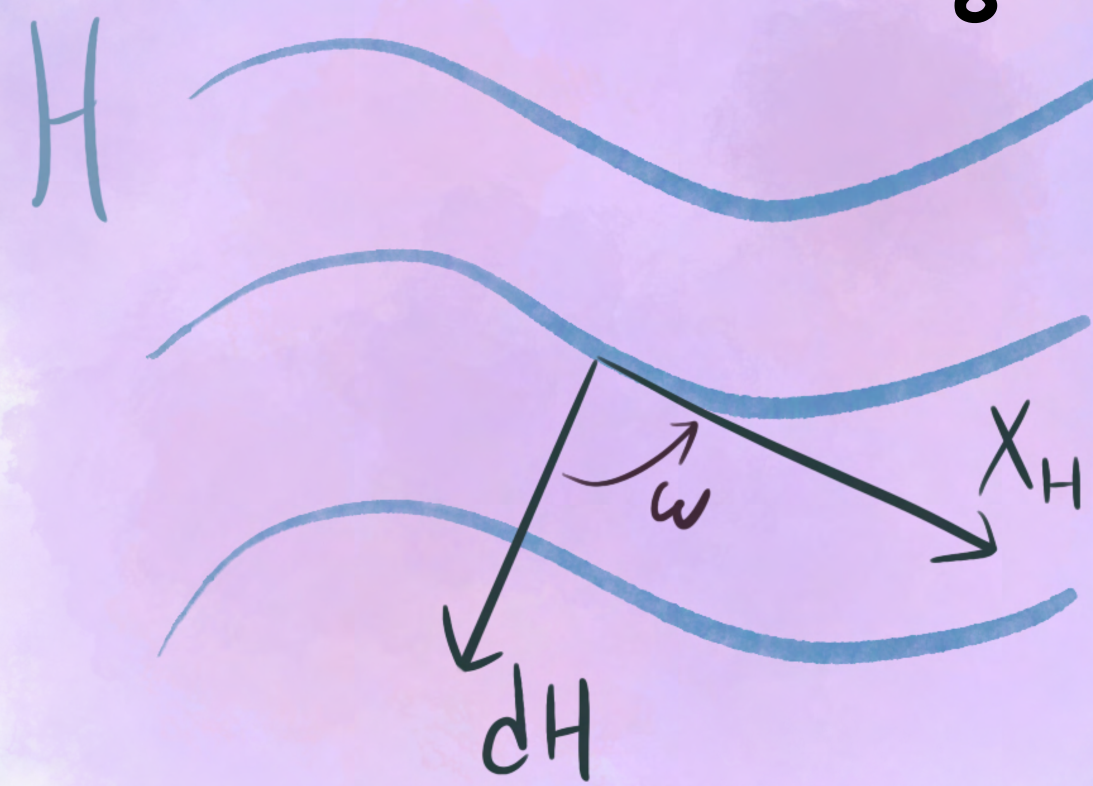
Quaternions!

$IJ = -JI$

$\mathbb{C}P^1$ of complex structures

Symplectic form $\omega \in \Omega^2(M)$
(closed, nondegenerate)

$$dH = \omega(X_H, \bullet)$$



Symplectic group action G on M
$\Rightarrow$ induced by hamiltonian

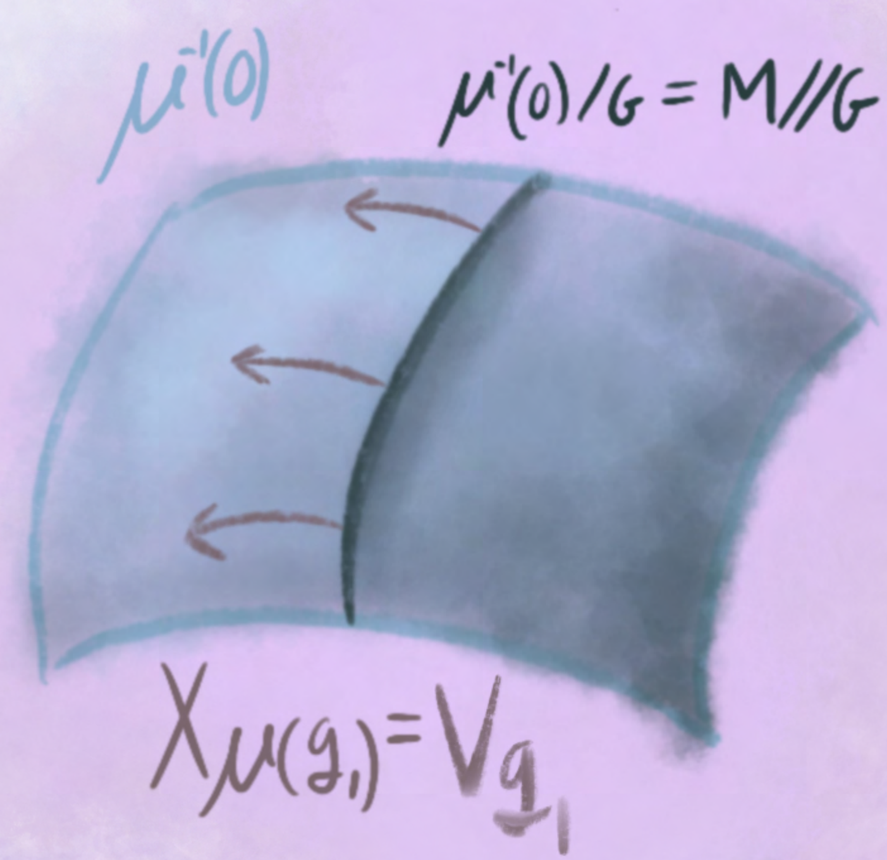
$\mu: M \to \underline{g}^*$ **Momentum Map**

for $g \in \underline{g}$, V_g induced by $\mu(g)$

$\{\mu(g_1), \ldots, \mu(g_n)\}$ hamiltonians $\{g_i\}$ basis of $\underline{g}$

$X_H(\mu(g_1)) = -X_{\mu(g_1)}(H) = 0$ for G-invariant H

$\Rightarrow \mu$ conserved quantity ("momentum")



$$M//G = \mu^{-1}(0)/G$$

Symplectic Quotient

HyperKähler Quotient

Symmetry: Gauge transforms $\mathcal{G}$

(Automorphisms of P trivial on C)

lie algebra $\Omega^0(C, \mathrm{ad}(P))$

$$\Omega^0(C, \mathrm{ad}(P))^* \cong \Omega^2(C, \mathrm{ad}(P))$$

$\mu_I = F_A - \phi \wedge \phi \qquad \mu_J = D_A \phi \qquad \mu_K = D_A * \phi$

$$T^*\mathcal{A} /\!/\!/ \mathcal{G} = \mu_I^{-1}(0) \cap \mu_J^{-1}(0) \cap \mu_K^{-1}(0) \,/\, \mathcal{G}$$

Reduce in stages:

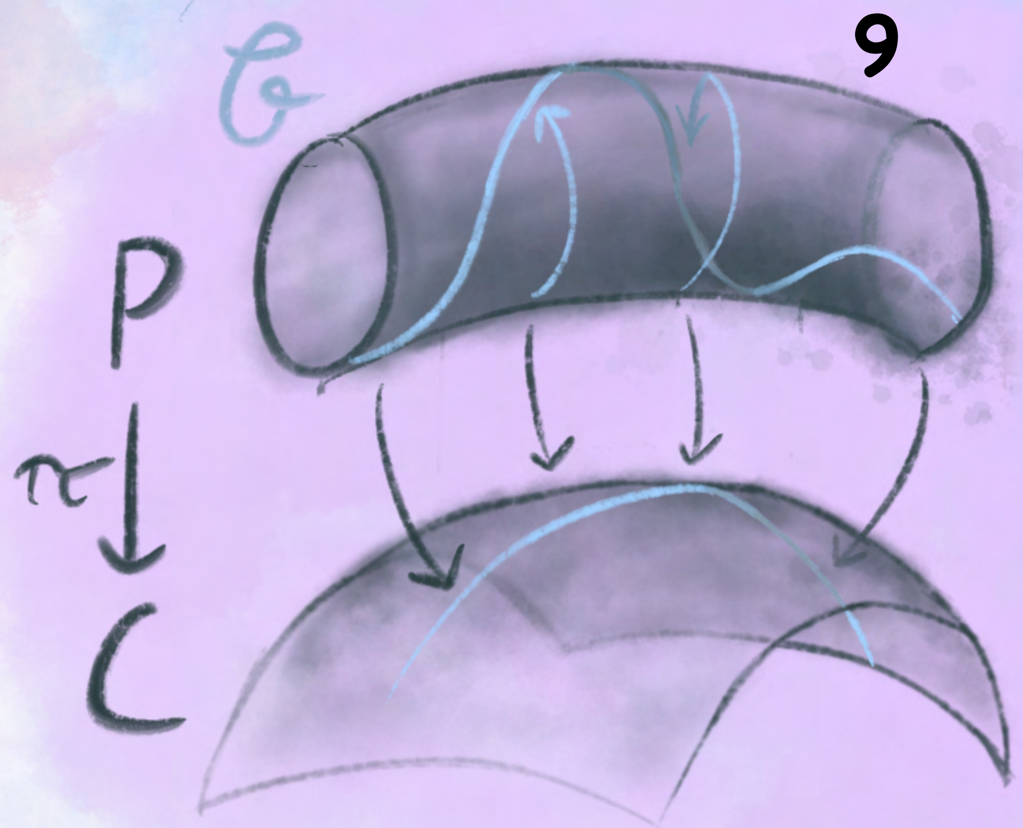
J: Ω_J has moment $\nu_J = \mu_K + i\mu_I$

