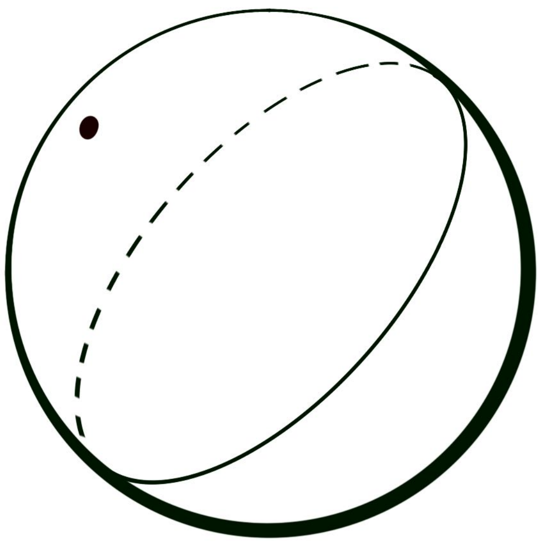
How To Draw a Sphere

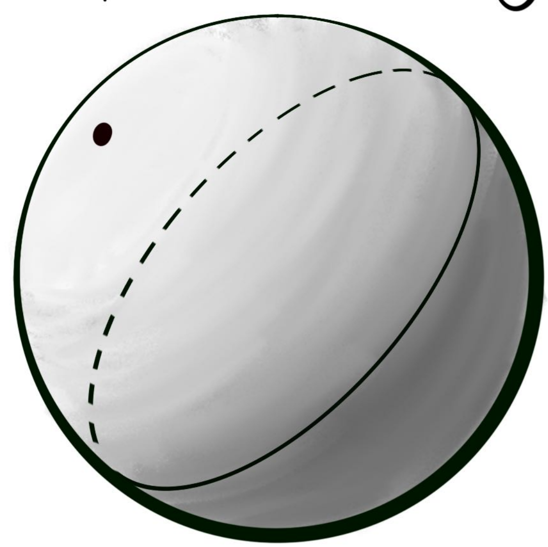
~a Synthetic, Pencil & eyeball construction~

we will break up the process into 4 easy steps

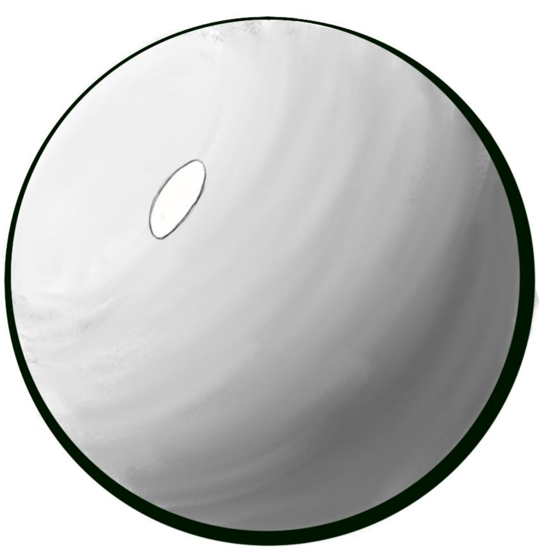
Step 1: Scaffolding



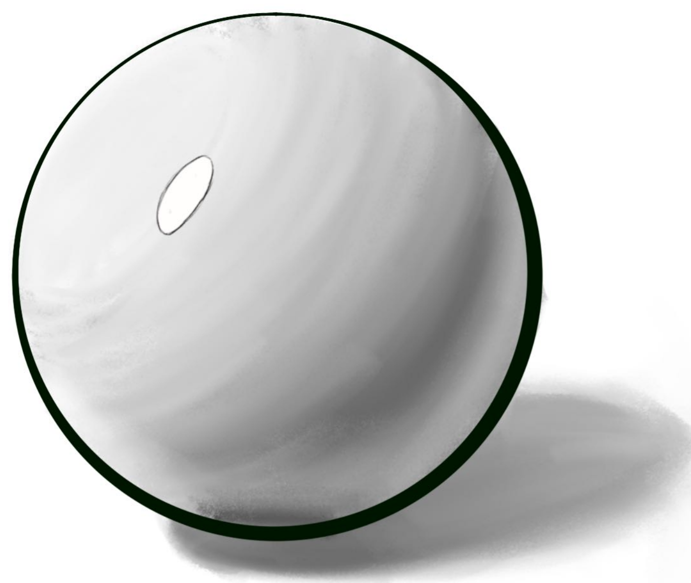
Step 2: shading



Step 3: highlights

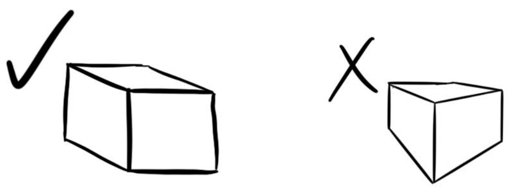


Step 4: Ambiance

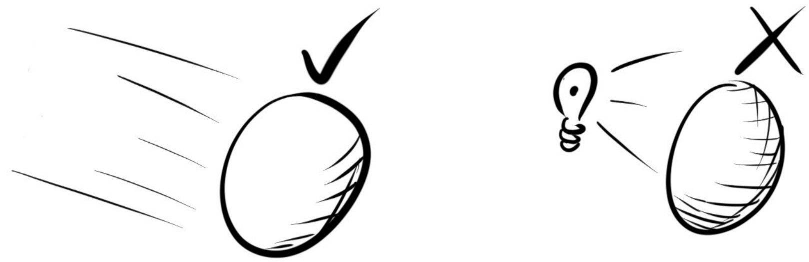


Goal: draw an unadorned sphere, sitting on a table, in orthographic projection, lit from a sunbeam.

Orthographic:
Parral lines appear parralel



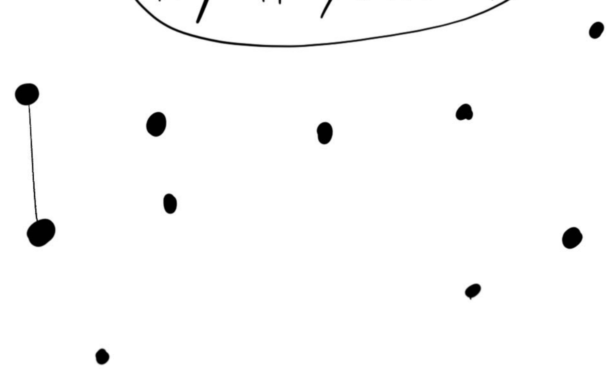
Sunbeam:
All light rays are parralel



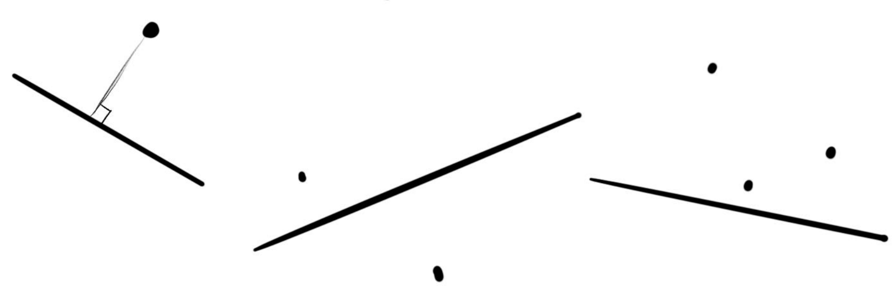
We want to draw this as accuratly as possible with no tools except a pencil and our eye. Fortunatly, the sphere with all its shadings is exactly constructable. We need to design this construction so that it may be accuratly eyeballed. This gives an interesting model of geometry.

Constructions w/ pencil & eyeball:

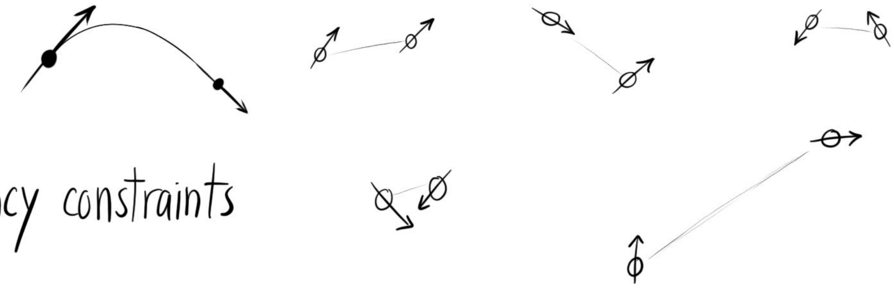
- line between 2 points.
  (can only extend a short distance)



- Perpindicular thru line

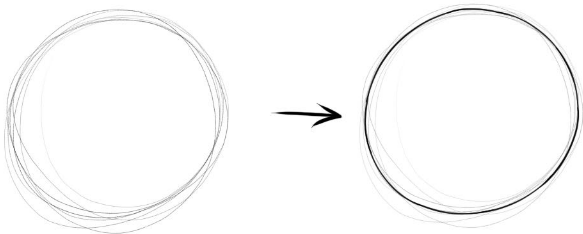


- Bezier curves



curves w/ point & tangency constraints

- Circles

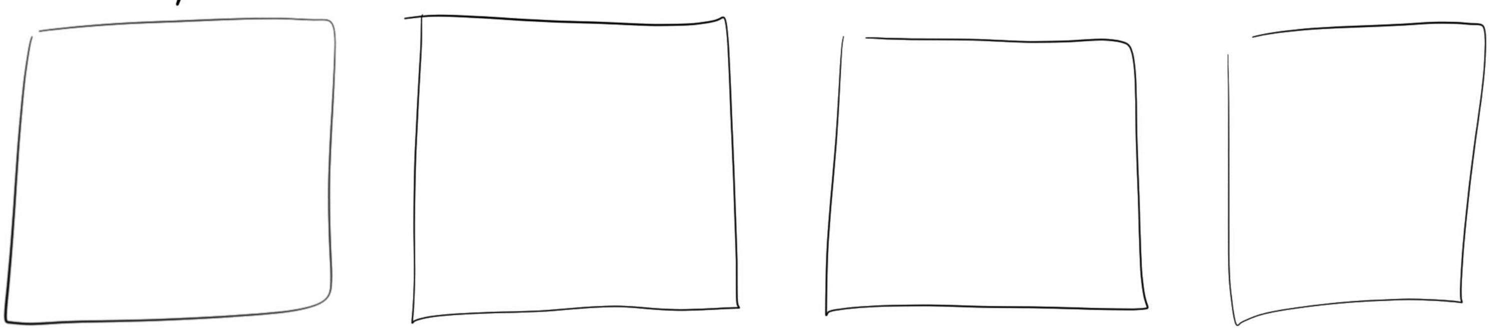


Tip: How to find lines

- Make many light attempts & see where they concentrate

- erase sketch, & make single dark line

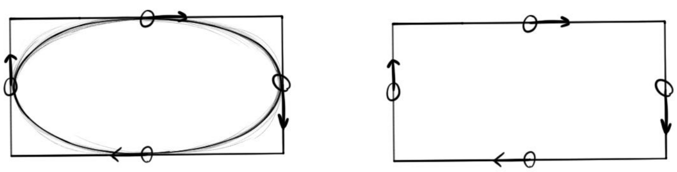
try it!