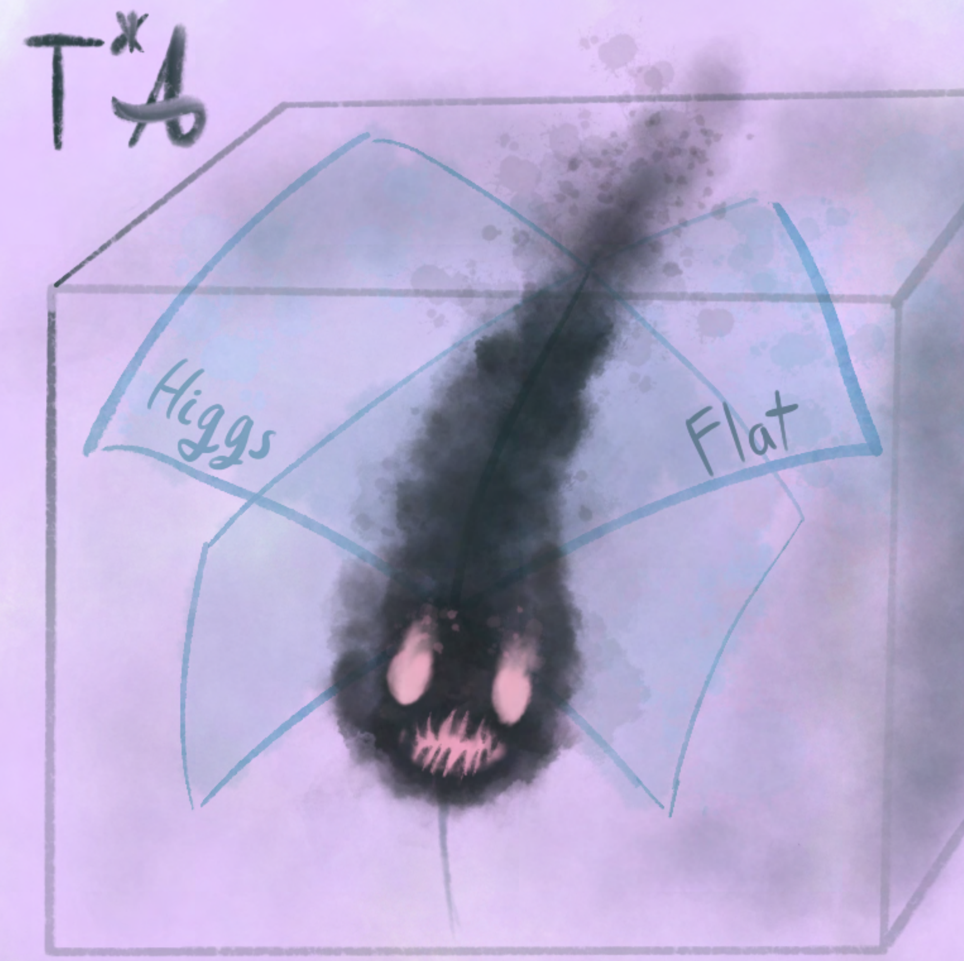
(Atiyah-Bott) $\nu_J = F_{A_{\mathbb{C}}}$!

Space of flat $G_{\mathbb{C}}$-bundles

I: $\Omega_I = 0 \Rightarrow D_A \phi = D_A^* \phi = 0 \Rightarrow \bar{\partial}_A \phi^{1,0} = 0$

space of higgs bundles (P, ϕ)





Nonabelian Hodge Correspondence

De-Rham

Singular manifolds

Dolbeault

Same real structure
diffrent complex structure

J I

C

Moduli of Higgs bundles

(P,ϕ) $\phi \in \Omega^1(C, ad(P))$

$H^0(C, K^* \otimes ad(P)) = H^1(C, ad(P))$ Def. space of Bun_G

Serre duality

$T^* Bun_G$

Modulii of flat $G_{\mathbb{C}}$ – connections

$Loc_{G_{\mathbb{C}}}$

Stability

dim @ $x \in M/G$ = dim M − dim G

x has stabilizer: dim @ x = dim M − dim G + dim Stab

$\Rightarrow$ cut out!

Stabilizer $\Leftrightarrow$ structure splits

- Flat: stable $\Leftrightarrow$ irreducible holonomy
- Higgs: $L < E$ holo. subbundle; $\dfrac{c_1(L)}{\mathrm{rank}(L)} \leq \dfrac{c_1(E)}{\mathrm{rank}(L)}$ Slope stability!

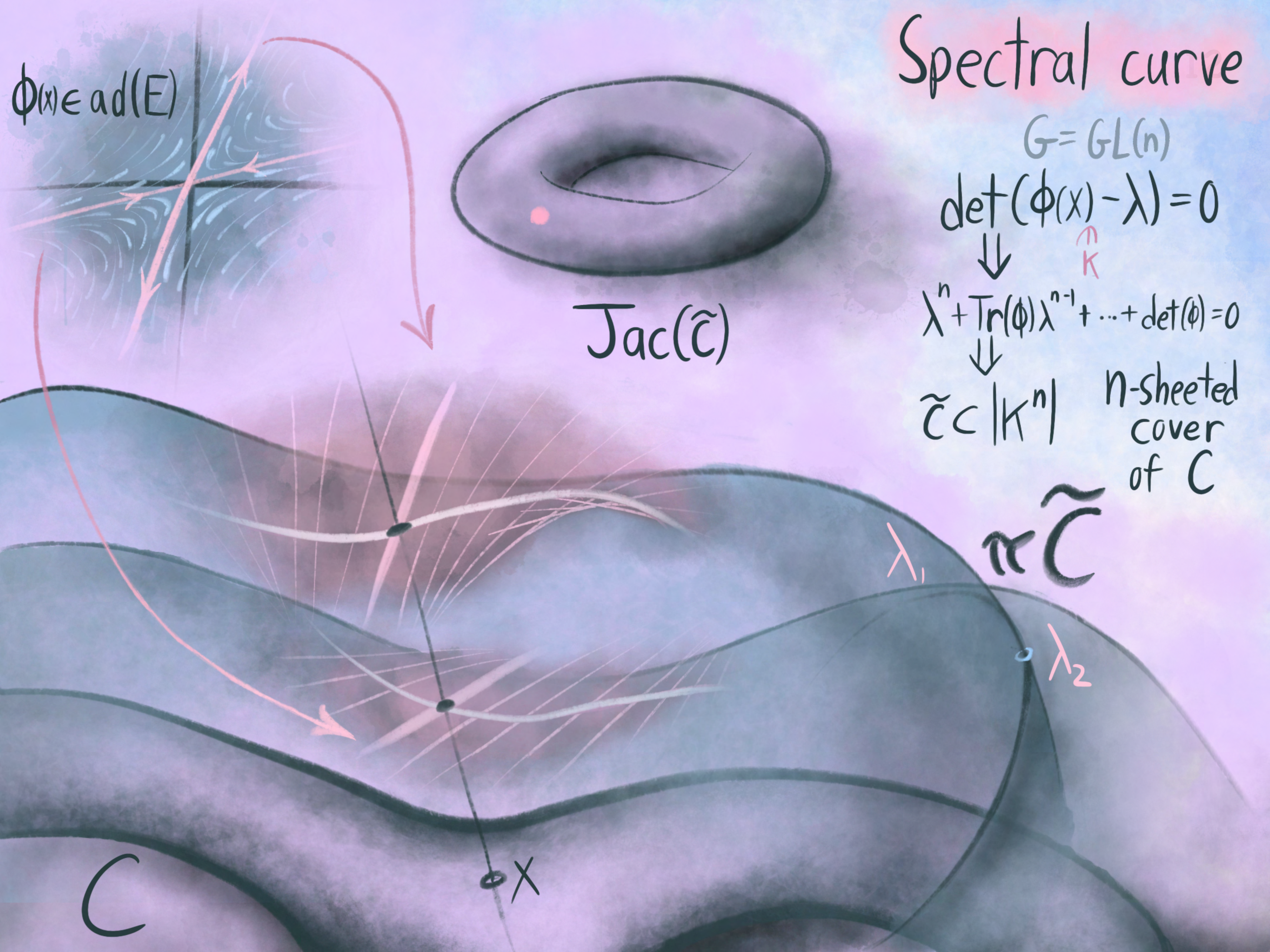
$\dfrac{\text{Irreducible}}{\text{Flat bundles}}$

$\dfrac{\text{Semistable}}{\text{Higgs bundles}}$

smooth manifolds

$\longleftrightarrow$

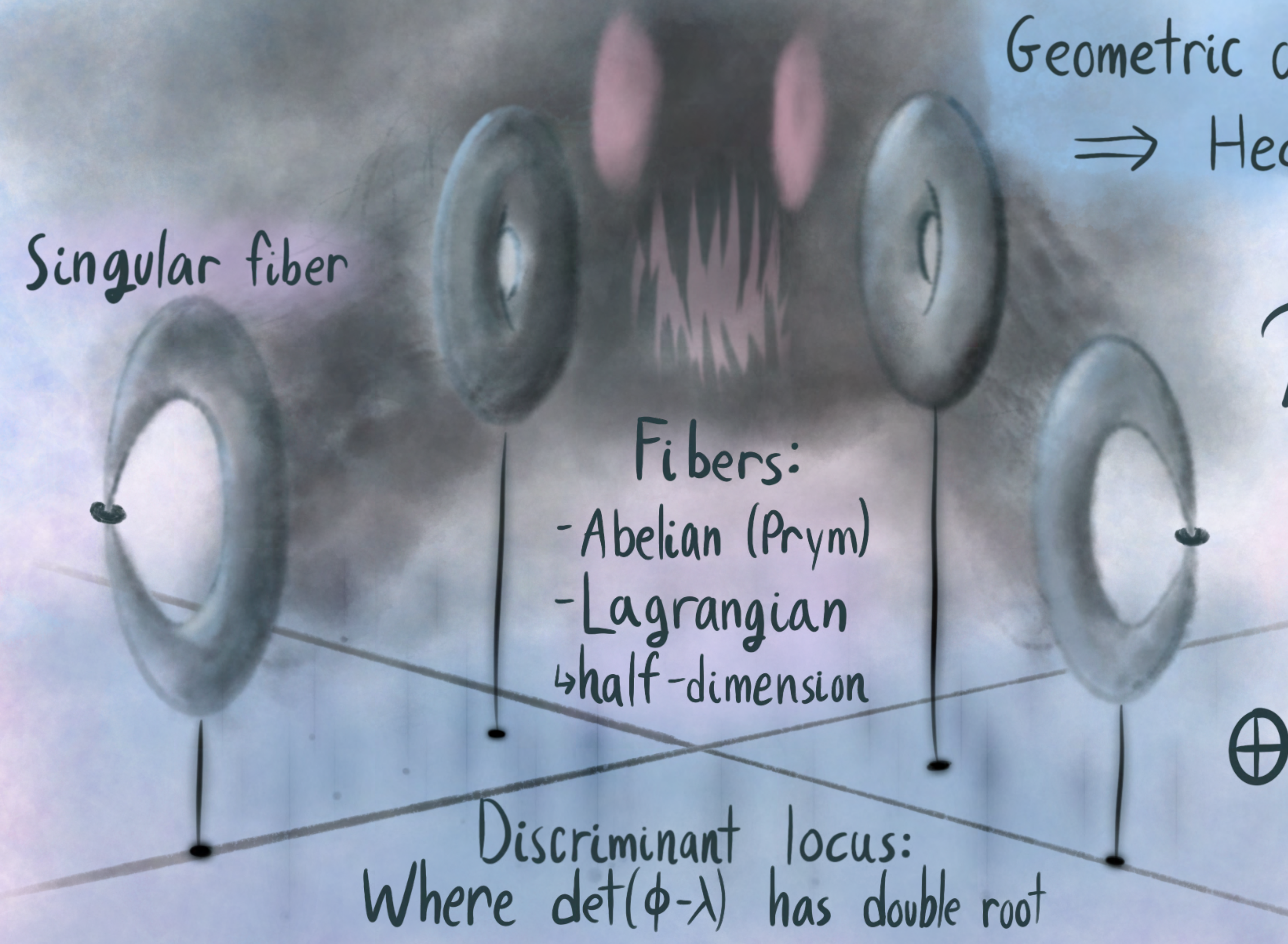
diffeomorphic!


Spectral curve
$G = GL(n)$
$\det(\phi(x) - \lambda) = 0$
K
$\Downarrow$
$\lambda^n + Tr(\phi)\lambda^{n-1} + \dots + \det(\phi) = 0$
$\Downarrow$
$\tilde{C} \subset |K^n|$
n-sheeted cover of C
π
$\tilde{C}$
$\phi(x) \in ad(E)$
$Jac(\tilde{C})$
λ_1
λ_2
x
C

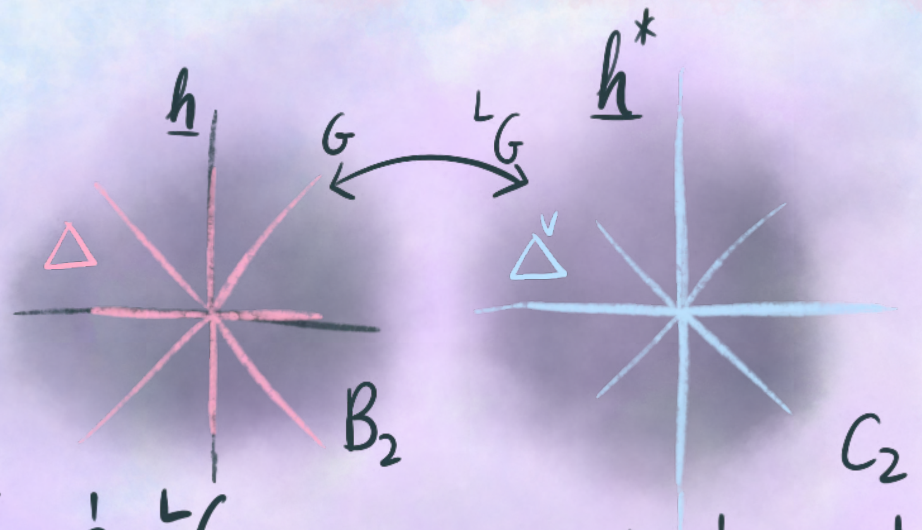
Hitchin Fibration

Completely integrable:
(Beilinson & Drinfeld '04)
Geometric quantization
$\Longrightarrow$ Hecke operators!

$$\mathcal{M}^{Higgs} \xrightarrow{\mathbb{C}[\underline{g}]^G} \bigoplus H^0(C, K^{d_i})$$



Langlands dual hitchin Moduli



G & ${}^LG$ swap roots & coroots

$$\mathbb{C}[\mathfrak{g}]^G \overset{\text{chevalley}}{\cong} \mathbb{C}[\mathfrak{h}]^W \xleftrightarrow{\langle,\rangle} \mathbb{C}[\mathfrak{h}^*]^W \overset{\text{chevalley}}{\cong} \mathbb{C}[{}^L\mathfrak{g}]^{{}^LG}$$

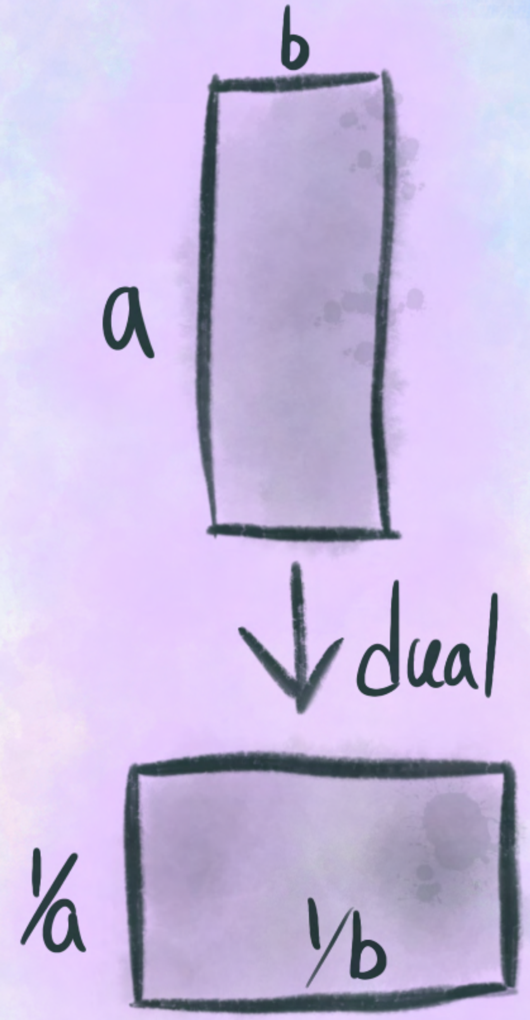
$$\Delta \longmapsto \check{\Delta}$$

$$\mathbb{C}[\mathfrak{g}]^G \cong \mathbb{C}[{}^L\mathfrak{g}]^{{}^LG}$$

T-Duality

fibers $\mathcal{M}_a$ & ${}^L\mathcal{M}_a$ are **Dual Abelian vareities**

$\mathcal{M}_a$ paramertizes line bundles on ${}^L\mathcal{M}_a$ & vice-versa



spectral curve of a

$G=GL(n)$: $\mathcal{M}_a \simeq {}^L\mathcal{M}_a \simeq \mathrm{Jac}(\tilde{C}_a)$

Jacobians of smooth $\tilde{C}_a$ are **self-dual**

$$\mathbb{C}^n/\Lambda \xleftrightarrow[\langle,\rangle]{\text{dual}} \mathbb{C}^n/\Lambda^* \quad :: \quad \underline{h} \xleftrightarrow[\langle,\rangle]{} \underline{h}^*$$

T-duality: $\Sigma \to M \times S^1_R \cong \Sigma \to M \times S^1_{1/R}$

target space

SYZ Mirror Symmetry

nonlinear σ-models

Mirror symmetry: $\sigma:\Sigma \longrightarrow X \;\approx\; \sigma:\Sigma \longrightarrow \check{X}$

S-duality: $\sigma:\Sigma \longrightarrow \mathcal{M}_H \;\approx\; \sigma:\Sigma \longrightarrow {}^L\mathcal{M}_H$

Symplectic X

$\Updownarrow$

Complex $\check{X}$

$\check{\mathcal{M}} = {}^L\mathcal{M}$?

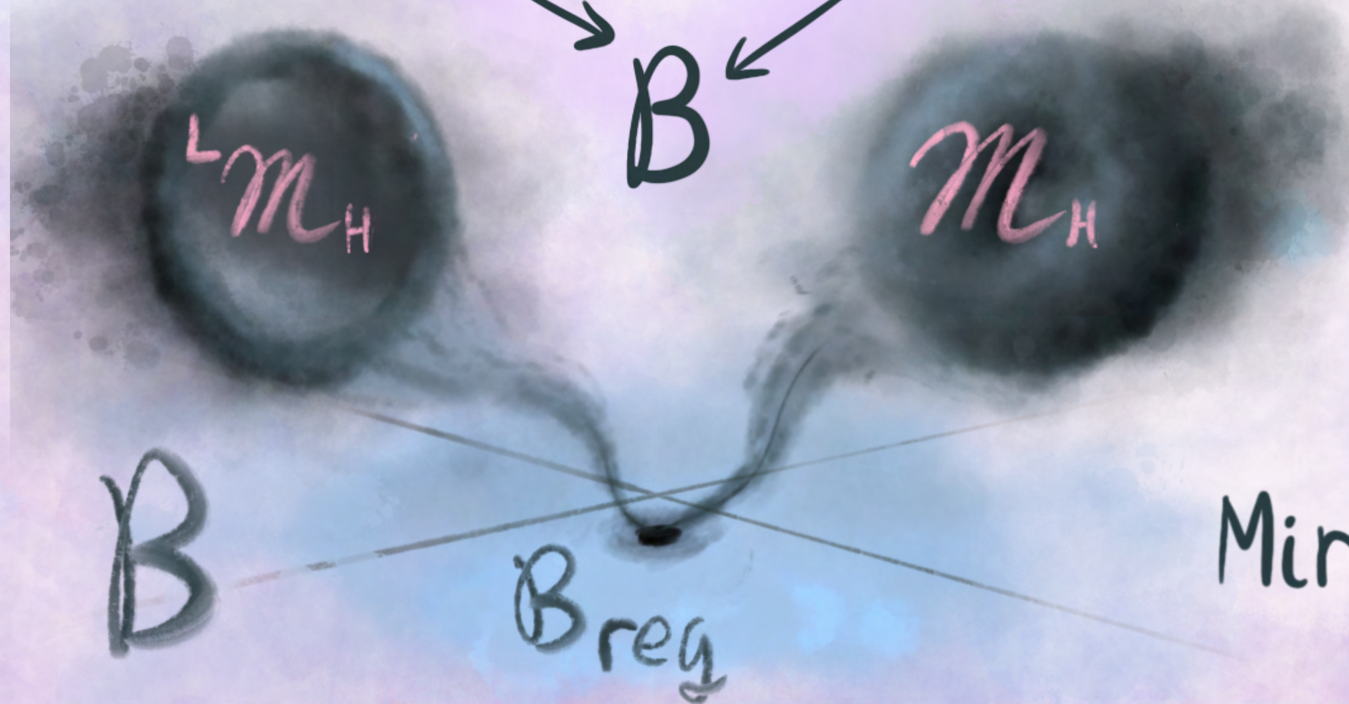
"Mirror symmetry is T-Duality"

On $B_{reg} \subset B$ dense: Special lagrangian tori fibration

↳ volume n-form real on fibers (area minimizing)

- $\pi^{-1}(b)$ & $\hat{\pi}^{-1}(b)$ dual

$$\begin{tikzcd} X \arrow[dr, "\pi"] & & \hat{X} \arrow[dl, "\hat{\pi}"'] \\ & B & \end{tikzcd}$$



Conjecture (SYZ '96):

Mirror C.Y. 3-folds are SYZ fibrations

S-duality

Hitchin's eq.