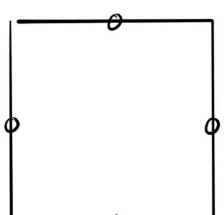


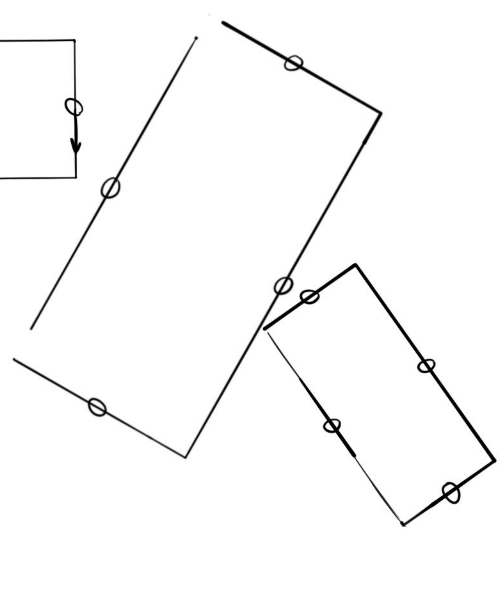
or, trace something circular.

- Ellipses:

a bunch of Bezier curves does the job

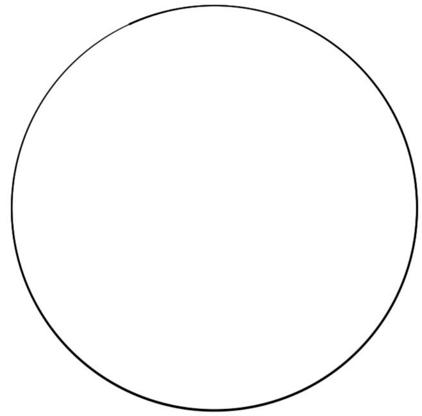






Step 1: Build the scaffolding

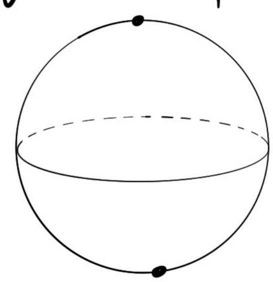
Try to draw a simple diagram of a sphere, showing the equator & north pole:



Some common pitfalls:

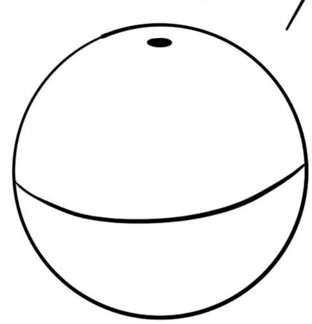
Seeing both poles at once

X



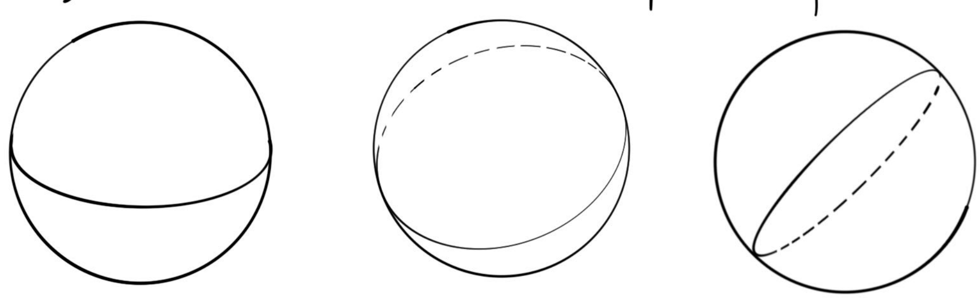
Equator not tangent to boundary

X

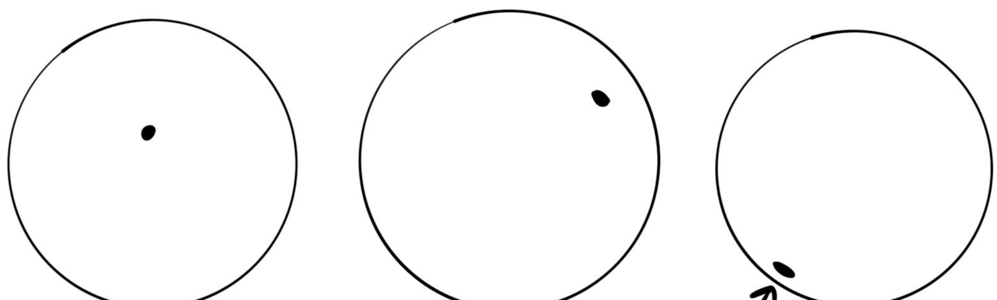


The trickiest part is matching the perspective. The curvature of the equator determines the rotation of the sphere, which determines the pole's position.

Place the poles w/ these equators:

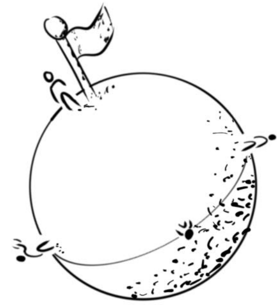


Place the equators w/ these poles:

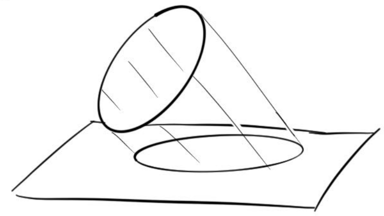


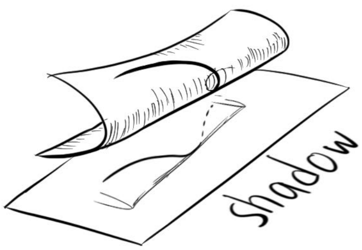
Tip: Draw pole as flat disc to show angle

A theory of equators:



- An equator forms a circle in 3D space, so its projection to the page is an _ellipse_

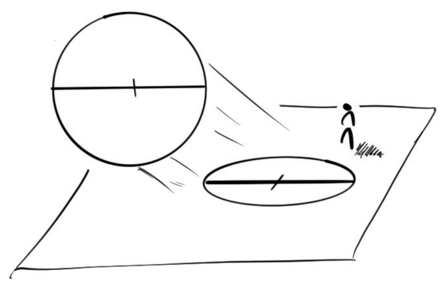


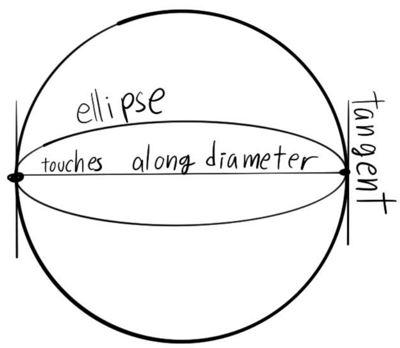


- Any differentible curve on a surface must project tangent to the folds of the surface

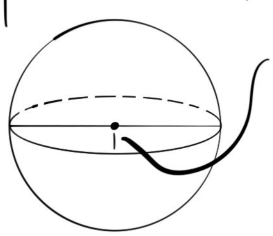
in particular, the equatorial ellipse must be tangent to the outline circle of the sphere

- The major axis of the projection of a circle equals the diameter of the circle. in particular, the equatorial ellipse has major axis equal to the diameter of the outline circle





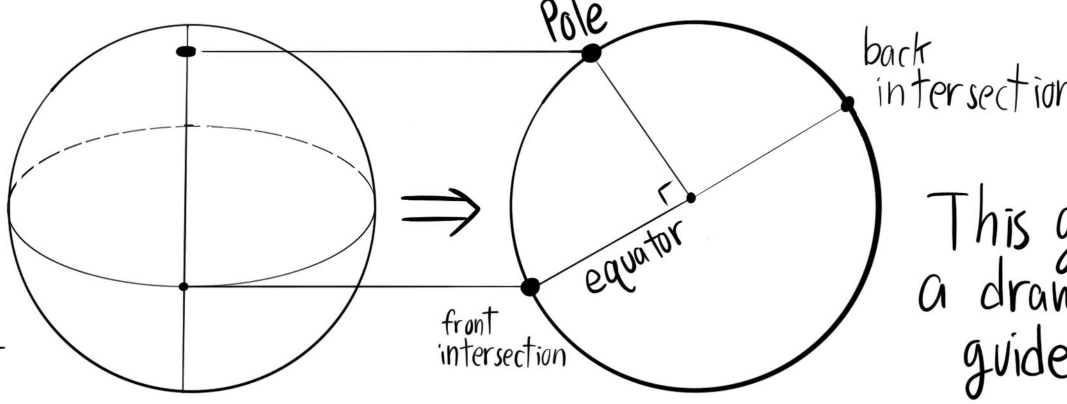
All together, the equator must be drawn as up to rotation of the drawing.



The minor axis controls the perspective

so how does minor axis control pole position?