Nonabelian hodge theory

T-duality

Mirror Symmetry

categories...?

To Be Continued


Insane in
D-BRANE

Categorical Geometric Langlands

Arithmetic:

flat ${}^LG$ - Bundle

$\downarrow$ $\mathcal{E} \in Loc_{{}^LG} \Rightarrow$ skyscraper sheaf $\mathcal{O}_{\mathcal{E}}$

(derived)
Category of $\mathcal{O}$-modules on $Loc_{{}^LG}$

Automorphic:

Hecke eigensheaf on Bun_G

$\downarrow$ e. sheaf $\Rightarrow$ $\mathcal{D}$-module

(derived)
Category of $\mathcal{D}$-modules on Bun_G

Daydream

Category of $\mathcal{O}$-modules on $Loc_{{}^LG}$ $\xleftrightarrow{\text{equivlence}}$ Category of $\mathcal{D}$-modules on Bun_G

Spectral decomposition of $\mathcal{D}$-modules on Bun_G into Hecke e.sheafs $F_{\mathcal{E}}$

TQFT

QFT assigns # to a riem. n-mfld
w/ boundry conditions

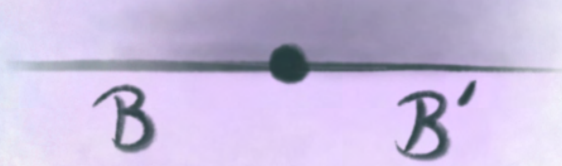
TQFT: local data (metric) doesn't matter
$\Rightarrow$ ONLY B.C.s matter!! $\Rightarrow$ Define by B.Cs

N-mfld $\rightarrow \mathbb{C}$ N-1 mfld $\rightarrow V$

Partition fn   States

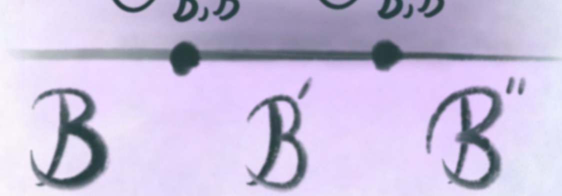
Boundry condition for
boundry conditions

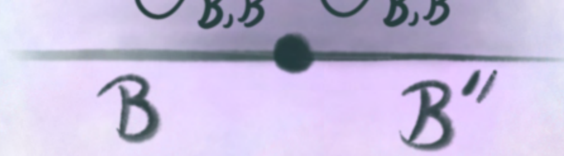
$\mathcal{O}_{\mathcal{B},\mathcal{B}'} : V_{\mathcal{B}} \rightarrow V_{\mathcal{B}'}$



category of B.Cs

$\mathcal{O}_{\mathcal{B},\mathcal{B}'}$ $\mathcal{O}_{\mathcal{B}',\mathcal{B}''}$ $\longrightarrow$ $\mathcal{O}_{\mathcal{B},\mathcal{B}'} \cdot \mathcal{O}_{\mathcal{B}',\mathcal{B}''}$





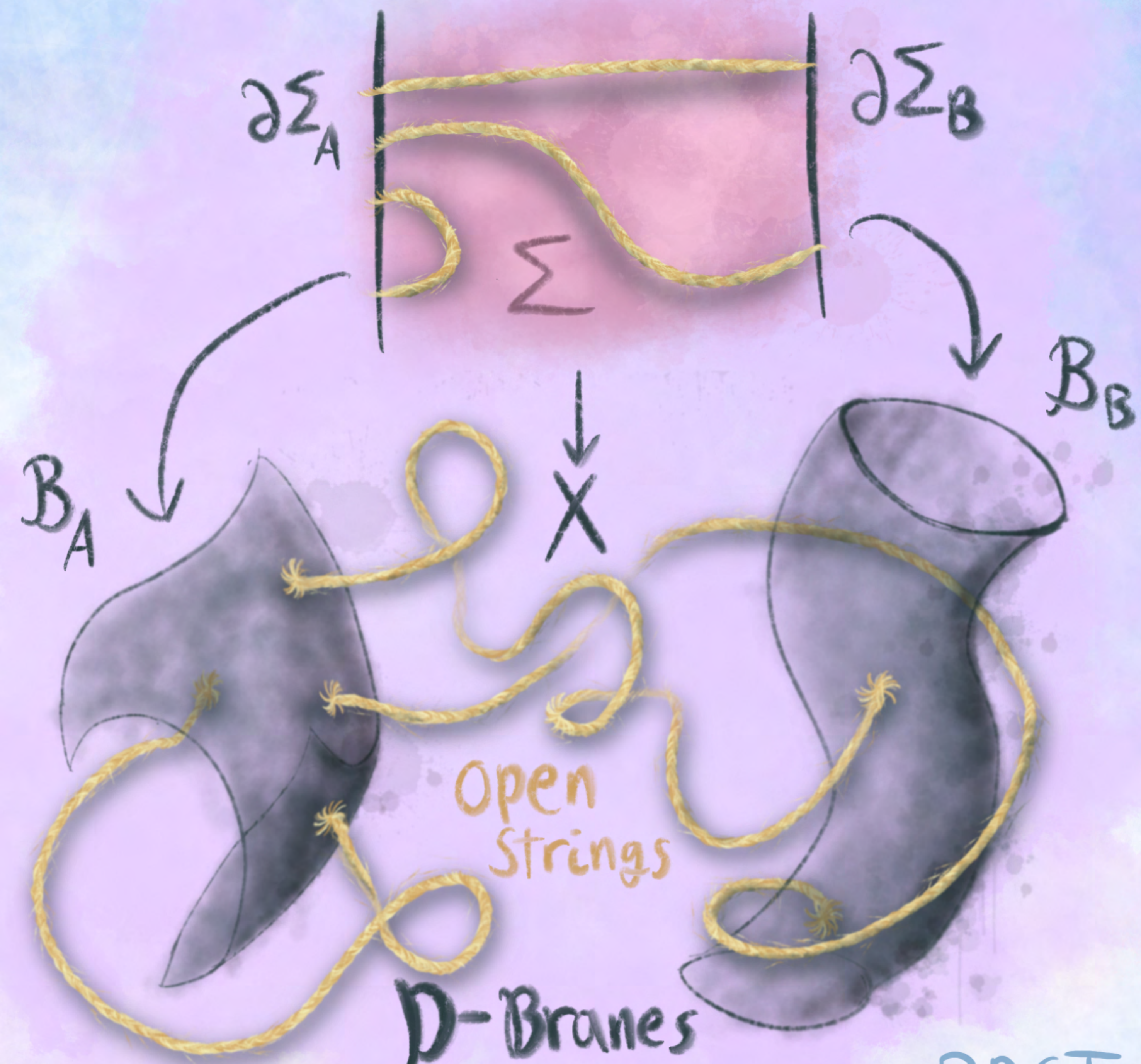
	n	$n-1$	$n-2$	$n-3$	
Extended TQFT:	$\mathbb{C}$	$V_{\mathbb{C}}$	category	2-category	$\circ\ \circ\ \circ$

D-branes

2D-CFT σ-model $\Sigma \to X$

$\Updownarrow$

Strings on X



Boundry conditions:

fields on $\partial\Sigma$ live on $B \subset X$
B has vector bundle (sheaf)
"chan - Paton"

Perserve Conformal invariance
$\Rightarrow$ D-brane!

Supersymmetry: want B w/ $\{Q, B\} = 0$ mod $B = \{Q, B_0\}$

BRST cohomology

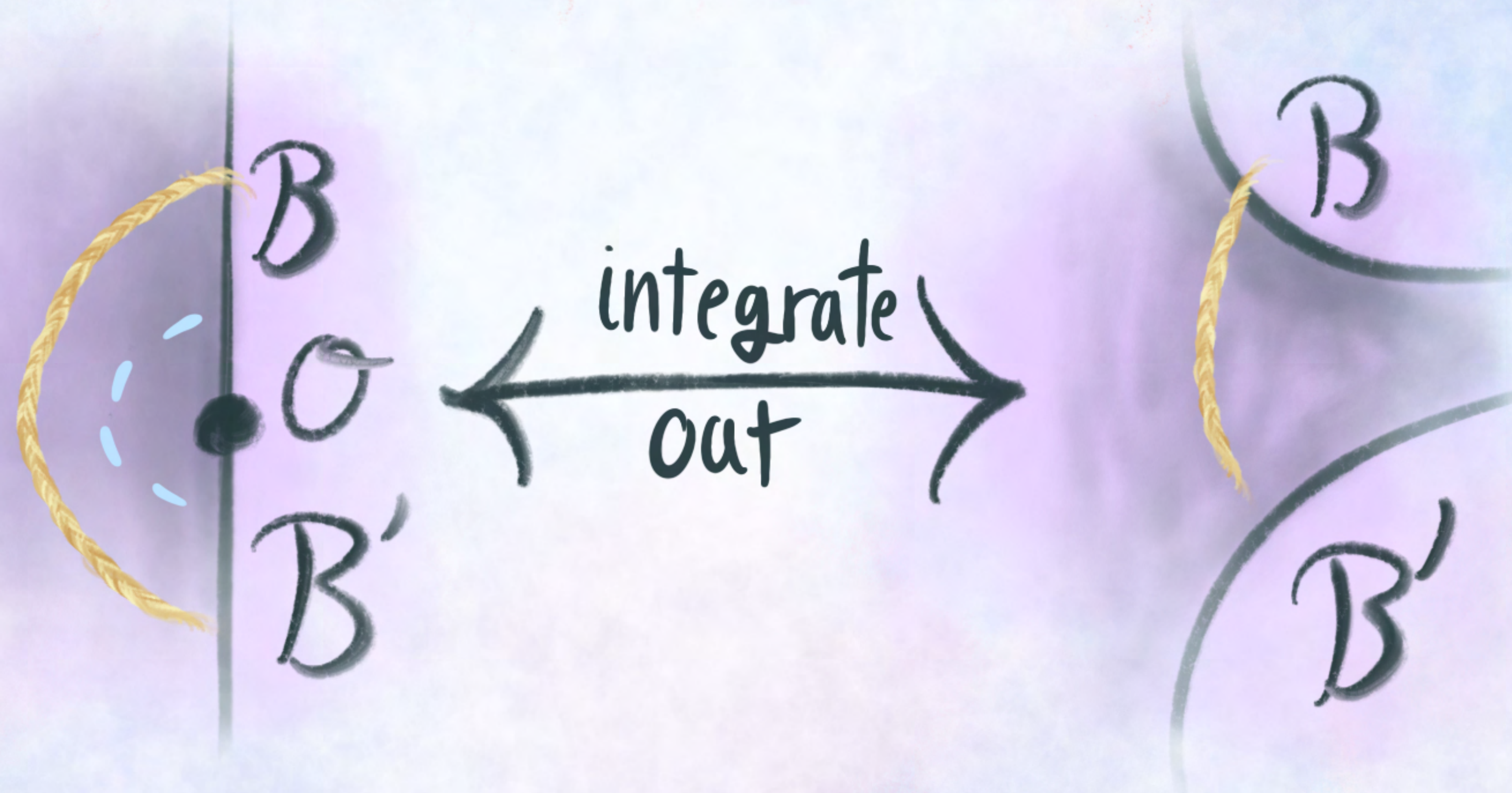
$$\cdots \xrightarrow{Q} B_0 \xrightarrow{Q} B_1 \xrightarrow{Q} B_2 \xrightarrow{Q} \cdots$$

BRST operator $Q^2 = 0$

Q-cohomology = Q-chain complex / Quasi-isomorphism

Derived categories!

Open Strings



$\mathrm{Hom}(\mathcal{B}, \mathcal{B}') \quad = \quad \mathcal{H}_{\mathcal{B},\mathcal{B}'}$ hilbert space of open strings

morphism in $\mathcal{B}$ category $=$ string between $\mathcal{B}$ & $\mathcal{B}'$

B-Branes

$\sigma : \Sigma \to X$, X calabi-Yau

B-twist: $Q \to \bar{\partial}_X$!

B-Brane: complex submfld of X w/ holomorphic V.B.

Coherent sheaves:

holomorphic V.B.s = sheaf of finitely generated $\mathcal{O}_X$-modules

category of v.b.s not abelian: dim $\mathrm{Ker}\,\phi : V_1 \to V_2$ can jump

abelianization: "Coherent sheaves"

contains all kernels $\Rightarrow$ contains V.Bs on submflds!

$\mathrm{Ker}\,\phi : \mathbb{R}^2 \times L \to \mathbb{R}^2 \times L$

$((x,y),t) \mapsto ((x,y), x\cdot t)$

B-Branes $\in$ $D^b(X)$ (derived) category of coherent sheaves

A-Branes (Y, ∇)

Defined w.r.t symplectic structure on X

Y is coisotropic: $\omega|_{Y^\perp} = 0 \Rightarrow \dim Y \geq \frac{1}{2} \dim X$

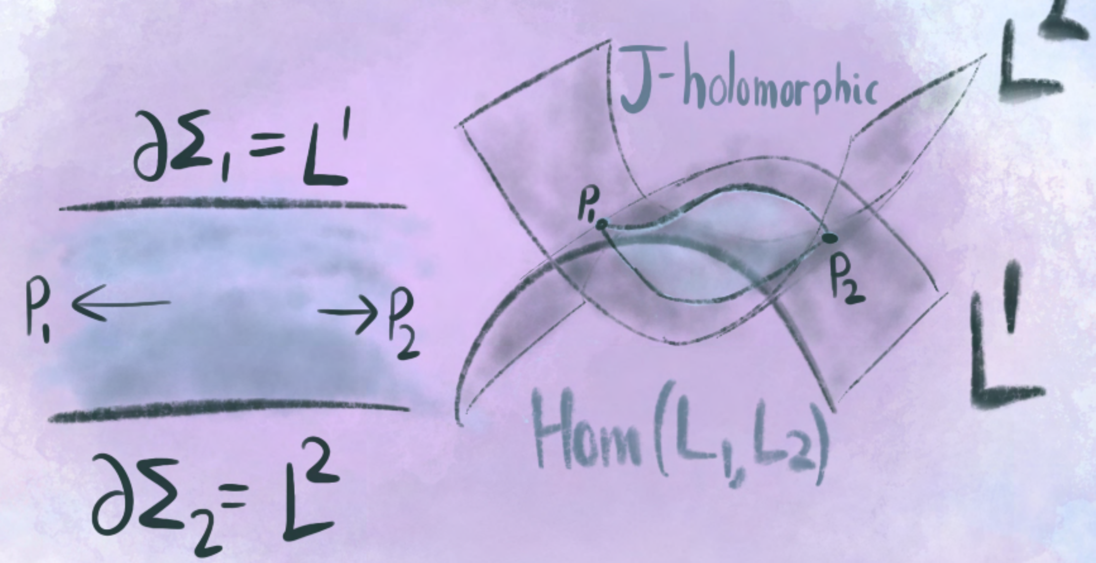
familiar case: $\dim Y = \frac{1}{2} \dim X$ "Lagrangian submanifold"

$\Rightarrow \nabla$ is flat

morphisms: open string states

complexes of pts $L_1 \cap L_2$ w/ differential counting curves — lagrangian Floer homology

Fukaya category



includes coisotropic branes

A-brane category: Derived, "extended" Fukaya category

$D(\mathcal{F}uK(X))$

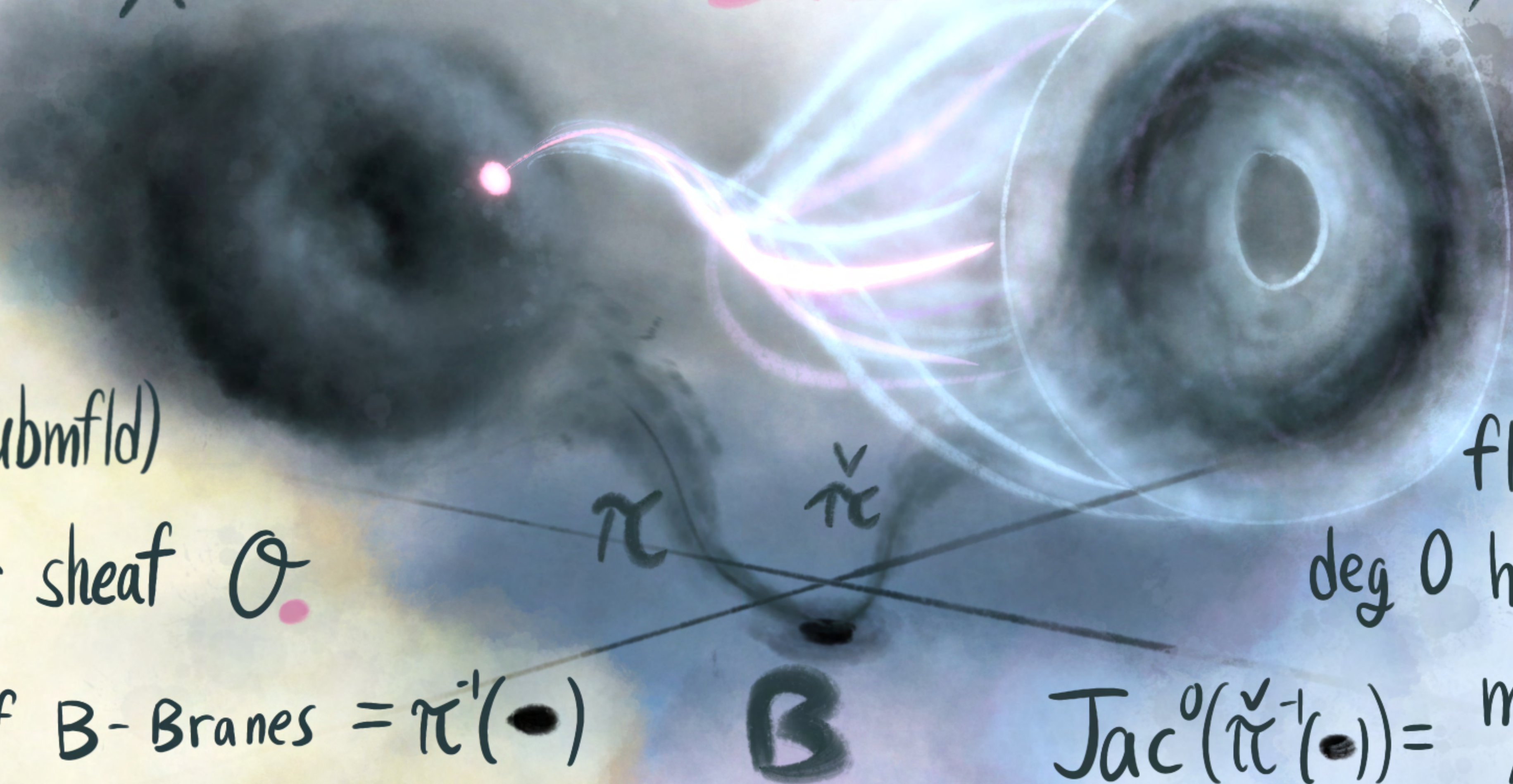
Homological mirror symmetry

$$D^b(X) \longleftrightarrow D\mathcal{F}uk(\check{X})$$

B-branes on X | A-Branes on $\check{X}$

SYZ

X | $\check{X}$



0-brane
type B:
(complex submfld)

skyscraper sheaf $\mathcal{O}_\bullet$

moduli of B-Branes $= \pi^{-1}(\bullet)$

n-brane
around $\check{\pi}^{-1}(\bullet)$
type A:
lagrangian

flat $U(1)$ connection
$\Updownarrow$
deg 0 holo. line bundle

$\mathrm{Jac}^0(\check{\pi}^{-1}(\bullet)) =$ moduli of A-Branes

Specalize to Hitchin moduli space:

$$\mathcal{M}^{Flat}_{({}^LG)} = \check{\mathcal{M}}^{Higgs}_{(G)}$$

A-branes on $\mathcal{M}^{Higgs}_{(G)}$

$\mathcal{D}\,\mathcal{F}uk(\mathcal{M}^{Higgs}_{(G)})$

mirror symmetry

B-Branes on $\mathcal{M}^{FLAT}_{({}^LG)}$

$\mathcal{D}^b(coh(\mathcal{M}^{FLAT}_{({}^LG)}))$

$\mathcal{O}$-modules on $Loc_{{}^LG}$

???