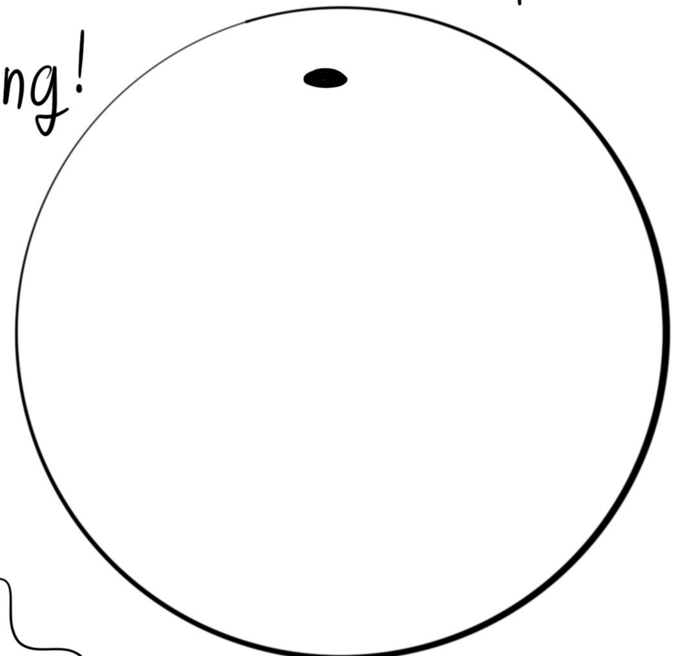
Take a vertical crossection, & see intersections with pole & equator



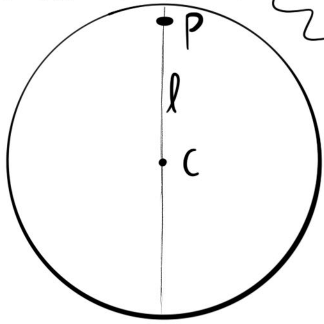
This gives a drawing guide !!

How to draw an Equator, from the pole

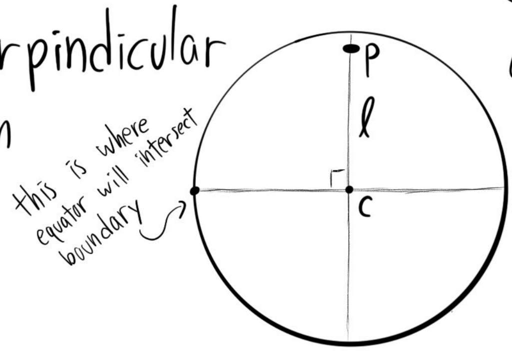
Follow along!



1: Draw line ℓ from pole to center

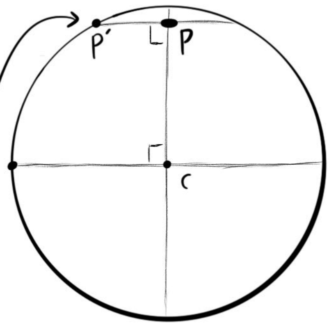


2: Draw perpindicular to ℓ, from center



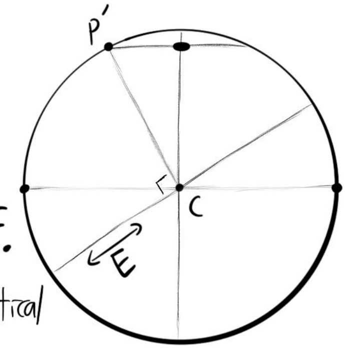
3: Draw perpindicular to ℓ from pole. intersect w/ outer circle @ P′

(we are using the outline circle as our vertical cross-section. This is the pole location)



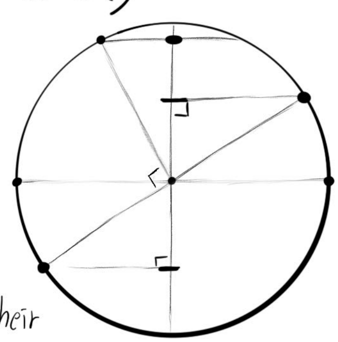
4. Draw line from P′ to center. Construct ⊥ through center. Call new line E.

(E represents equator in the vertical crossection)

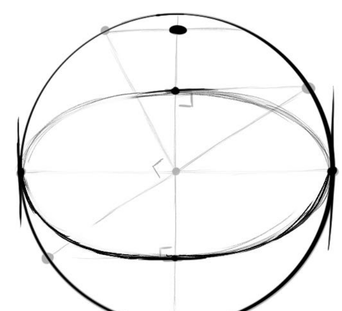


5. Intersect E with circle, then drop ⊥ to ℓ.

these lines are the vertical extent of the equatorial ellipse. We know the ellipse is tangent to these lines & (by symmetry) the tangency points are their intersection with ℓ.

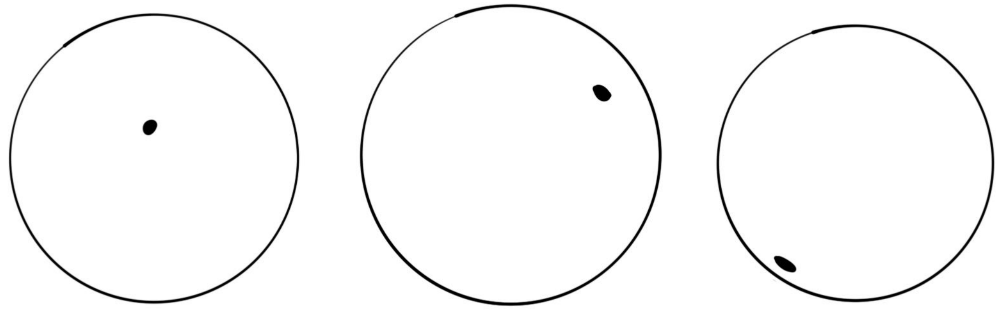


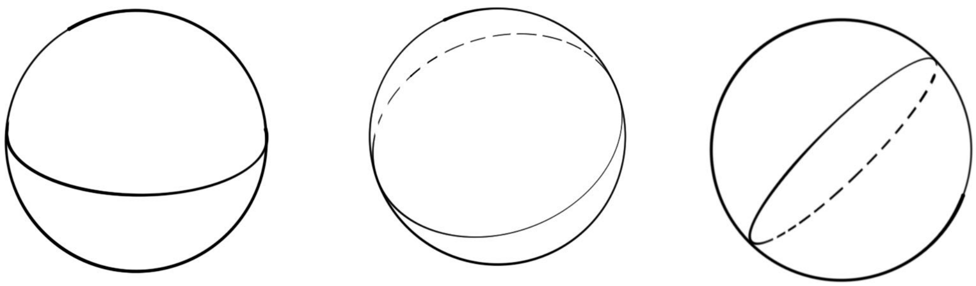
6. construct the equatorial ellipse



Practice!

Draw eqators





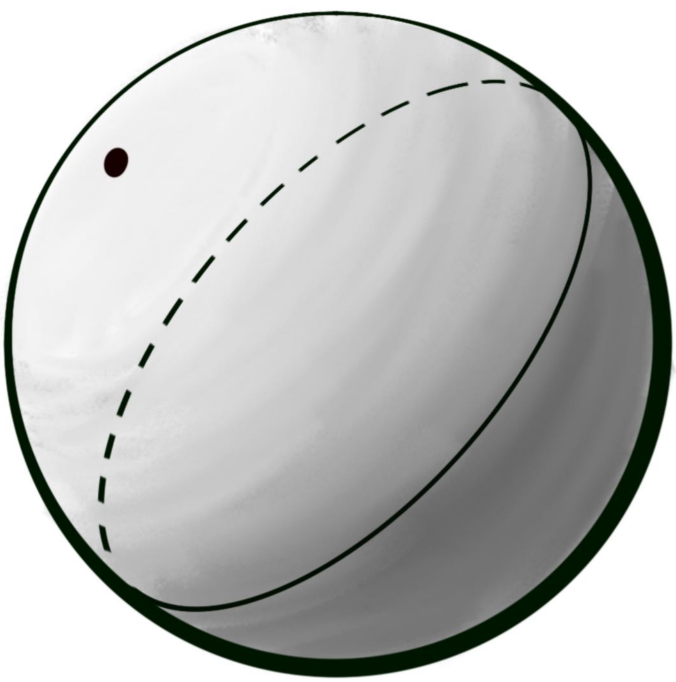
Draw poles

Shading:

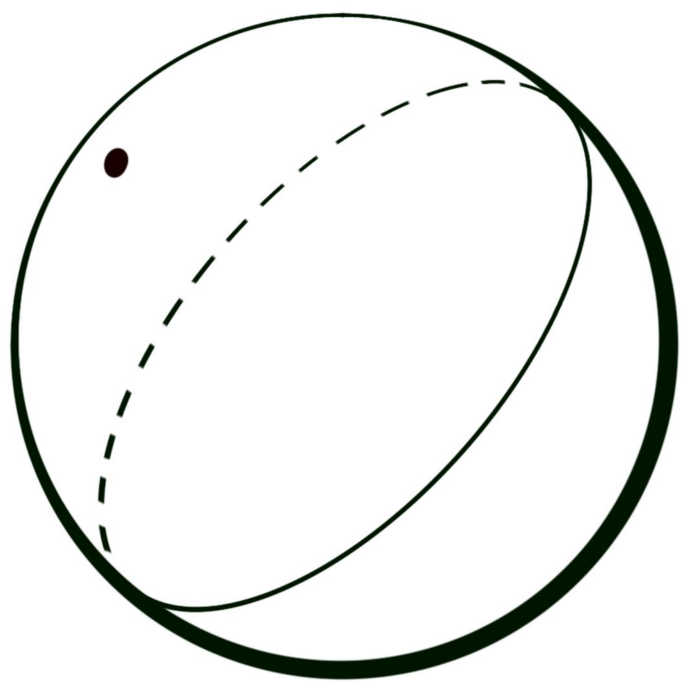
imagine light shining on a sphere, w/ north pole facing towards the light. Everything below equator is facing away, so is in shadow.

Try to shade:
- Uniform dark color below equator
- soft gradient above equator, w/ brightest pt @ pole.

Example:



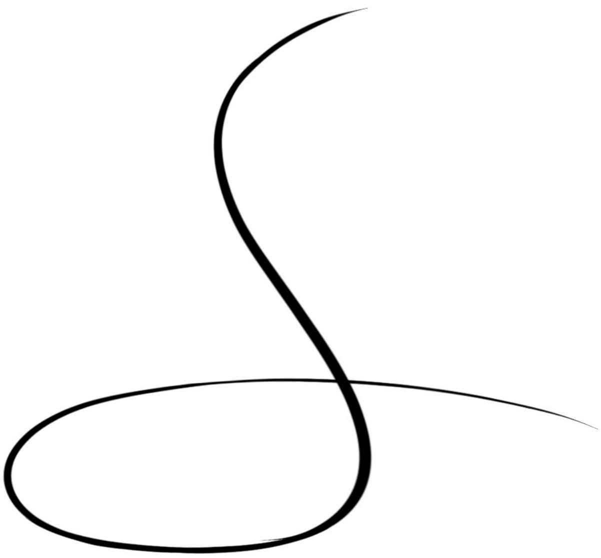
Your turn:



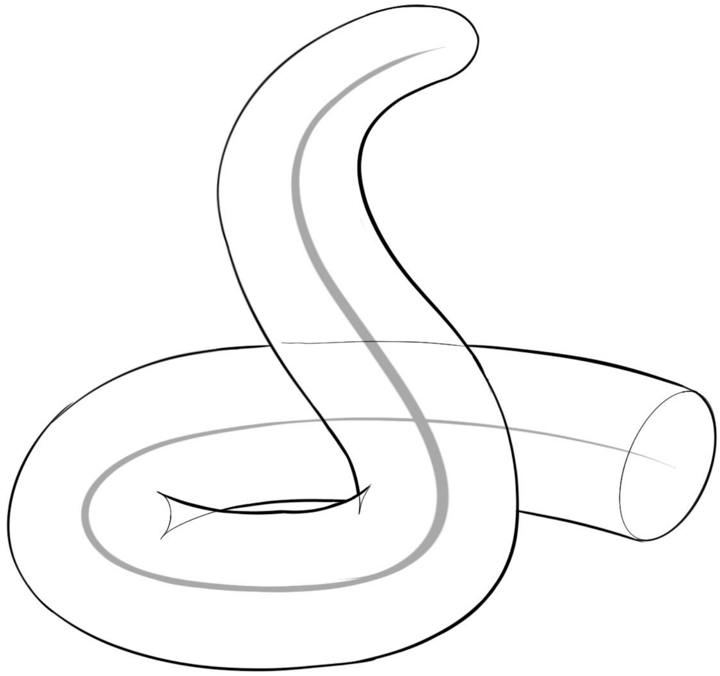
try shading your equators above! on some, put the light behind the sphere.

How to draw a Worm

Step 0: Guiding curve



Step1 : Outline



Step 2: Lattitudes

Step 3: Shading

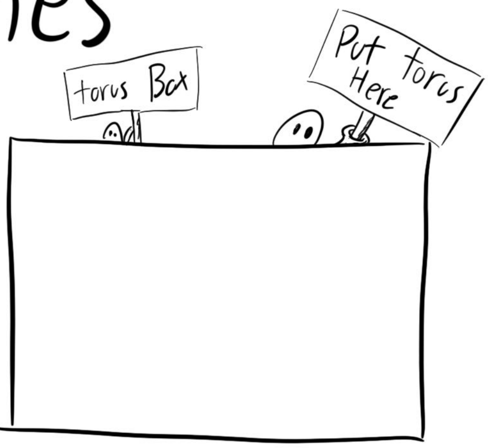
As mathematicians, we like to draw tori 

A Torus is just a circular worm

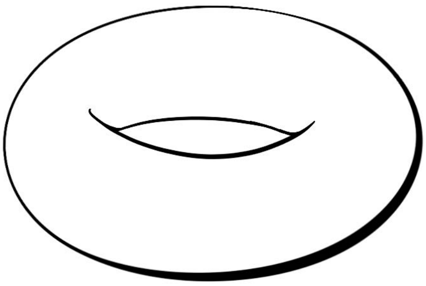
To draw a torus, we must first draw a worm.

Step 1: Outlines

Draw a picture of a torus here →

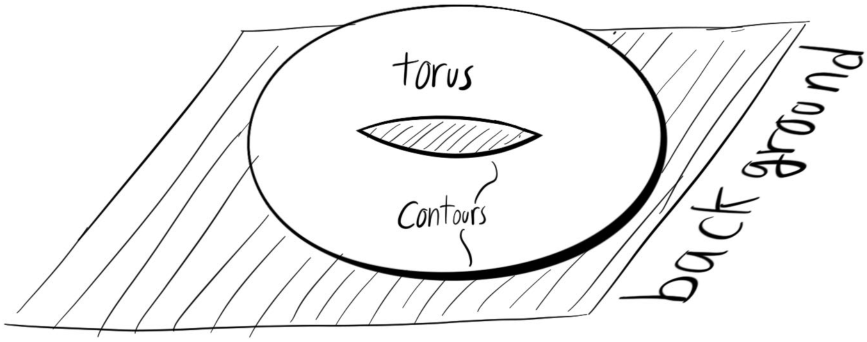


I bet you drew something like this:

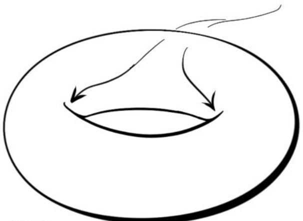


What do these penstrokes represent?

- Some of them are the boundary between the torus & the background

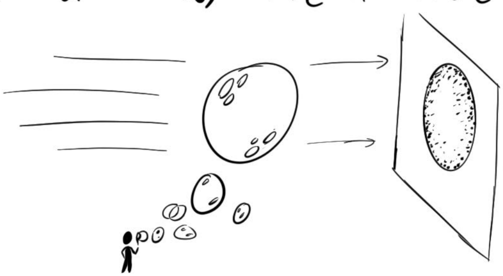


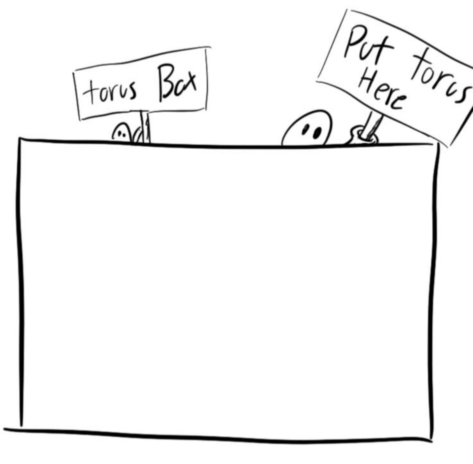
But what are these lines?



Try to draw a "Ghost torus"

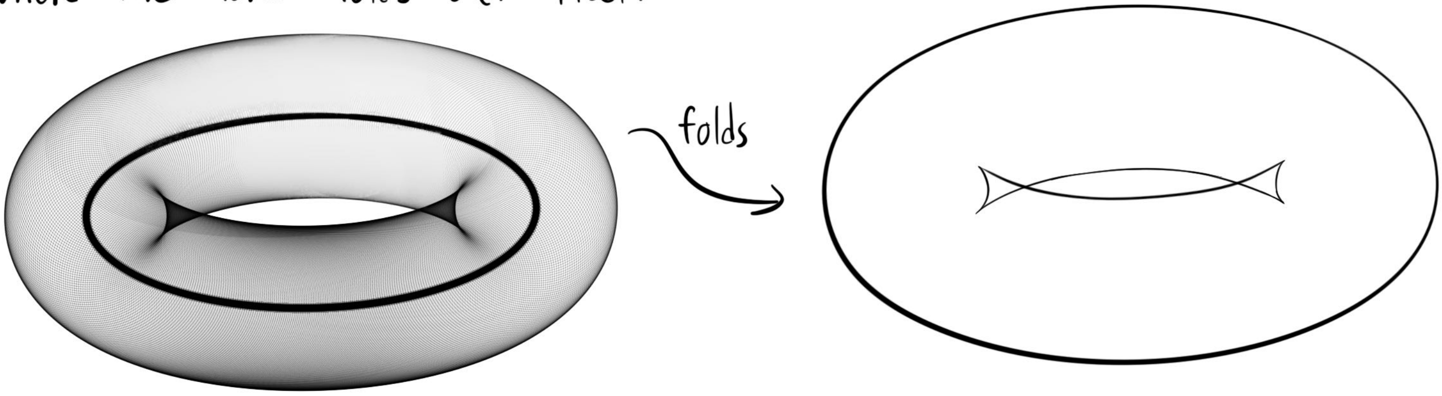
The shadow of a translucent shell of a torus, like a bubble in sunlight

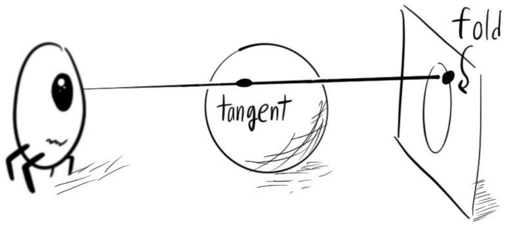




Here's a "perfect" ghost torus. It is darkest along the edges, where the torus folds over itself.

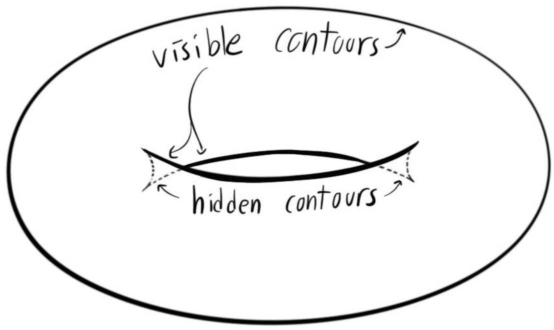
The folds include all the contours from the line diagram, & then some!