$\mathcal{D}$-modules on Bun_G

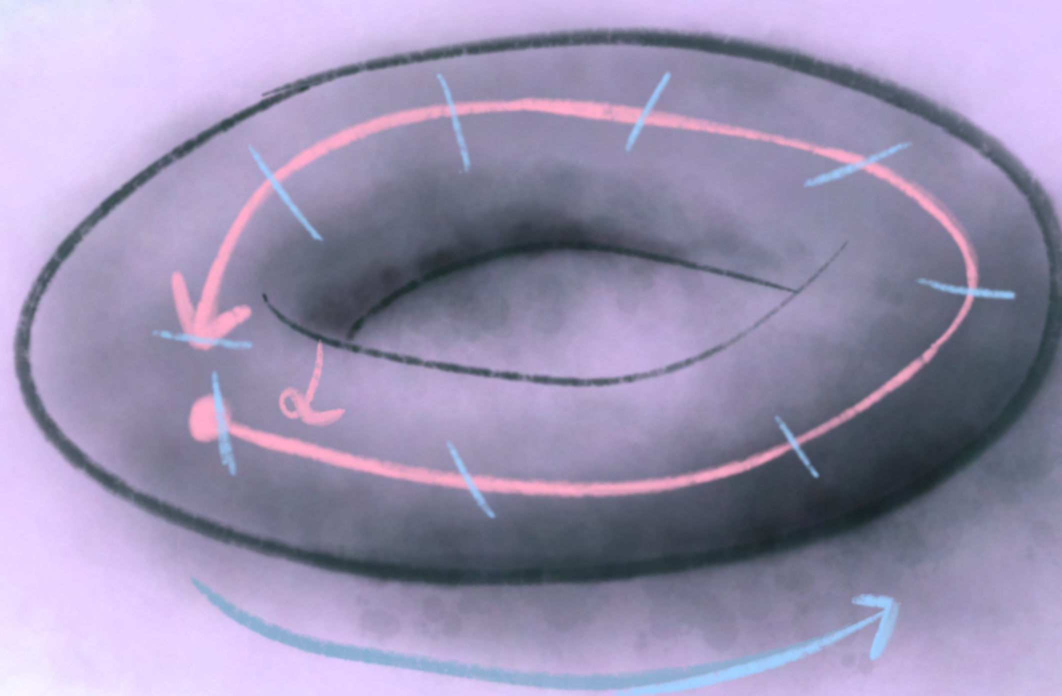
$$\mathcal{M}^{Flat}_{({}^LG)} = Loc_{{}^LG}$$

Geometric Langlands

$$T^*Bun_G^{stable} \underset{dense}{\subseteq} \mathcal{M}^{Higgs}_{(G)}$$

$$Bun_G^{stable} \underset{dense}{\subseteq} Bun_G$$

Wilson operator



Parralel transport by A along α

Pick loop α, irrep R

$$\mathrm{Tr}_R\, P\{\exp(-\oint_\alpha A)\}$$

NOT BRST invariant!!

$$= \mathrm{Tr}_R\, \mathrm{hol}_A(\alpha)$$

$$\Downarrow$$

$$W_R(\alpha) = \mathrm{Tr}_R\, P\{\exp(-\oint_\alpha A_{\mathbb{C}})\} \qquad A_{\mathbb{C}} = A + i\phi$$

higgs field $\phi \in T^*_A \mathcal{A}$

hitchin equations $\Rightarrow$ $A_{\mathbb{C}}$ flat $\Rightarrow$ $W_R(\alpha)$ invariant

`T Hooft operator

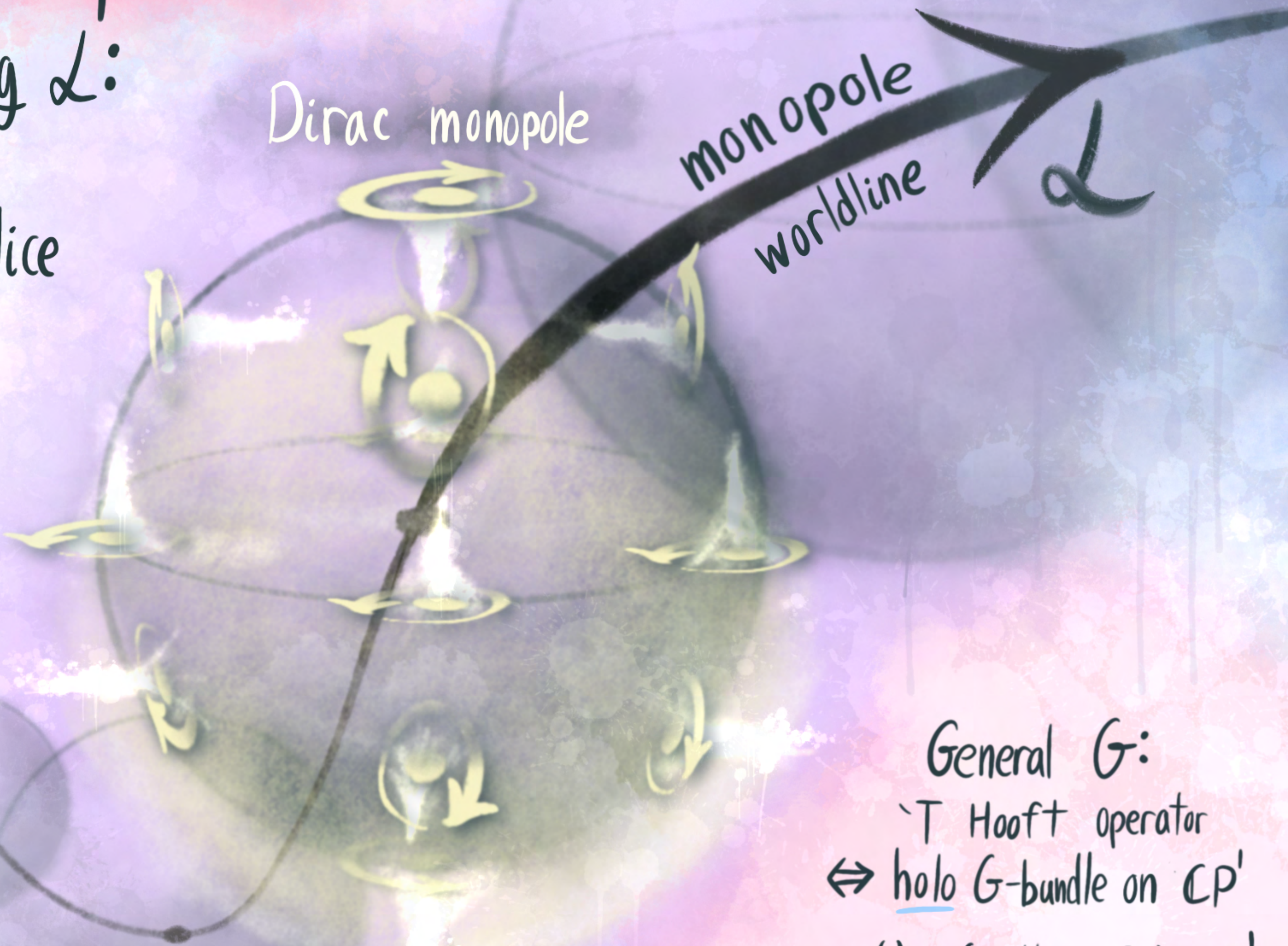
U(1) monopole along $\mathcal{L}$:

- cut out $\mathcal{L}$
- Twist bundle on slice

c_1 = magnetic charge

Yang-mills solution

$\Rightarrow F_A$ has pole @ $\mathcal{L}$



`T hooft op. T:

$$\int_{\mathcal{A}} T e^{S(A)} = \int_{\mathcal{A}_{\text{sing } @ \mathcal{L}}} e^{S(A)}$$

Disorder operator

General G:

`T Hooft operator

$\Leftrightarrow$ holo G-bundle on $\mathbb{CP}^1$

$\Leftrightarrow \rho : U(1) \to G$ thru $\mathcal{O}(1) \to \mathbb{CP}^1$

T_ρ makes dirac monopole w/ ρ

'magnetic charge' $= \rho : U(1) \to G / \text{conj.}$

S-Duality on line operators

electric
magnetic
Duality

'T Hooft

{magnetic charges}

$\{U(1) \to G\}/\sim$

$\{U(1) \to T\}/\mathcal{W}$

$\{\underline{u} \to \underline{t}\}/\mathcal{W}$

$\Lambda_{cw}(G)/\mathcal{W}$

$\Lambda_{w}({}^{L}G)/\mathcal{W}$

{irreps of ${}^{L}G$}

inserts magnetic particle

Wilson

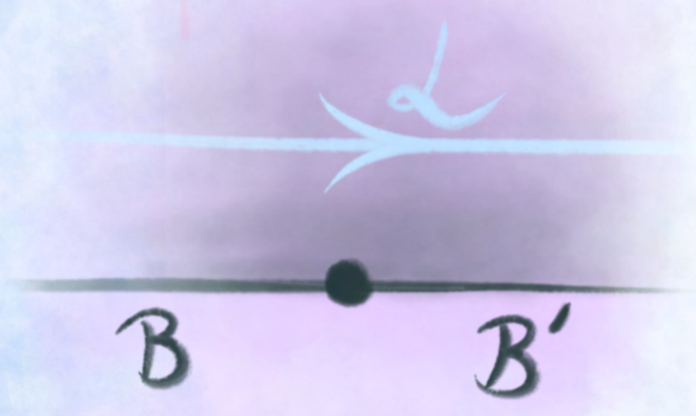
{Electric charges}

{irreps of G}

inserts electric particle

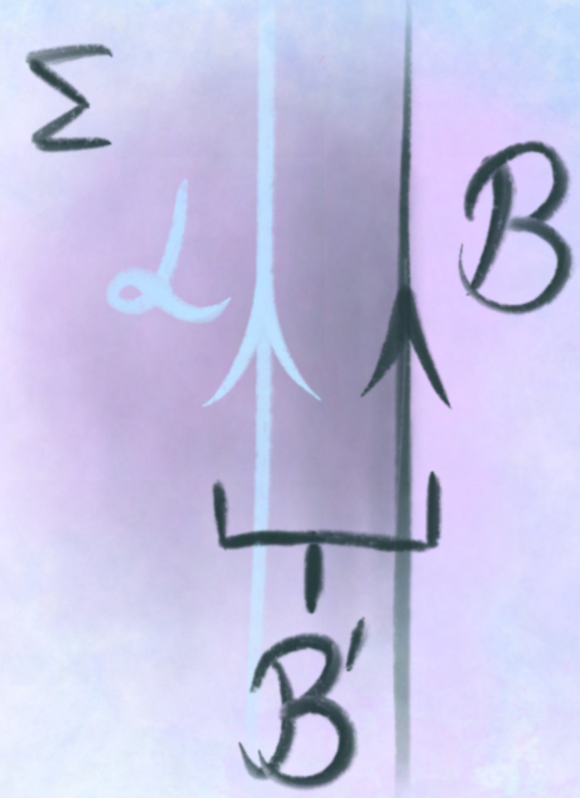
Line operators as functors

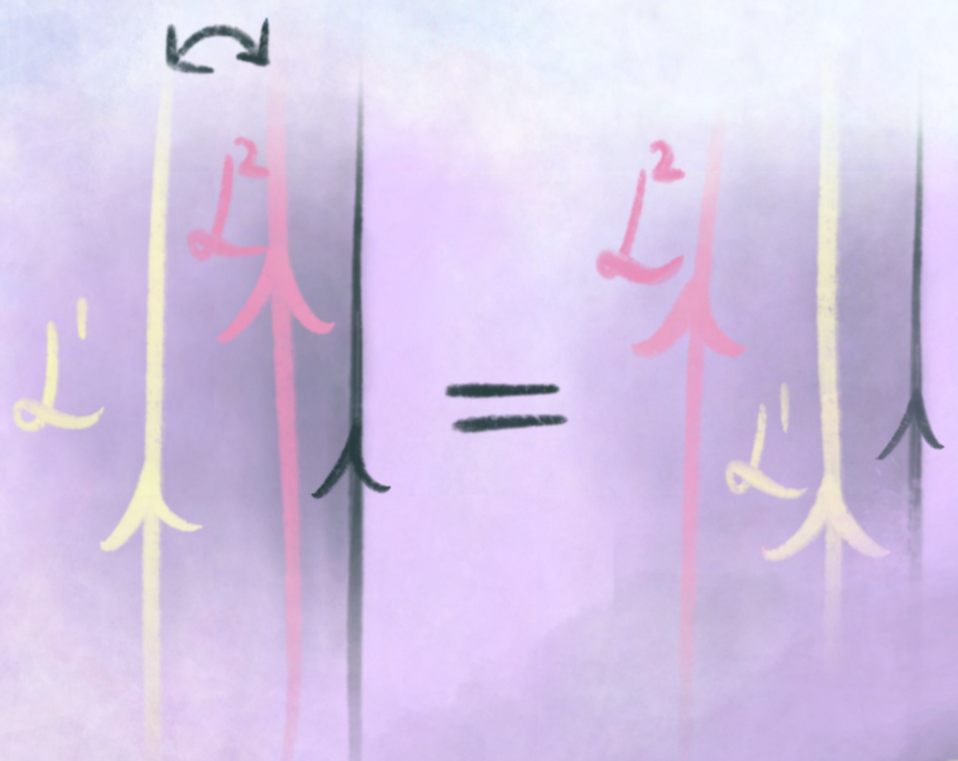
line operators 'modify' B.C.s $\hat{\mathcal{L}} : \mathcal{B} \to \mathcal{B}'$



functorial

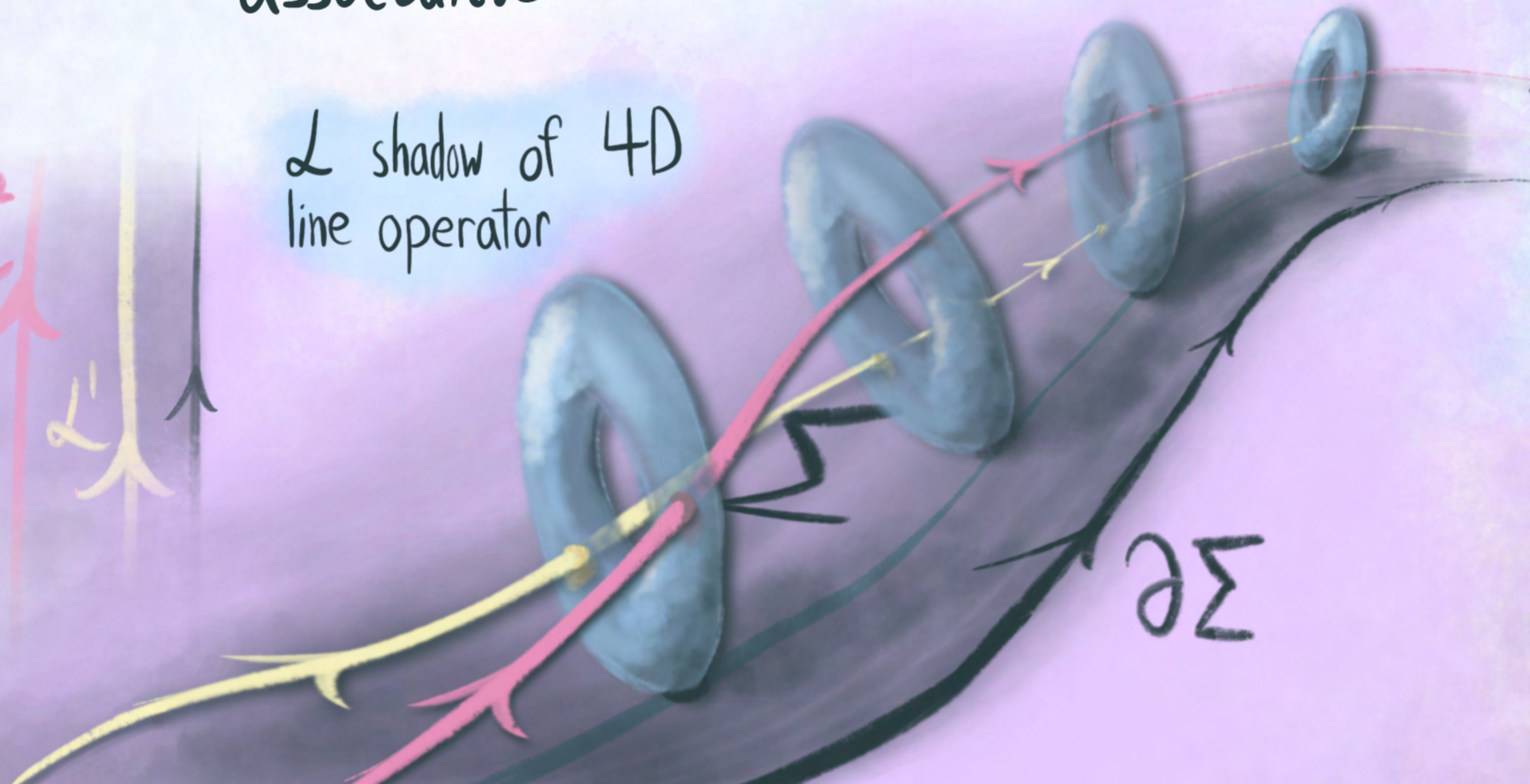
associative





Communative!

$\mathcal{L}$ shadow of 4D line operator



Eigenbrane

A brane $\mathcal{B}$ is an *Eigenbrane* of a line operator $\hat{\mathcal{L}}$ if

$$\hat{\mathcal{L}}\mathcal{B} = \mathcal{B} \otimes V$$

For any other brane $\tilde{\mathcal{B}}$, $\mathrm{Hom}(\tilde{\mathcal{B}}, \mathcal{B} \otimes V) = \mathrm{Hom}(\tilde{\mathcal{B}}, \mathcal{B}) \otimes V$

$\mathcal{B} \otimes \mathbb{C}^n =$ n copies of $\mathcal{B}$

for D-branes, $\otimes$ acts soley on chan-paton sheaf

$W_R(\mathcal{L})\mathcal{B} = \mathcal{B} \otimes V$

$\Downarrow$ S-Duality

$T_{{}^LR}(\mathcal{L})\check{\mathcal{B}} = \check{\mathcal{B}} \otimes V$

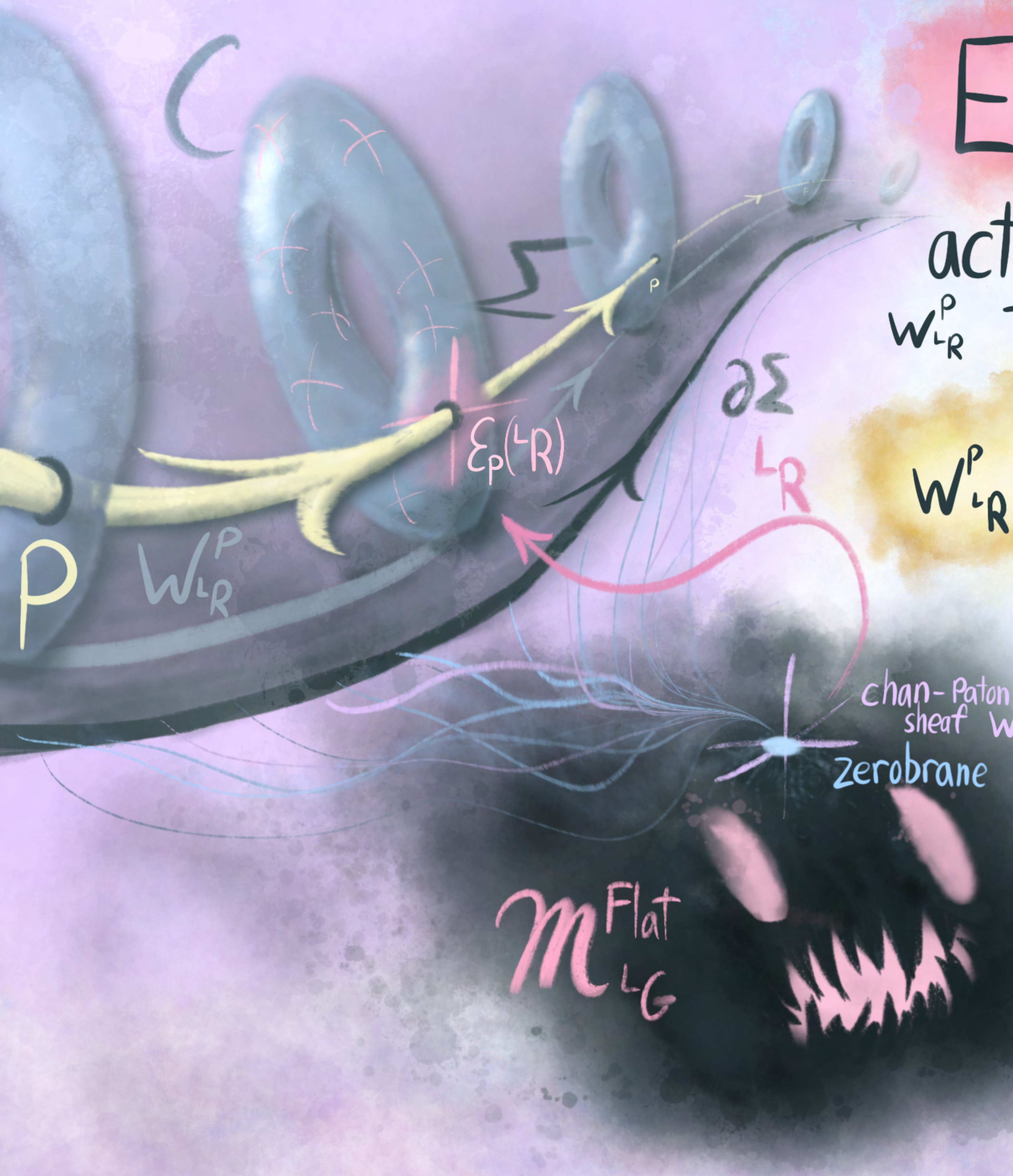
Dual e.brane is e.brane of dual operator w/ same e.value!