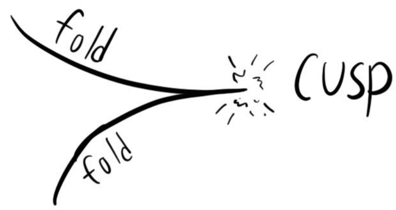


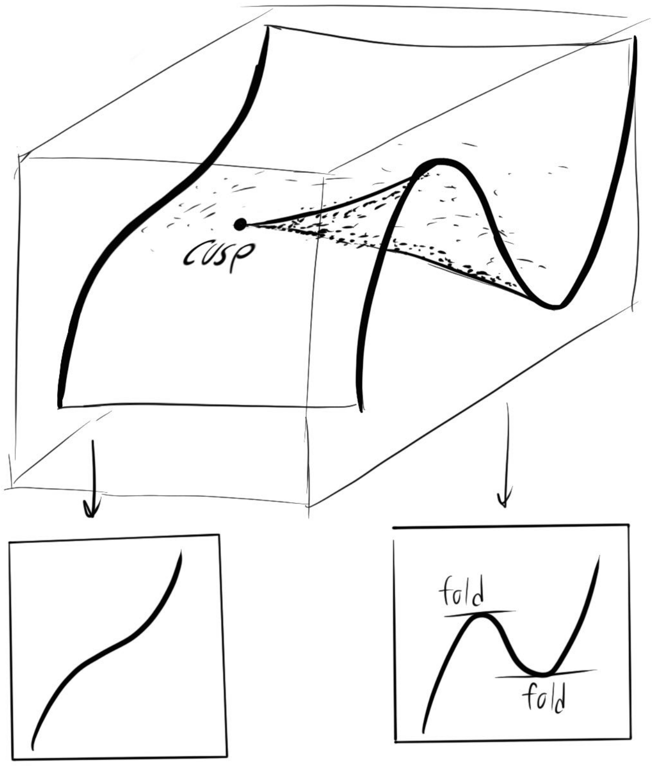
The folds are points where the line from our eye to the picture plane is tangent to the surface

For an opaque surface, folds may be hidden behind the surface. The "smile" shows the visible contours of the torus.



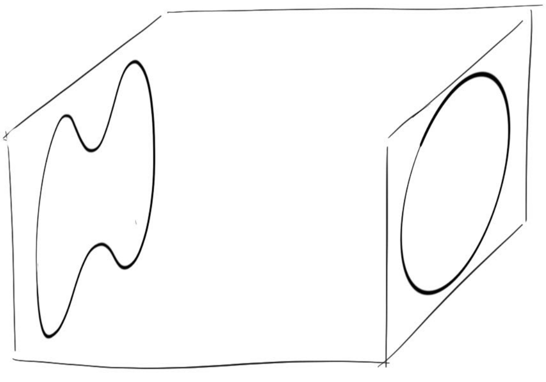
Two folds meet at a "Cusp"





Cusps appear generically when projecting surfaces in $\mathbb{R}^3$ onto $\mathbb{R}^2$.

Try it yourself!



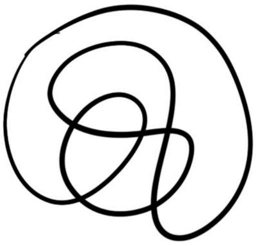
interpolate between these two curves. mark the folds & cusps you produce.

How to Draw Knots Bounding surfaces

Draw A knot!

step 1: Loop

Draw loop w/o picking up pencil (start & end at same point)

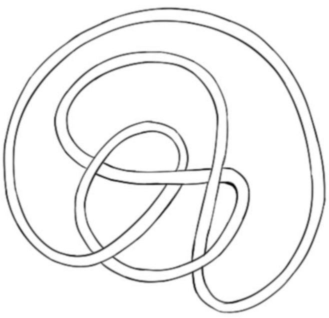


step 2: Crossings

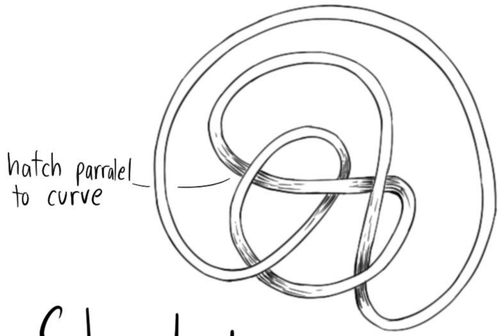
choose over/under strands at each crossing

{Fun Fact: you can always weave a curve, altrenating over/under crossings while drawing the knot.}

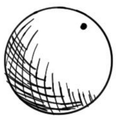
Step 3: thicken

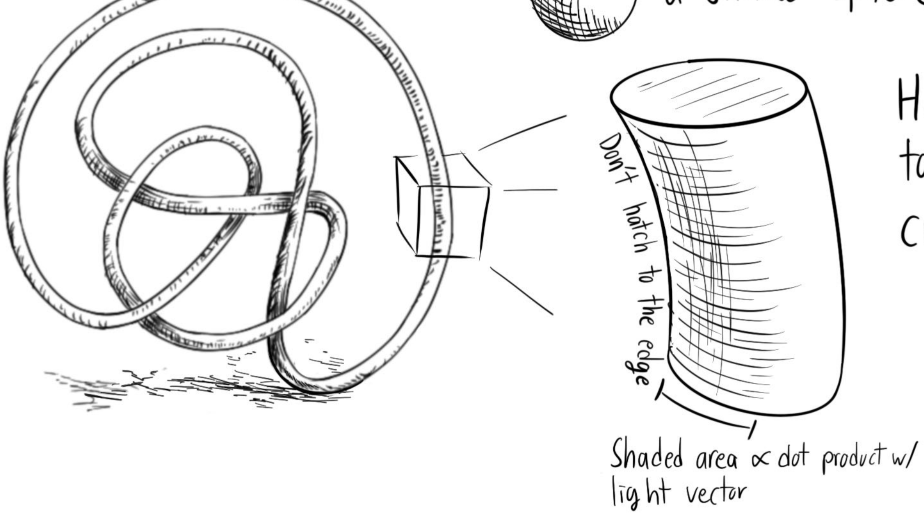


step 4: shade crossings

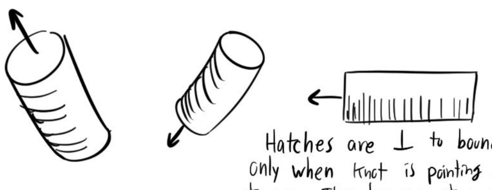


Step 5: Shade!

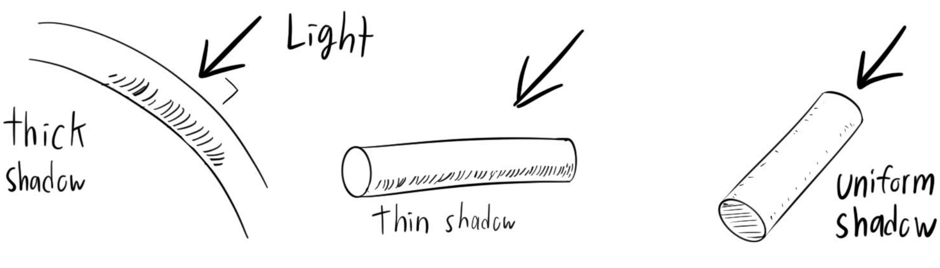




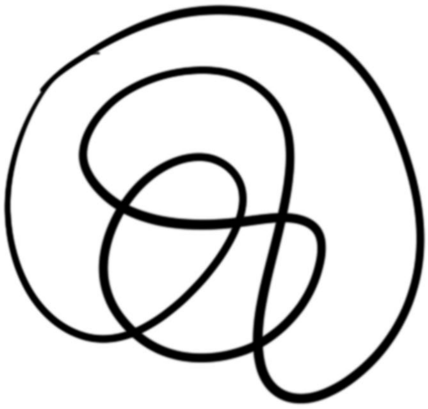
Hatch along ellipses tangent to boundary. Ellipses are orthogonal crossections to the knot tube



Hatches are ⊥ to boundary only when knot is pointing ⊥ to you. This happens at maximal/minimal distance of knot from plane.



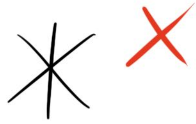
A soapy surface:



1: Draw a loop:

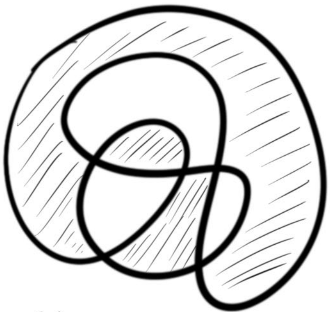
no tangencies

no triple points



start & end @ same point

2. shade every other region

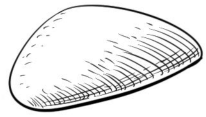


Always possible!

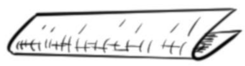
3. choose crossings

The concavity of a fold is determined by the gaussian curvature

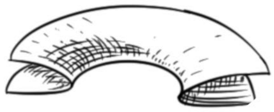
convex fold ⟺ Positive curvature



straight fold ⟺ zero curvature



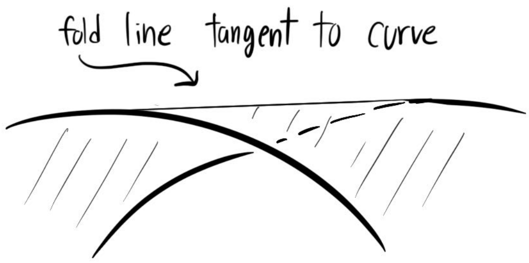
concave fold ⟺ negative curvature



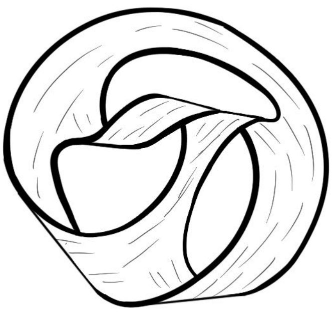
4. Draw crossing as Twist

Choose your material!!