Line op.s commute

$\Downarrow$

simutanious eigenbranes

Electric e.brane



action of Wilson operator:

$W^p_{{}^LR}$ from $\mathcal{L} \subset \Sigma \times C$ w const. $p \in C$

$$W^p_{{}^LR}(\bullet, W) = (\bullet, \mathcal{E}_p({}^LR) \otimes W)$$

0-brane are e.branes

eigenvalue:

vary p over C

$\Rightarrow$ V.B of eigenvalues

$\mathcal{E}({}^LR) \longrightarrow C$

Magnetic eigenbranes

Hecke eigensheaf w/ eigenvalue $\mathcal{E}$: $\mathcal{D}$-module $\mathcal{F}$ on Bun_G

$$H_{{}^LR,p}(\mathcal{F}) \simeq \mathcal{E}_p({}^LR)\otimes\mathcal{F} \rightsquigarrow H_{{}^LR}(\mathcal{F}) = \mathcal{E}({}^LR)\otimes\mathcal{F}$$

Hecke functor $H_{{}^LR,p}$ increases deg by 1, poles @ p associated w/ $\left\{\begin{array}{l} {}^LR \\ U(1)\to G \end{array}\right.$

e.g. $G = U(1)$ $\quad H_{1,p}: \mathcal{L}\to\mathcal{L}(x)$

'T hooft operator!!

magnetic eigenbrane $\longleftrightarrow$ hecke eigensheaf

but, 't hooft acts on A-branes...

Q: A-Brane $\longrightarrow$ D module ?

Canonical coisotropic brane

$\mathrm{Hom}(\mathcal{B},\tilde{\mathcal{B}})$ is a $\mathrm{Hom}(\tilde{\mathcal{B}},\tilde{\mathcal{B}})$-module

i.e, $\mathrm{Hom}(\mathcal{B},\tilde{\mathcal{B}}) \circ \mathrm{Hom}(\tilde{\mathcal{B}},\tilde{\mathcal{B}}) \in \mathrm{Hom}(\mathcal{B},\tilde{\mathcal{B}})$

$\mathcal{B}_{c.c}$: canonical coisotropic brane on X:

- support $Y = X$
- $U(1)$ bundle w/ curvature F satisfying $(\omega^{-1}F)^2 = -1$

$\Rightarrow$ $\omega - iF$ holo. symplectic form

consider $\mathcal{M}^H$ w/ symplectic form ω_K

$\omega_K - i\omega_J = \overline{\Omega_I} = \int_C \delta A \wedge \delta\phi$ for $(A,\phi) \in T^*\mathcal{A}$ (think $dq \wedge dp$!)

take quotient & restrict to $T^*\mathrm{Bun}_G^{\mathrm{stable}}$

What is $\mathrm{Hom}(\mathcal{B}_{c.c}, \mathcal{B}_{c.c})$?

$\mathrm{Hom}(\mathcal{B}_{c.c}, \mathcal{B}_{c.c})$

fields

$f : \Sigma \to T^*\mathrm{Bun}_G^{\text{stable}} \subset \mathcal{M}^{\text{Higgs}}$

A-model Action: $S = \frac{1}{\hbar}\int_\Sigma f^*(\omega - iF) + \{Q, \bullet\} \simeq \frac{1}{\hbar}\int_{f_*\Sigma} dq \wedge dp$

$S \simeq \frac{1}{\hbar}\int_{f^*\partial\Sigma} q\,dp$ classical action of particle on $T^*\mathrm{Bun}_G^s$!!

$dq \wedge dp = d(q\,dp)$ louiville 1-form

Path integral of $S \rightsquigarrow$ deformation quantization of $T^*\mathrm{Bun}_G^s$

↳ noncommunative deformation of sheaf of algebras $\mathcal{O}_X$

↳ differental operators $q \longmapsto q$ holomorphic fns

$p \longmapsto i\hbar \frac{\partial}{\partial q}$ 1st order differentials

nontrivial cocycle on Quantized sheaf

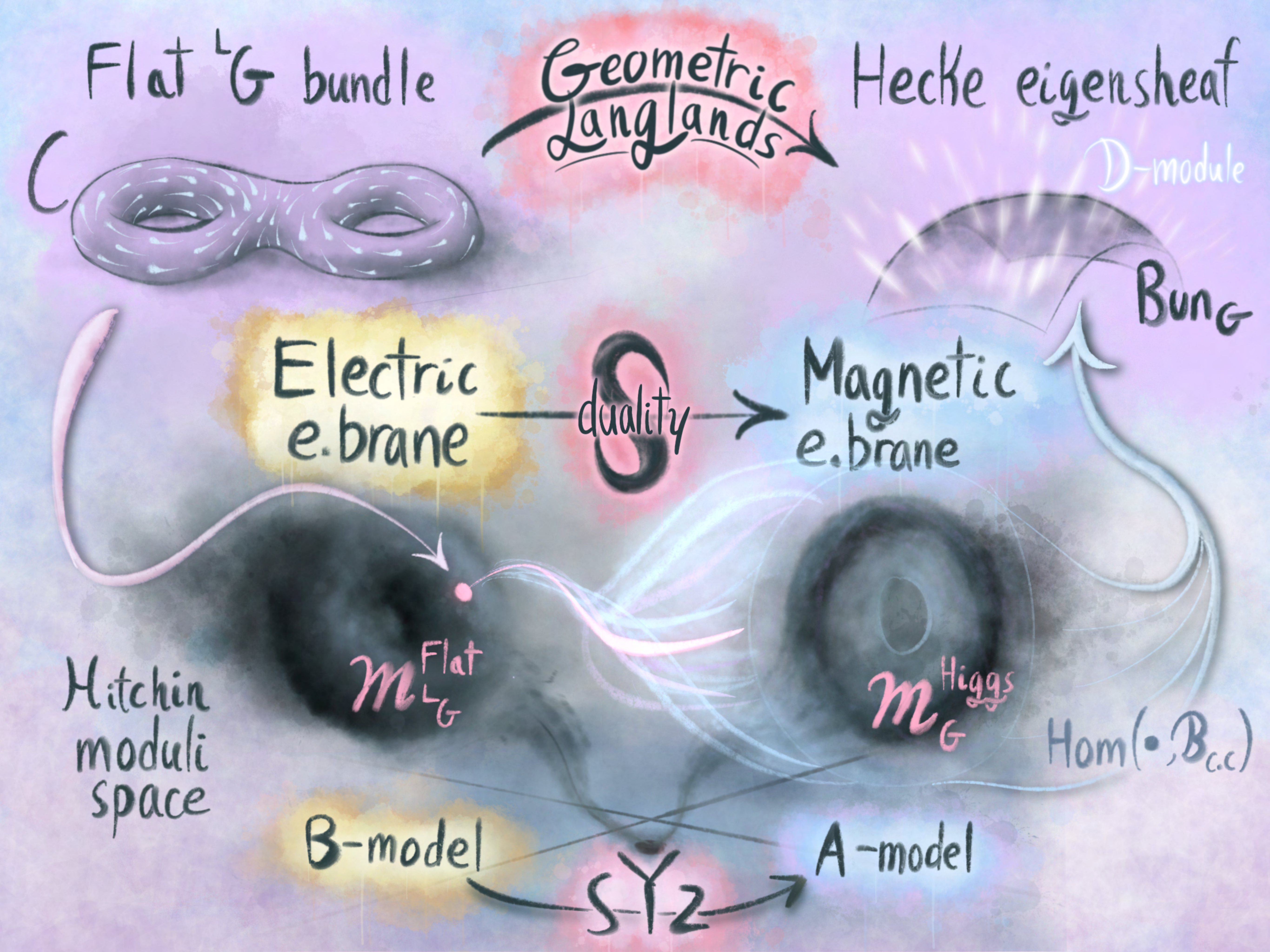
$\mathrm{Hom}(\mathcal{B}_{c.c}, \mathcal{B}_{c.c}) \xrightarrow{\text{sheafify}}$ sheaf of differential operators on $K_X^{1/2}$

$\Longrightarrow \mathrm{Hom}(\mathcal{B}, \mathcal{B}_{c.c})$ sheaf of $\mathcal{D}$-modules!!

$\mathcal{B}$
electric eigenbrane
w/ e.val $\mathcal{E}$

$\Longleftrightarrow$

$\mathrm{Hom}(\mathcal{B}, \mathcal{B}_{c.c})$
Hecke eigensheaf
w/ e.val $\mathcal{E}$


Flat ^LG bundle
Geometric Langlands
Hecke eigensheaf
C
D-module
Bun_G
Electric e.brane
duality
Magnetic e.brane
$\mathcal{M}^{Flat}_{^LG}$
$\mathcal{M}^{Higgs}_{G}$
Hitchin moduli space
$Hom(\bullet, B_{c.c})$
B-model
SYZ
A-model

References

- Kapustin, "Langlands Duality and Topological Field Theory"
- Frenkel, "Gauge Theory and Langlands Duality"
- Witten, "Quantization by Branes and Geometric Langlands"
  - Slides with some useful perspective
- Clay institute, "Dirichlet Branes and Mirror Symmetry"
  - De-facto tome on mirror symmetry by branes. Long, but individual parts are readable, and has everything you need

References

- Kapustin, Witten, "Electric-Magnetic Duality And The Geometric Langlands Program"
    - Original paper in subject. Very long, but readable if you focus on the parts you care about
- Swoboba, "Moduli Spaces of Higgs Bundles — Old and New"
    - One of many sets of notes explaining Higgs bundles

Specific references

- Hitchin, "Hyperkahler Metrics and Supersymmetry"
    - Lays out connection between N=4 and hyperkahler mflds
- Hitchin, "Hyperkähler manifolds"
    - Has Useful account of hyperkahler quotients and hitchin modulii space.