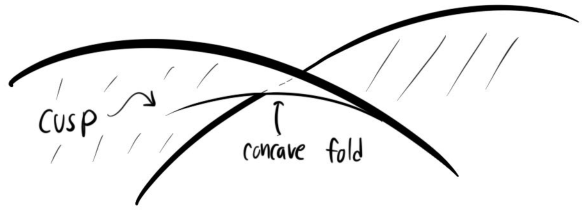
Paper: zero gaussian curvature



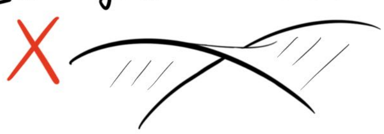
⚠ may need to rearrange knot to allow tangent lines



Soap: negative gaussian curvature

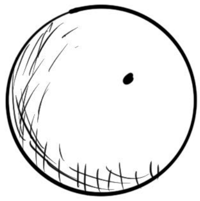


⚠ fold goes on "inside"

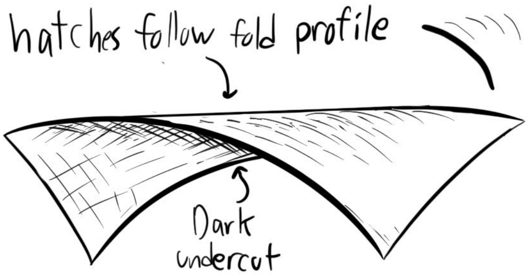


⚠ include the cusp!

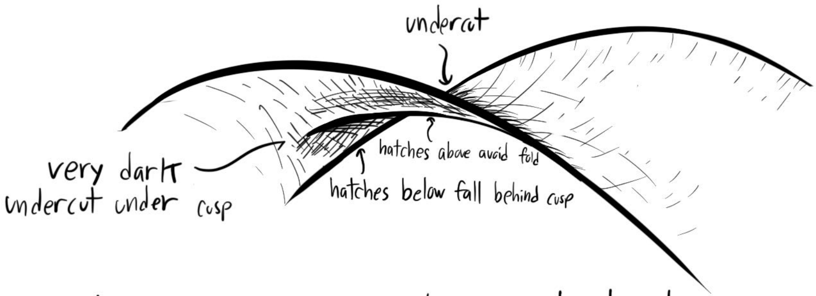
5. Shade twists



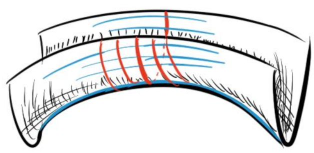
Paper:



Soap:

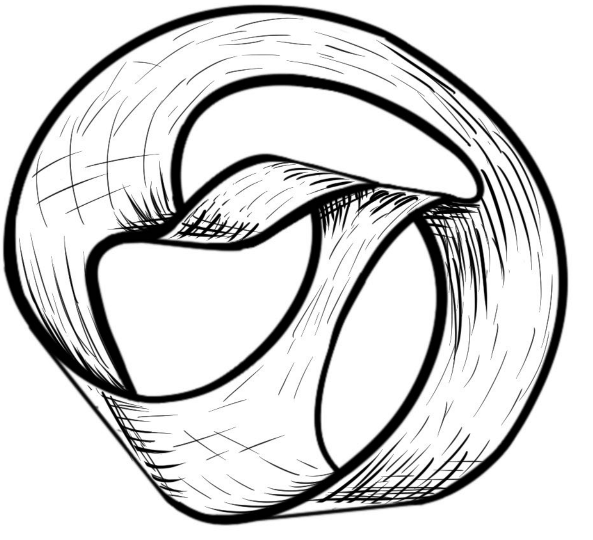


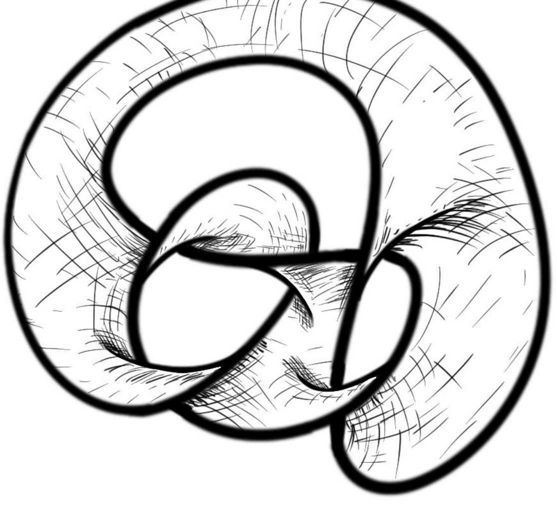
- Darkness indicates Depth. if one sheet on front of another, shade back sheet near overlap.
  ↳ shade the area behind a cusp espically dark. (cusps are important)
- Crosshatching establishes a coordinate grid on surface. conform this to the geometry. I crosshatch along lines of principal curvature
  ↳ hatches are tangent to folds, but pass transversally behind cusps.



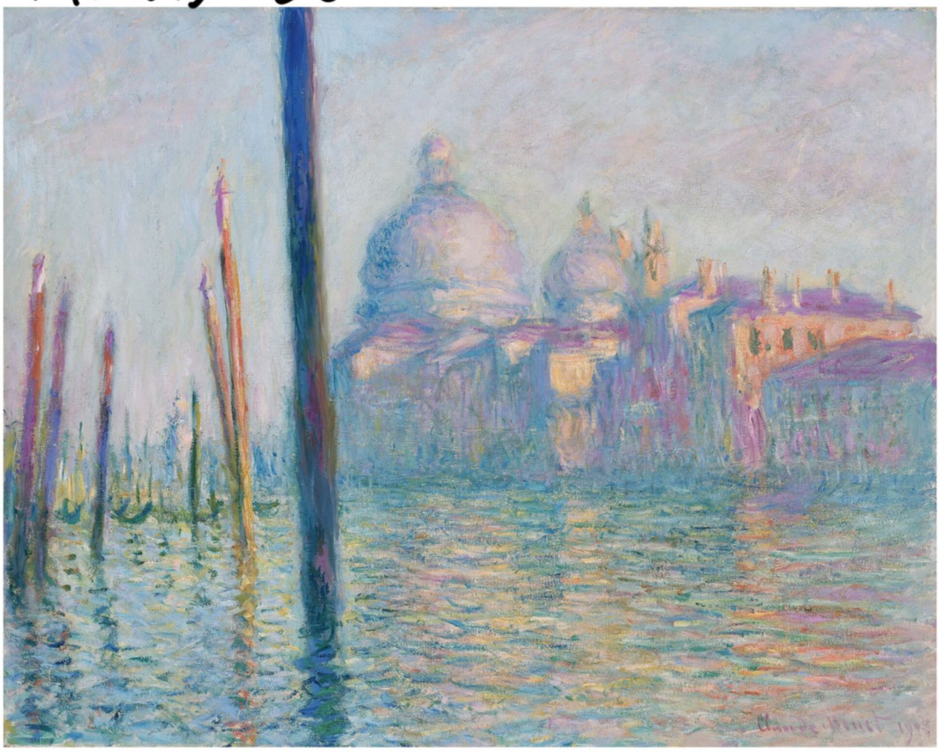
Along a fold, lines of principal curvature are Parallel to The fold (Blue) and its crossection (Red)

Shade each crossing and interpolate to shade your surface!





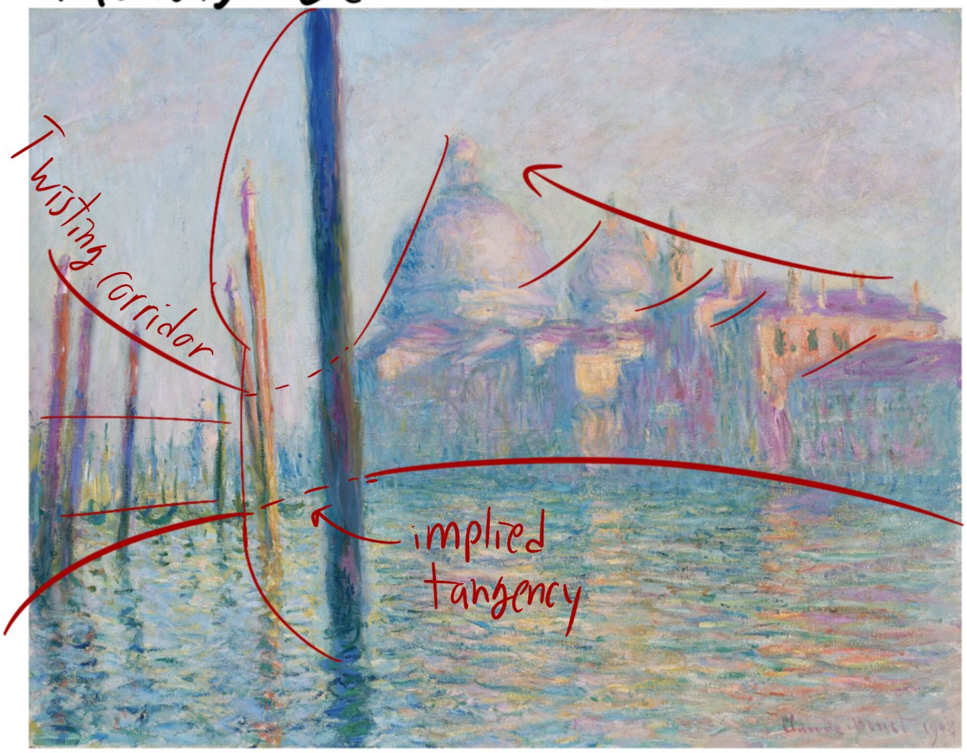
Monet, Le Grand Canal



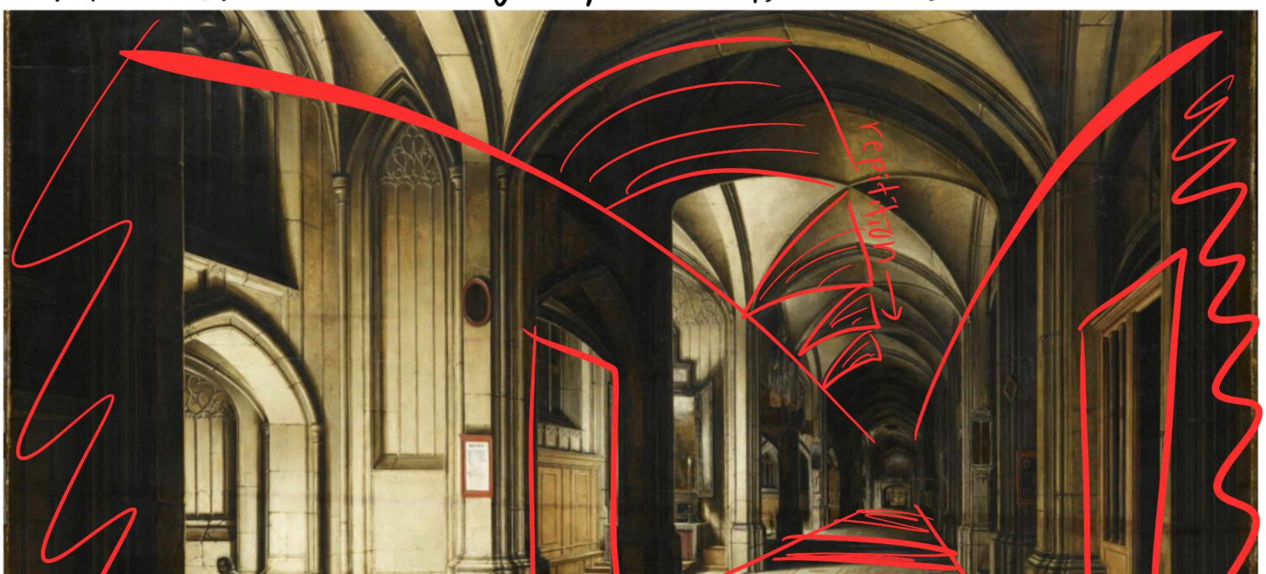
interior of an imaginary church, @ night. Steenwyck II

Rough forms

Monet, Le Grand Canal



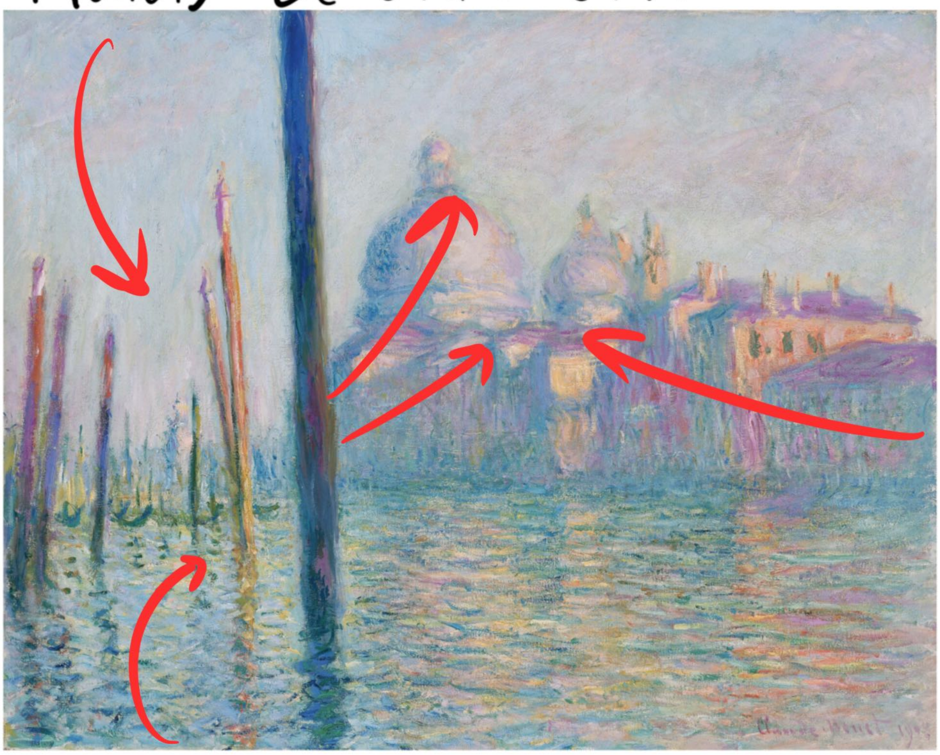
interior of an imaginary church, @ night. Steenwyck II



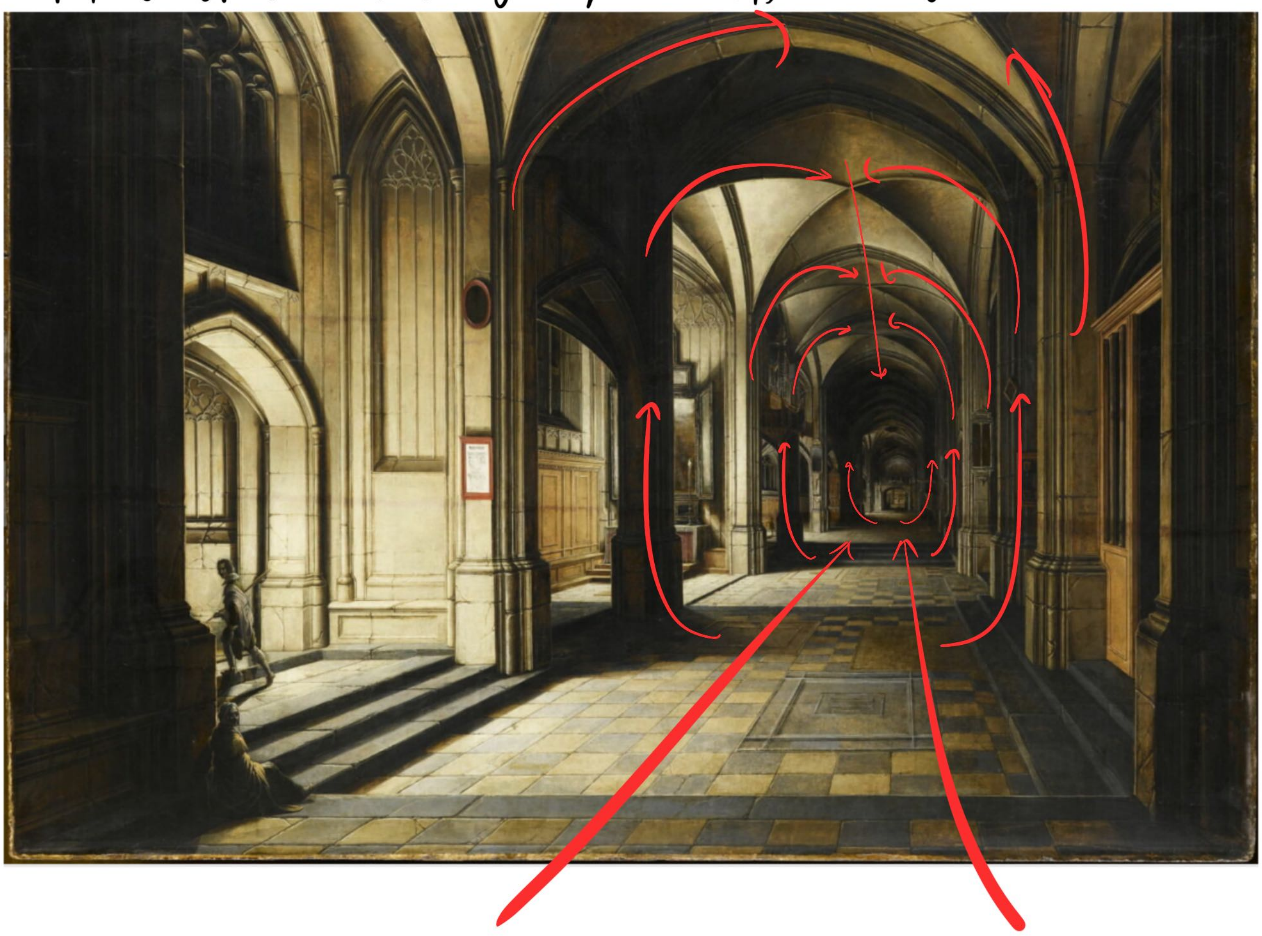
Dark frame

Sightlines (where eyes move)

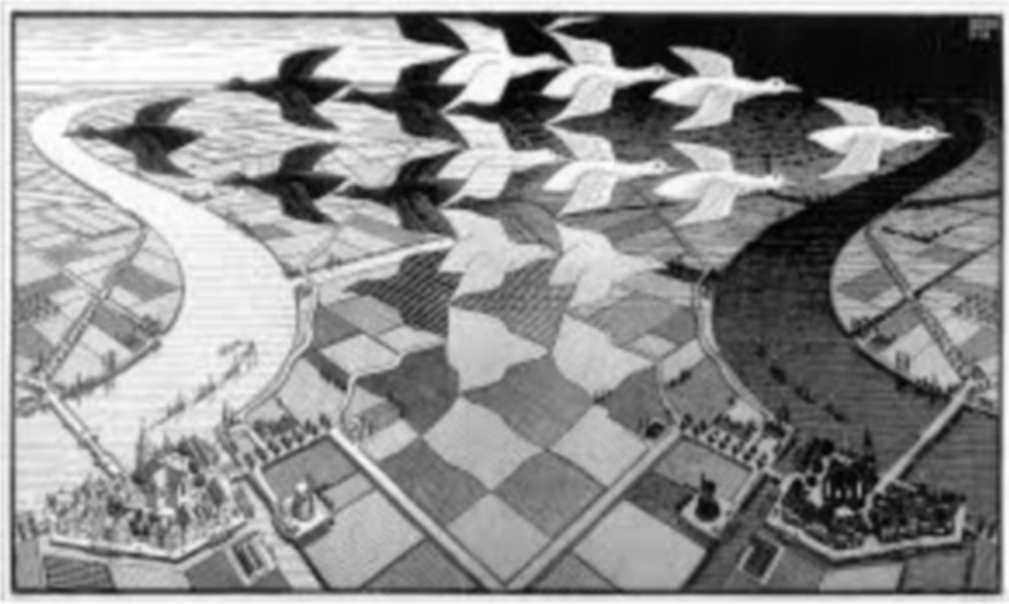
Monet, Le Grand Canal



interior of an imaginary church, @ night. Steenwyck II

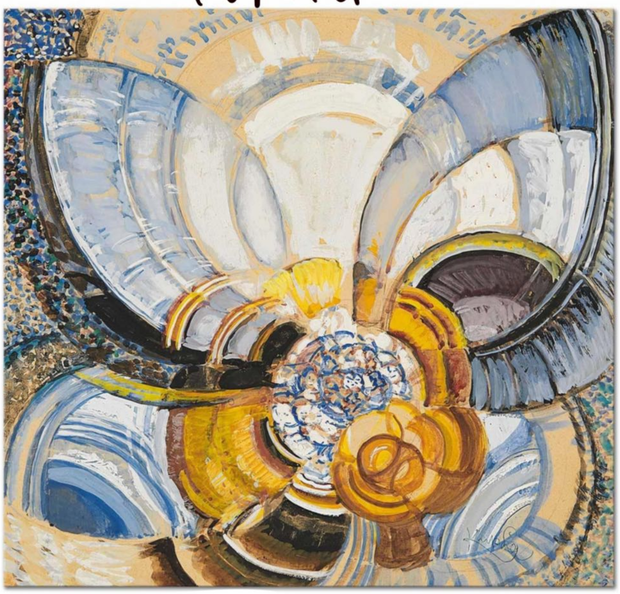


Try it yourself:



Escher

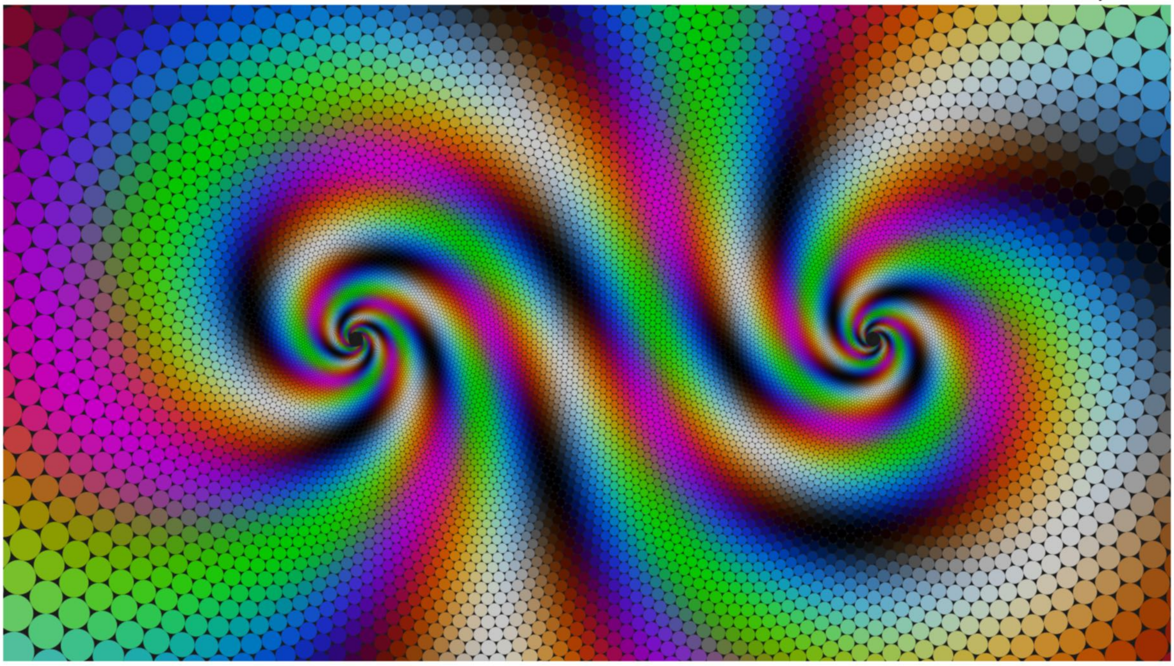
Kupka



Kupka

Kadinsky

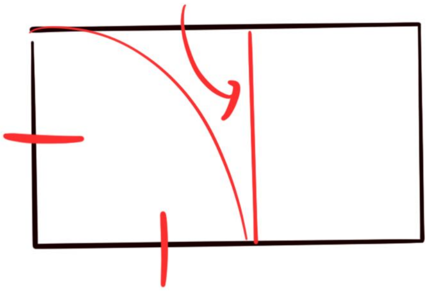
Loxodromic spiral (wikipedia commons)



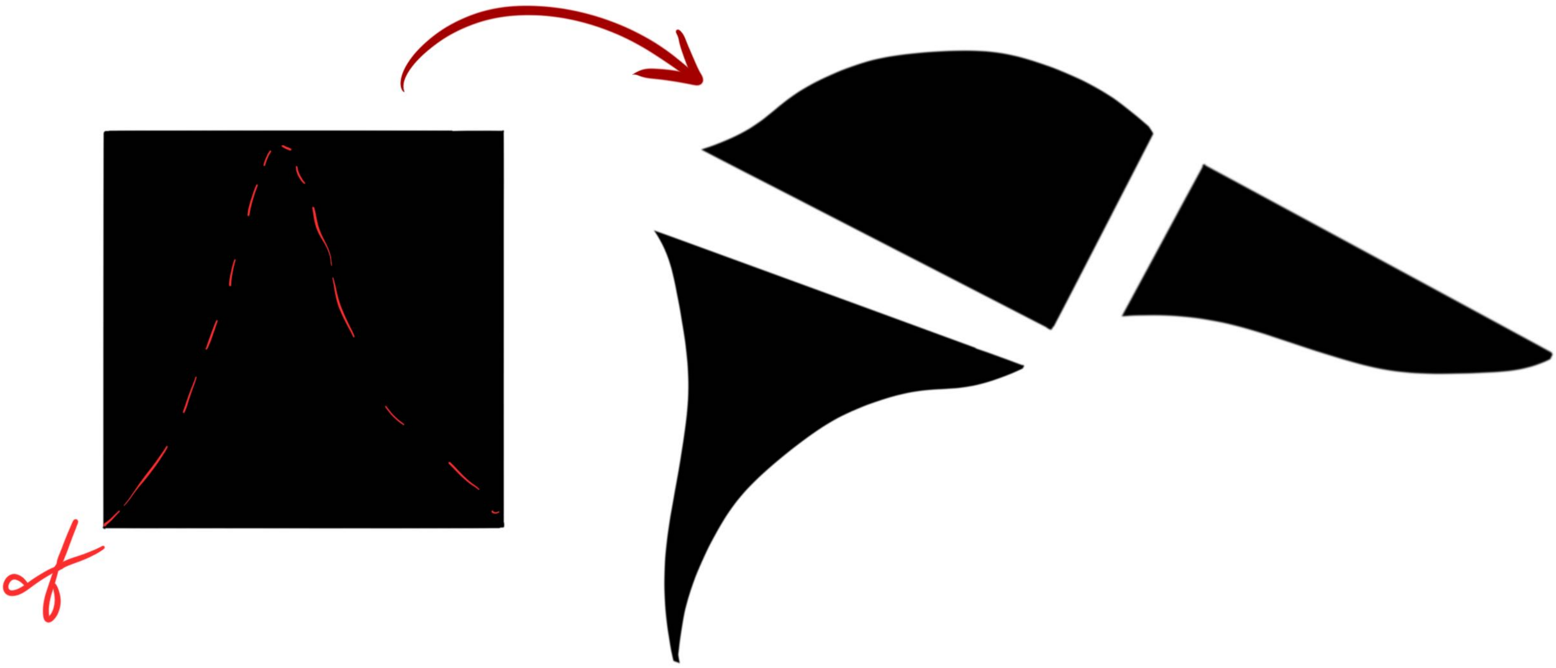
Algebraic starscape (steven brooks)

Artist in his studio (rembrandt)

rabatment (of rectangle)



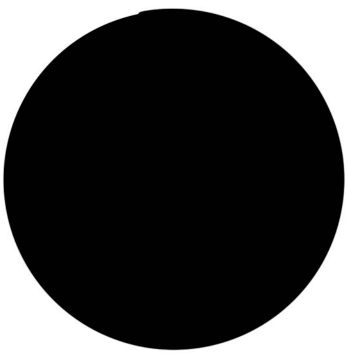
Exercise: Square disections



cut up a square any way you like. Rearrange the piece (without overlap) to make compositions which feel:

- Rigid
- disordered
- free flowing
- has a strong focal point
- has 2 focal points
- has one main piece, with 2 smaller pieces
- Draw a curve on a paper, and make the square match the curve

Now try w/ a circle

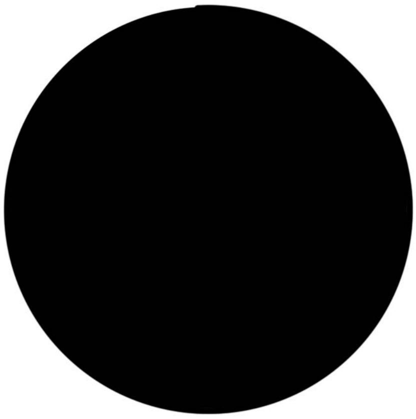


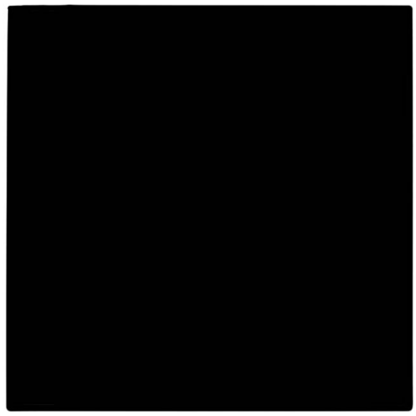
which are easier?

which are harder?

try this one!

Cut up and rearrange a square & a circle to swap their aspects.





Make a composition using your pieces which has the following eye paths: